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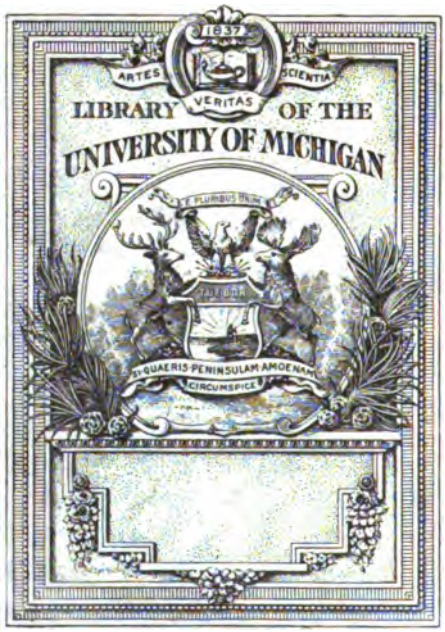

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To the READER.

AT *the Requeſt of my Friend I undertook the Reviſal and Improvement of this Book, the doing whereof has coſt me a great deal of Time and Pains, but I hope it will redound to thy great Benefit and Satisfaction.*

I have nicely Examined the ſeveral Problems *and* Propoſitions *throughout the whole Work, and have made ſuch Corrections, Alterations and Improvements as I thought neceſſary, and do hope and think that this is now render'd the moſt uſeful Book that was ever yet Publiſhed on the Subject, in ſo Plain, Compendious and Practical a manner, but that I muſt ſubmit to the Judicious and Candid* Reader.

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his hearty Well-wiſher
and Servant,

W. Alingham.

Chanel Row, Nov. 1ſt. 1706.

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The PREFACE.

HOW admirably profitable the Study of the Mathematicks has been to theſe Britiſh Iſlands, and to all other Parts of the Univerſe in which any kind of good Learning has been eſteem'd and practis'd, is well known to all wiſe and judicious Men. And indeed it is an undeniable Truth, that among all humane Arts and Sciences whatſoever, the Noble Science Mathematical has obtain'd the greateſt evidence of certainty, as being the Queen of Truth that impoſeth nothing on her Subjects but what ſhe proves by moſt infallible Demonſtrations.

Now this Prerogative reſults from the Verity and Perſpicuity of its Principles, which conſiſt of Definitions, Poſtulats and Axioms. Hence comes it to paſs that all Propoſitions that are prov'd by thoſe moſt infallible Precepts are call'd certain demonſtrative Truths; for which cauſe it has been the endeavour of ſundry Philoſophers, to make the force of their Arguments (as far as the quality of their diſcourſe would admit) amount unto Mathematical Demonſtrations, as being the moſt convincing Proof of a Propoſition that by humane reaſoning can be given.

Now having for divers Years (amongſt my other Studies) been converſant in the Study of the Mathematicks, and for my own private uſe compil'd this Treatiſe, never in the leaſt intending it ſhould have appear'd in publick in this nice and Critical Age; but it by chance falling into the Hands of ſome of my Mathematical Friends and Acquaintance, I have at their Requeſt condeſcended to Publiſh it, tho' not without a great Averſion in my own mind to expoſe my ſelf in any publick thing.

But this Difficulty being overcome, I ſhall give the Impartial Reader to underſtand, that I have faithfully compiled this Treatiſe from the beſt of Authors (and my own Experience) that I have contracted their various Works into this little Cabinet or choice Compendium of the Mathematicks, in which he will find the whole Subject clearly and intelligibly handled: I have us'd a plain and eaſie Method: I have labour'd to be as plain and perſpicuous as poſſible: I have apply'd ſuch Examples to each as may beſt demonſtrate

their

their Operation, be moſt eaſie for memory, and applicable to practice; here is indeed Multum in parvo, *the whole* Marrow *of the* Mathematicks *in this* Tract *afforded thee, which is as a true and* Golden Key *to unlock the choiceſt Myſteries in thoſe Arts contain'd.*

Thus, Reader, *I have laid my Labours before thee, and muſt entreat thee to uſe me as thou wouldſt be done by, which is the* Spontaneous *act of every good Man.*

But if this ſhall chance to fall into the Hands of any curious conceited Perſon, who thinks himſelf wiſer than the reſt of the World, and ſo he beginneth enviouſly to carp hereat, and like a Country Cur *bark at my back ſide, to him I ſhall in modeſty only ſay,* ——Facillus eſt unicuivis noſtrum aliena curiosè obſervare: quam propria negotia rectè agere. *'Tis much eaſier for thoſe* captious Readers *to carp than to copy.*

If the Collections *of* Authors *ſhall offend any, and ſo procure the ſame cenſure with the* Jackdaw, *as our learned Poet hath long ſince caution'd againſt*——

Ne, ſi fortè ſuas repetitum venerit olim
Grex avium plumas, moveat cornicula riſum,
Furtivis nudata coloribus.

I would have ſuch contentious Readers *know, that I have robbed no Man of the Honour of his* Works, *but have given to each his due; only I have borrow'd ſome choice things of them, which is no more than what the moſt learned have always done.*

Thus, Courteous *and* Impartial Reader, *'tis only for thee that I have taken theſe Pains, and have ſubmitted to the Publication hereof, and it ſhall be to thee the future parts of my Study ſhall be ſerviceable, hoping that thou wilt find ſucceſs in all thy Studies according to thy deſire and endeavour; which are and ſhall be the hearty wiſhes of him who is*

Thine and *Urania's*

Servant,

John Taylor.

TO

To the READER.

Being desired to Peruse this *Mathematical Treasury*, accordingly, to gratifie the Request of my Friend, I did, and I must confess, with no small Satisfaction to my self to see so much Practical Matter of useful *Mathematical Arts* so neatly and compendiously digested into this Portable Volume: 'twill be useful not only to *Learners* and meer *Tyro's*, but to others also who have made some considerable Progress in these Studies. 'Tis well *Methodiz'd*, very *Concise*, yet *Plain* and *Perspicuous*, so that any Person of a pregnant Fancy, may without a Tutor (in some reasonable time) wade through the whole or any Part thereof, and such as would be more expeditious may take the Assistance of a Teacher to instruct them.

The Author is wholly a Stranger to me, but to give him his due, in my opinion he has discharg'd himself like a Master in these Arts, and an Ingenious *Mathematician*, to whom I return Thanks for this his generous Offer in presenting his *Mathematical Treasury* to the Publick, and remain

A true Lover of the Mathematical Sciences, and all such that really delight in those pleasing (but useful) Speculations,

Henry Coley.

Courteous Reader,

I Have Perused this *Treatise*, and find that the Author has in every respect discharged himself like an *Artist*; the Work throughout the whole, is very *Plain* and *Easie*, nothing being omitted that might render it Intelligible to the meanest Capacity: And indeed, I know not any Treatise of this nature extant that is more Practically handled, so that I doubt not but that it will be very serviceable to the Publick; and that thou in particular mayst find Incouragement in the Perusal thereof, is the hearty Wish of him who is

Thine and Truth's Servant,

John Hawkins, *Philomath.*

The

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A D-

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AT the Houſe late the Lord *Weymouth's*, in *Chanel-Row*, *Weſtminſter*, are Taught *Arithmetick*, *Geometry*, *Trigonometry*, *Navigation*, *Surveying*, *Meaſuring*, *Fortification*, *Throwing* of *Bombs*, with ſeveral other Parts of the *Mathematicks*; alſo *Merchants* and other Accompts.

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Some few Errata's *having eſcaped the* Preſs, *the* Reader *is deſired to Correct the ſame with his Pen as followeth.*

PAge 60 Line 14. read *Figure* 94. p. 65 l. 21 r. *Figure* 95. p. 84 l. 17 r. *Sum* for *Difference.* p. 105 l. 17 for HE r. HL p. 164 l. 11 for N.W. *by* W. r. N.W. *by* N. p. 193 in the Margin for 45 r. 54. p. 196 l. 22 for 0.0 r. 7.50. p. 202 l. 18 for 24 r. 42. p. 255 l. 5. for A t a r. A T A. p. 259 l. 7 r. from P to A. p. 260 l. 25 r. from P to C. p. 261 l. 27 r. *which make* M T *equal to* 150. p. 267 l. 22 r. *Centimate.*

Arithmetick.

CHAP. I.
Of ARITHMETICK.

SECT. I.

ARITHMETICK is an Art of numbering well: For as Magnitude, or Greatneſs is the Subject of *Geometry*, ſo is Multitude, or Number that of *Arithmetick.*

Omnia quæcunque à primæva rerum natura conſtructa ſunt, Numerorum videntur ratione formata. Hoc enim fuit principale in animo conditoris exemplar. Boetius *Arith. lib.* 1 *cap.* 2.

I ſhall not in this place trouble you with the firſt Rudiments of Arithmetick, as Numeration, Addition, Subſtraction, Multiplication and Diviſion: Becauſe they are already largely handled by many, as Mr. *Leybourn*, Mr. *Wingate*, and divers others, and alſo that then this Book would ſwell to too big a Bulk for the Pocket, and ſo my deſign would be fruſtrated; I ſhall therefore only propoſe and operate ſome

principal Propoſitions, that are of Special Moment in Arithmetick, and which moſt immediately concern the other following Parts of this Treatiſe.

SECT. I.

§ 1. *The Explication of ſome Arithmetical Propoſitions.*

PROP. I.

To three Numbers given, to find a fourth in a direct Proportion.

To operate this proportion Multiply the third Term, by the ſecond Term, and their Product divide by the firſt Term, the Quotient ſhall be a fourth Term required. *Examp.* 1. Admit the Circumference of a Circle whoſe Diameter is 14 Parts be 44 Parts, what is the Circumference of that Circle, whoſe Diameter is 21 Parts? Now according to the Rule if you multiply the third Term 21, by the ſecond Term 44, it produceth 924; which divided by the firſt Term 14, the Quotient is 66, and ſo the Circumference of the Circle, whoſe Diameter is 21, will be 66 Parts, and ſo for any other in a direct Proportion.

PROP. II.

To three Numbers given, to find a fourth in an Inverſed Proportion.

To operate this Propoſition, multiply the firſt Term by the ſecond Term, and their Product divide by the third Term, the Quotient is the fourth

fourth Term required: *Examp.* Admit that 100 Pioneers, be able in 12 Hours to caſt a Mote of a certain Length, Breadth and Depth; in what time ſhall 60 Pioneers do the ſame? Now if according *to* the Rule, you multiply the firſt Term 100, by the ſecond Term 12, their Product is 1200; which divided by the third Term 60, the Quotient is 20, ſo I ſay that in 20 Hours, 60 Pioneers ſhall do the ſame, and ſo for any other in an Inverſed Proportion. § 1.

PROP. III.

To three Numbers given, to find out a fourth in a Duplicate Proportion.

The nature of this Propoſition is to diſcover the Proportion of Lines to Superficies, and Superficies to Lines; for like Plains are in a Duplicate Ratio; that is, as the Quadrate of their *Homologal* ſides; therefore to operate any Example in this Proportion, ſquare the third Term, and its Square multiply by the ſecond Term, their Product divide by the Square of the firſt Term, the Quotient is the 4th. Term ſought. *Examp.* Admit there be two Geometrical Squares; now if the ſide of the greater Square be 50 Feet, and require 3000 Tiles to pave it; what Number ſhall the leſſer Square require, whoſe ſide is 30 Feet? To operate this according to the Rule, I ſquare the third Term 30, whoſe Square is 900: Then I multiply it by the ſecond Term 3000, its Product is 2700000, which divided by 2500, the Square of the firſt Term 50, the Quotient is 1080, and ſo many Tiles will pave the leſſer Square, whoſe ſide is 30 Feet.

§ 1.

PROP. IV.

To three Numbers given, to find a fourth in a Triplicate Proportion.

The Nature of this Propoſition is to diſcover the Proportion of Lines to Solids, and Solids to Lines; for like Solids, are in a Triplicate Ratio, that is to the Cubes, of their Homologal ſides: Therefore to operate any Queſtion in this Proportion, Cube the third Term, and this Cube multiply by the ſecond Term, and their Product divide by the Cube of the firſt Term; the Quotient is the fourth Term ſought. *Examp.* Admit an Iron Bullet whoſe Diameter is 4 Inches, weigh 9 Pounds; what is the Weight of that Bullet whoſe Diameter is 6 Inches? Now to operate this Proportion; firſt according to the Rule, I Cube the third Term 6 whoſe Cube is 216, then I multiply its Cube by the ſecond Term 9, the Product is, 1944, which divided by 64, the Cube of the firſt Term; the Quotient is $30\frac{24}{64}$ Pounds which is equal unto 30 *l.* 6 Ounces: Which is the Weight of the propounded Shot; and ſo for any other.

PROP. V.

To two Numbers given, to find out a third, fourth, fifth, ſixth, &c. *Numbers in a continual Proportion.*

To operate this Proportion, you muſt multiply the ſecond Number by it ſelf, and that Product divide by the firſt Term, the Quotient is a third proportional: Again you muſt multiply

ply the third Term by it ſelf, and its Quadrate divide by the ſecond Term, the Quotient is a fourth proportional, and ſo after this manner a fifth, ſixth; or as many more Proportionals as you pleaſe may be found: *Examp.* Let it be required to find ſix Numbers in a continual Proportion to one another; as 4 to 8. To operate this firſt according to the Rule, I multiply the ſecond Term 8 by it ſelf, the Product is 64, which divided by the firſt Term 4, the Quotient is 16: So is 4, 8, and 16 in a continual Proportion; And ſo obſerving the Rules preſcribed, proceed in your Operation until you have found your ſix Numbers in a continual Proportion; which in this Example will be 4, 8, 16, 32, 64, and 128, and ſo will you have form'd ſix Numbers in a continual Proportion. § 1.

PROP. VI.

Between two Numbers given, to find out a mean Arithmetical Proportional.

This Propoſition might be performed without the help of the Rule of Proportion: Nevertheleſs becauſe it conduceth to the Reſolution of the next enſuing Propoſition, I inſert it in this Place. To operate it this is the Rule: Add half the Difference of the given Terms, to the leſſer Term, ſo that Aggregate is the Arithmetical mean required: *Examp.* Admit 20 and 50 to be the two Numbers propounded: Now to operate this Propoſition, firſt according to the Rule, I find that the difference of the two given Terms 20, and 50, is 30, whoſe half is 15, which being added to the leſſer Term 20, it makes 35,

§ 1. ſo is 35, a mean Arithmetical Proportion betwixt 20, and 50, given.

PROP. VII.

Between two Numbers given, to find out a mean Muſical Proportional.

Boetius hath this Rule for it, wherefore take his own Words: *ſaith he, "*"*Differentiam terminorum in minorem terminum multiplica, & poſt junge terminos, & juxta eum qui inde confectus eſt, committe illum numerum, qui ex differentiis & termino minore productus eſt, cujus cum latitudinem inveneris, addas eam minori termino, & quod inde colligitur medium terminum pones.* That is, multiply the difference of the Terms, by the leſſer Term, and add likewiſe the ſame Terms together: This done, if you divide the Product, by the Sum of the Terms, and to the Quotient thereof, add the leſſer Term; the laſt Sum is the Muſical mean deſired: *Examp.* Admit the two Numbers given be 6, and 12. I ſay that if the difference of the Terms which is 6, were multiplied by the leſſer Term 6, it would produce 36; then if you add the two Terms 6, and 12, together, their Sum would be 18: Now if you divide 36, by 18, the Quotient is 2. Laſtly, if to the Quotient 2, you add the leſſer Term 6, the Sum thereof will be 8, which is a Mean Muſical Proportional required.

* *In his ſecond Book of his Arithmetick, and the 38 Chapter; where he ſaith that this Proportion hath,* Magnam vim in Muſici Modulamintis temperamentis, & in Speculatione naturalium quæſtionum, *i. e. Great Force in Muſical Compoſition, (or in the Compoſure of Muſick) and in the Diſcovery of the Secrets of Nature.*

PROP.

PROP. VIII.

How to find the Square-Root of any whole Number, or Fraction. § 1.

Defin. To extract the Root of any Square Number propounded, is to find out another Number, which being multiplied by it ſelf, produceth the Number propounded. Now for the more eaſie and ready Extraction of the Square-Root of any Number given, This Table here under annexed will be uſeful; which at firſt ſight giveth all ſingle ſquare Numbers, with their reſpective Roots.

ROOT.	1	2	3	4	5	6	7	8	9
SQUAR.	1	4	9	16	25	36	49	64	81

The Explication of the TABLE.

In the uppermoſt Rank of this Table is placed the reſpective Root of every ſingle Square-number, *and in the other* the ſingle Square-numbers themſelves; ſo that if the Root of 25 were demanded, the Anſwer would be 5, ſo the Square Root of 49 is 7, of 81 is 9; and ſo for the reſt, and ſo contrarily the Square of the Root 5 is 25, of 7 is 49, of 9 is 81, *&c.*

Example: If the ſquare Root of 20736 were required, firſt they being wrote down in order as you ſee, draw the crooked Line, * then to prepare this or any other Number for Extraction,

* *As you uſe to do in Diviſion, to repreſent the Quotient.*

§ 1. ction, make a Point over the place of *Unites*; and ſo on every other Figure towards the Left-hand, as you ſee in the Margin. Then find the Root of the firſt Square 2, which is 1; place it in the Quotient, and alſo under 2; then draw a Line, and ſubſtract 1 from 2, there remains 1; which place under the Line, then to the laſt remainder 1, bring down the next Square 07; and then there will be this Number 107, which Number I call a Reſolvend: Then double the Root in the Quotient 1, whoſe double is 2, which 2 place under the place of Tens in the Reſolvend, under 0; ſo is this 2 called a Diviſor; and 10 called a Dividend.

```
  . . .
  2 0 7 3 6 ( 1 4 4
  1
  ---------
  24 ) 1 0 7
       9 6
  ---------
  284 ) 1 1 3 6
        1 1 3 6
  ---------
  : : : :
```

Then demand how often the Diviſor 2, can be had in the Dividend 10, it permitteth but of 4, which place in the Quotient, and under 7 the place of *Unites* in the Reſolvend, and there will appear this Number 24; Then multiply this 24 by 4, (the laſt Square placed in the Quotient) it produceth 96, which place orderly under 24, as you ſee, and this 96 is called an *Ablatitium*; (but ſome call it a *Gnomon*:) then draw a Line under it, and ſubſtract 96, the *Ablatitium*, out of the Reſolvend 107, there remains 11, which place orderly under the laſt drawn Line, then thereunto bring down the next Square 36, ſo will there be a new Reſolvend

ſolvend 1136; then double the whole Root 14 in the Quotient, whoſe double is 28: Place it under the Reſolvend 1136 as was a-fore directed; ſo ſhall 28 be a new Diviſor, and 113 be a Dividend; then I find the Diviſor 28 can be had in the Dividend 113, 4 times, which four place in the Quotient, and under the place of *Unites* in the Reſolvend, ſo there appeareth this Number 284, which Number multiplied by 4, the laſt Figure in the Quotient produceth a new *Ablatitium* 1136; which place orderly under the Reſolvend 1136, and then draw a Line, then ſubſtract the *Ablatitium* 1136, from the Reſolvend 1136; and the remainder is 00, or nothing: And thus the Work of Extraction being finiſhed, I find the Root of the ſquare Number 20736, to be 144; and ſo muſt you have proceeded gradually ſtep by ſtep, if the Number propounded, had conſiſted of 4, 5, 6, or more Squares; ſtill obſerving the aforegoing Rules and Directions. § 1.

NOTE.

But when a whole Number hath not a Root exactly expreſſible by any rational or true Number, *then to find the* fractional part of the Root very near; To the given whole Number annex Pairs of Cyphers, as 00, 0000, or 000000, then eſteem the whole Number, with the Cyphers both annexed thereunto, as one intire whole Number: And extract the Root thereof according to the foregoing Directions, then as many Points as were placed over the Integers, ſo many of the firſt Figures in the Quotient muſt

§ 1. muſt be taken for Integers; and the remainder for the Roots fractional part in Decimal parts, and ſo you may proceed infinitely near the true Root of a Number.

To extract the Square-Root of a Vulgar or Decimal Fraction, and a Mixt-Number.

Firſt, if the Fraction propounded be not in its leaſt Terms, reduce it, and then by the Rules aforegoing, find the Root of the Numerator for a new Numerator; and of the Denominator for a new Denominator; ſo ſhall this new Fraction be the Square-root of the Vulgar Fraction propounded, ſo the Square-root of $\frac{16}{25}$ is $\frac{4}{5}$.

The manner of extracting the Square-root of a Decimal Fraction.

But many times the Numerator and Denominator of a Vulgar Fraction hath not a perfect Square-root; to find whoſe Root infinitely near, you muſt reduce it into a Decimal Fraction, whoſe Numerator muſt conſiſt of an equal Number of Places, to wit, 2, 4, 6, *&c.* Then extract the Square-root of that Decimal, as if it were a whole Number, and the Root that proceedeth from it is a Decimal Fraction, expreſſing the Square-root of the Fraction propoſed, infinitely near: So the Root of $\frac{13}{16}$ (whoſe Decimal is .81250000) will be found to be $\frac{9}{10}$ which is very near, for it wanteth not $\frac{1}{10000}$ of an Unite of the exact Square-root, of $\frac{13}{16}$ propounded.

The manner of extracting the Square-root of a Mixt-Number.

Now having a Mixt Number propounded whoſe Root is required, to find which reduce it into an improper Fraction, and then extract the

the Root thereof as before. Suppoſe the Number propounded be $75\frac{24}{25}$; its improper Fraction is $\frac{679}{9}$, whoſe Square-root I find to be $\frac{26}{3}$ or $8\frac{2}{3}$, very near, *&c.* But if it had not an exact Square-root, then reduce the Fractional part of the given Mixt-number into a Decimal Fraction, of an even Number of Places, and then annex this Decimal to the Integers, and ſo extract the ſame as a whole Number; and obſerve that ſo many Points as were ſet over the Integers, ſo many of the firſt Figures in the Quotient muſt be eſteemed Integers; and the remainder for the Root's Fractional part. § 1.

PROP. IX.

How to find the Cube-Root of any whole Number, or Fraction.

Defin. To extract the *Cube-Root* of any Number propounded, is to find out another Number, which being multiplied by it ſelf, and that Product by the Number again, ſhall produce the Number propounded; Now for the more eaſie and ready Extraction of the Cube-root of any Number propounded, this Table hereafter annexed will be uſeful, which at firſt ſight giveth *the Cube-root of any* whole Number under 1000; which are called ſingle Cube-numbers.

ROOT.	1	2	3	4	5	6	7	8	9
CUBE.	1	8	27	64	125	216	343	512	729

The

§ 1.

The Explication of the TABLE.

In the uppermoſt Rank of the Table is placed the reſpective Roots of every ſingle Cube, and in the other the reſpective ſingle Cube-numbers; for if the Cube-root of 512 were deſired, the Anſwer would be 8, of 64 is 4; and ſo of the reſt: And if the Cube of the Root 7 were deſired, it would be found 343; of 9 it would be 729, *&c.*

Examp. Admit the Cube-root of the Number 262144 were required, firſt they being wrote down in order as you ſee, draw the crooked Line. * Then place a Point over the place of Unites, and another over the place of Thouſands; and ſo on, ſtill intermitting two places between every adjacent Point; and obſerve, that as many Points as in that Order are placed over any Number propounded, of ſo many Figures doth the Root conſiſt: So that in this Example, there being two Points, therefore the Root conſiſteth of two places, as you ſee in the Quotient; Now firſt find the Root of the firſt Cube 262; which permitteth but of 6, place 6 in the Quotient, and ſubſcribe its Cube 216, under 262, and then draw a Line under it, and ſubſtract 216 out of 262, and the remainder is 46, which place in order under the laſt drawn Line as you ſee; Then to the Remainder 46, bring down the next Cube-number 144, ſo will there appear 46144, which I call a Reſolvend: Then draw a Line under it, and ſquare the Number in the Quotient 6, whoſe Square

* *As you uſe to do in Diviſion, to repreſent the Quotient.*

§ 1.

Square is 36; then triple it, and it will be 108, then subscribe this triple Square 108, under the Resolvend, *so that* the place *of* Unites in the triple Square 8, may stand under 1 the place of Hundreds in the Resolvend: Then triple the Root in the Quotient 6, whose Triple is 18; then subscribe the Triple 18, under the Resolvend, so that the place of Unites 8 in the Triple, may stand under 4 the place of Tens in the Resolvend, and so draw a Line underneath it, and add the triple Square 108, and the Triple 18 together in such order as they stand; their Sum is 1098, which may be called a Divisor, and the whole Resolvend 46144, except 4 the place of Unites a Dividend; then draw another Line.

```
2 6 2 1 4 4 (6 4
2 1 6
-----------
  4 6 1 4 4
  -----------
  1 0 8
      1 8
  -----------
  1 0 9 8 Divisor.
  -----------
  4 3 2
    2 8 8
        6 4
  -----------
  4 6 1 4 4
  -----------
```

Then seek how many times 1098 the Divisor can be had in 4614 the Dividend, it permitteth but of 4, which subscribe in the Quotient; now multiply the triple Square 108, by 4, it produceth 432, which in order subscribe under the triple Square 108: Then square 4, the Figure last placed in the Quotient, whose Square is 16; and multiply it by 18 the Triple, it produceth 288, which subscribe under the Triple

§ 1. Triple orderly, then ſubſcribe the Cube of 4 (laſt placed in the Quotient) which is 64, in order under the Reſolvend. Then draw a Line underneath it, and add the three Numbers, *viz.* 432, 288, and 64, together in ſuch order as they are placed, their Sum is 46144: Then draw another Line under the Work, ſubſtracting the ſaid total 46144, from the Reſolvend 46144, there remains 00, or nothing, which remainder ſubſcribe under the laſt drawn Line, thus the Work being finiſh'd I find the Cube-root of 262144 the Number propounded, to be 64: And thus you muſt have proceeded orderly ſtep by ſtep, if the Number propounded had ariſen to ſome 3, 4, 8, 10, or more places, obſerving the Direction preſcribed until all had been compleated.

NOTE.

But when a whole Number hath not a Cube-root expreſſible by any true or rational Number, then to proceed infinitely near the exact Truth, Annex to the Number a certain Number of Tenaries of Cyphers as 000, 000000, 000000000, *&c.* then eſteeming the whole Number with the Cyphers annexed as one intire whole Number, extract the Root thereof, as is afore taught. Then as many Points as were placed over the whole Number, ſo many places of Integers will there be in the Root, and the reſt expreſſeth the Root's Fractional part very near.

§. 1.

To Extract the Cube-Root, of any Vulgar or Decimal or Mixt Fraction conſiſting of a Whole Number and a Fraction.

To Extract the Cube-root of any Vulgar Fraction, you muſt firſt reduce it into his leaſt terms, and then, according to the former Directions, Extract the Cube-root of the Numerator, the Root found ſhall be a new Numerator; ſo likewiſe the Root of the Denominator ſhall become a new Denominator; ſo ſhall this new Fraction be the Cube-root of the Fraction propounded, ſo I find the Cube-root of $\frac{8}{125}$ to be $\frac{2}{5}$, and ſo for any other Vulgar Fraction.

The manner of Extracting the Cube-Root of a Decimal Fraction.

But many times the Numerator, and Denominator hath not a true Root: Then to find the Root thereof infinitely near, you muſt reduce the Fraction given, into a Decimal, whoſe Numerator is Tenaries of Places, and then extract the Root according to the former Directions, ſo ſhall the Root found, be a Decimal Fraction expreſſing nearly the Cube-root of the Fraction propounded, ſo I find the Root of $\frac{8}{125}$ or $\frac{2}{5}$, whoſe *Decimal* is, *666666666*, to be, $\frac{873}{1000}$ very near the Root of $\frac{2}{5}$ or $\frac{4}{10}$ propounded.

The manner of extracting the Cube-Root of a Mixt Number.

Now having a Mixt Number propounded, whoſe Root is required, firſt reduce it into an Improper Fraction, and then extract the Cube root thereof, as is afore directed, ſo the Cube-root of $12\frac{10}{27}$ Improper $\frac{343}{27}$, will be found to be $\frac{7}{3}$ or $2\frac{1}{3}$.

But

§ 1. But if it hath not an exact Cube-root, then reduce the Fractional part of the given mixt Number into a Decimal Fraction, which ſhall conſiſt of Tenaries of places, then to the whole Number annex the Decimal Fraction, and extract the Cube-root of the whole, and obſerve that ſo many Points as are over the Integers, ſo many of the firſt places in the Quotient muſt be eſteemed Integers, and the reſt expreſſeth the Fractional part of the Root in Decimal parts of a Fraction, ſo the Cube-root of $2\frac{1}{8}$, whoſe Decimal is 2, 375000000, *&c.* will be found to be 1, 334, or $1\frac{334}{1000}$, and is very near the true Root, and ſo for any other mixt Number of this nature.

Having thus given you the Practice of extracting the ſquare and Cube-root, I ſhall give you the reaſon of pointing them.

The Square of any Number under 10 will be leſs than 100, and the Squares of all Numbers from 10 under 100 will be leſs than 1000; alſo the Squares of all Numbers, from 100 to under 1000, will be leſs than 1000000, *&c.*

And on the contrary 'tis evident, that the Square-root of any Number under 100 will be leſs than 10, and the Square-root of all Numbers from 100 to under 10000 will be leſs than 100; and of all Numbers, from 10000 to under 1000000, will be leſs than 1000, *&c.*

So that the Square-root of all Numbers under 100 will conſiſt but of one place, under 10000 they will have but two places, and under 1000000 the Root will conſiſt but of three places at the moſt; from whence may be ſeen the reaſon of pointing every other Figure before you begin

begin the Operation: For the Root encreaſing a Place, encreaſes the Square of ſuch Number two Places. § 1.

So alſo it holds in the Cube, where 'tis evident, that 9x9x9 will be leſs than 1000, that is, the *Cube of* any Number under 10 will be leſs *than* 1000; and the Cube of all Numbers, from 10 to under 100, will be leſs than 1000000, *&c.* So on the contrary 'tis evident, that the Cube-roots of all Numbers, from 1000 to under 1000000, will be under 100, *&c.* Whence you ſee, that the Root encreaſing one Place, augments the Cube of ſuch Number three Places: For ſuppoſe 1, 10, 100, 1000, *&c.* be Roots, their Cubes are 1, 1000, 1000000, 1000000000, which encreaſes by 3 Places.

The ſame would alſo follow, if it were 9, 90, 900, 9000, *&c.* that is, their Cubes would exceed each other by 3 Places.

From which is evident the reaſon of pointing the 1ſt, 4th, 7th, *&c.* Places in a propoſed Cube, to find its Root: For the firſt three Places will give but one Place for its Root. If the Cube have four Places in it, the Root will have two Places; and the ſaid Root will have no more than two, if there be five or ſix Places in *the given Cube: But if* there be ſeven Places, then will the Cube-root of ſuch Number conſiſt of three Places.

NOTE, This way of arguing will not only hold in the Square and Cube, but alſo in higher Powers.

Having given you this ſhort Diſcourſe, to ſhow the reaſon of pointing them, in order to their Extraction, I ſhall now give a Taſte of the Uſe

of theſe Operations in the Solution of ſome uſeful, and neceſſary Problems.

SECT. II.

§ 2. *The Extraction of the Square-Root apply'd to the Solution of ſeveral Queſtions in* Arithmetick, Geometry, Trigonometry, Navigation, Embattelling of Armies, *&c.*

PROP. I.

TWO Numbers, *viz.* 24 and 384, given to find a mean Proportion.

Multiply the two Numbers together, and extract the Square-root of the Product, it gives 96 *the mean Proportional required.*

PROP. II.

Let the Area of a Triangle, Rectangle, Circle, *&c.* be 71824 Foot.

Extract the Square-root of the given Area, it gives 268 *Feet, the ſide of a Square equal to the given Area.*

PROP. III.

The ſide of a Square is 26 Foot; What's the ſide of another ſix times as big?

Find the Area of the Square whoſe ſide is given; this Area multiply by 6, *then extract the Square-root of the Product, it gives* 637 *the ſide of a Square ſix times as big.*

NOTE 1. If you would have a Square but $\frac{1}{2}$, $\frac{1}{10}$ $\frac{1}{45}$ *&c.* as big. Take $\frac{1}{2}$, $\frac{1}{1}$, $\frac{1}{7}$, of the Area found

found, and extract the Square-root thereof, and 'twill give you the Area of a Square that is but $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, *&c.* ſo big. § 2.

NOTE 2. So if you would have a Square any number of times as big, or any part of a time as little as any Figure, 'tis but multiplying the given Area by the Number of times, you would have it as big; or taking the part of a time, you would have it as little, and extracting the Square-root of the Product, or Quote, will give you the Area required.

PROP. IV.

There is a Cable, or Rope, whoſe Diameter is 7 Inches, what's the Diameter of another four times as ſtrong.

And here note, That that Rope which has its Section four times as much when cut thro' is four times as ſtrong; and be but half the Superficies, 'tis but half as ſtrong.

Now the Proportion of the Area's of Circles to each other, are as the Squares of their Diameters. *Square* 7 *therefore it makes* 49, *which multiply by* 4 *it gives* 196: *The Square-root of which is* 14, *the Diameter of the Cable four times as ſtrong.*

NOTE 1. Whence 'tis obſervable, That a Rope four times as ſtrong as another, has its Diameter but only double to it.

NOTE 2. The like Operation will ſerve to find the Strength of Timber; that is, to augment or diminiſh any Strength in a given Ratio. As for Example:

§ 2.

PROP. V.

Suppoſe the ſide of a large Beam, or Girder, be 30 Inches; What's the ſide of another ſquare Beam, or Girder, whoſe Strength is but $\frac{1}{5}$ of the former?

Square 30, *the Product* 900 *divide by* 6, *the Quote is* 150; *the Square-root of which is* 12, 2 *the ſide of the Girder, or Beam, that will bear but* $\frac{1}{5}$ *of the Weight of the former.*

PROP. VI.

In Triangle BAC (Plate 9th, Fig. 96.) the BA is 48 Foot, and the Perpendicular CA 40 Foot; What's the Length of the Hypotenuſe BC.

The reaſon of the Solution of this Propoſition is given in the 47th of the 1ſt Book of *Euclid*; and in the 9th *Theorem* of my *Epitome* of Geometry, where 'tis prov'd in a right angled Triangle, that the Square-root of the Sum of the Squares of the Baſe and Perpendicular is equal to the Hypotenuſe.

Square therefore 48 *and* 40, *and add thoſe two Squares together, it makes* 3904; *the Square-root of which is* 62. 5 *near for the Hypotenuſe* BC.

This is commonly the laſt Caſe in right Angled, plain Triangles, and in Navigation, finds the Diſtance ſailed by having the Difference of Latitude (which is the Perpendicular) and Departure (which is the Baſe) given: For the Square-root of the Sum of the Squares of the Diffe-

Difference of Latitude, and Departure is the Diſtance ſailed. § 2.

This Propoſition is alſo of uſe in Fortification: For it finds the Length of a Scaling Ladder that *will* reach from the Edge of a Moat to the *Top* of a Rampire, when the Breadth of the Moat and Height of a *Rampire* is given.

It's alſo of great uſe to Artificers, in finding the Length of *Hyp-Rafters*, Strings of Stairs, *&c.*

PROP. VII.

Again, if the *Hypotenuſe* and *Baſe* be given, then the *Perpendicular* is eaſily found: For if from the Square of the *Hypotenuſe* you take the Square of the *Baſe*, and extract the Square-root of the Remainder, it will give the Length of the *Perpendicular*. So if the *Hypotenuſe* and *Perpendicular* be given, 'tis but ſubſtracting the Square of the *Perpendicular* from the Square of the *Hypotenuſe*, and extracting the Square-root of the Remainder for the Length of the *Perpendicular*.

This Propoſition is uſeful in Navigation: For by it the Difference of Latitude may be found, *having the* Diſtance ſailed and Departure given; or by it the Departure may be found, having the Diſtance ſailed and Difference of Latitude given.

PROP. VIII.

To place any Number of Soldiers. Suppoſe 132496 in Square *Battalia*; that is, to place them in ſuch order that there may be as many in Rank as in File.

§ 2. *Extract the Square-root of the Number propoſed, it gives* 364 *the Number to be placed in Rank and File.*

PROP. IX.

To place any Number of Men, ſuppoſe 35912 in double *Battalia*, that is, ſo that the Men in Rank may be double to thoſe in File.

Extract the Square-root of 17956, *which is half the Number of Men propoſed, it gives* 134 *the Number that muſt be placed in Files: This doubled gives the Number that muſt be placed in Rank, and theſe two Numbers multiply'd together, will make up the whole Army if the Work be right, elſe not.*

PROP. X.

Any Number of Men. Suppoſe 48224 with their Diſtance in Rank and File, *that is*, that their Diſtance in *File* ſhould be 8 Foot, and Rank 3 Foot, to order them in a ſquare Battaile of Ground, 'tis required to know how many Men muſt be in Rank and how many in File.

Say as 8 *to* 3 *ſo is* 48224 *to* 18084, *the Square-root of which is* 167 *near: Then to find the Number of Men to be placed in Rank, divide* 48224 *by* 167, *the Quote* 288 *is the Number of Men to be placed in Rank, beſide* 128 *Men over and above.*

PROP.

§. 2.

PROP. XI.

The Uſe of the Extraction of the Cube-root.

Suppoſe the ſide of a Cube be 84 Foot; What's the ſide of another double to it?

Cube 84 *the Number given, it makes* 592704; *then extract the Square-root of* 1185408, *it doubled, and 'twill give* 105 *the ſide of a Cube double to the former.*

After the ſame manner may you find the ſide of a Cube three, four, or more times as big, or $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, or any other part of a time ſo little, by multiplying the Cube of the given ſide by the Number of times propoſed, or taking ſuch part of the ſaid Cube as you would have it to be leſs, and extracting the Square-root of ſuch Product or Quote.

PROP. XII.

To find the ſide of a Cube equal to any given Solid, whether *Parallelepipedon*, *Sphere*, *Cylender*, *Cone*, &c.

Admit the Solidity of a *Parallelepipedon* were 5451776, What's the ſide of a Cube equal thereto?

Extract the Cube-root of 5451776 *the Content of the given Solid, what comes out is* (176) *the ſide of the Cube required.*

From this Propoſition the Proportion of the Diameter of a Sphere to the ſide of a Cube equal thereto may be found. For the Solidity of a Sphere, whoſe Diameter is 1, is .523608805 the Cube-root of which .806 is *near*.

§ 2. So that if the Diameter of a Sphere be 1, the ſide of the Cube equal thereto is 806.

And therefore as 1 to .806 ſo is any Diameter of a Sphere to the ſide of the Cube equal thereto.

To ſhow all the Uſes of the Square and Cube-root would fill a large Volume. Theſe are enough to let the Reader ſee the Neceſſity of learning this uſeful Operation.

To make this Treatiſe more perfect, I have added a Table of the Squares of all whole Numbers under 1000, and the Cubes of all Numbers under 200; which will be neceſſary on ſeveral Occaſions.

A TABLE

A TABLE shewing the Squares of all Numbers under 1000.

Root	Square	Root	Square	Root	Square	Root	Square
1	1	40	1600	79	6241	118	13924
2	4	41	1681	80	6400	119	14161
3	9	42	1764	81	6561	120	14400
4	16	43	1849	82	6724	121	14641
5	25	44	1936	83	6889	122	14884
6	36	45	2025	84	7056	123	15129
7	49	46	2116	85	7225	124	15376
8	64	47	2209	86	7396	125	15625
9	81	48	2304	87	7569	126	15876
10	100	49	2401	88	7744	127	16129
11	121	50	2500	89	7921	128	16384
12	144	51	2602	90	8100	129	16641
13	169	52	2704	91	8281	130	16900
14	196	53	2809	92	8464	131	17161
15	225	54	2916	93	8649	132	17424
16	256	55	3025	94	8836	133	17689
17	289	56	3136	95	9025	134	17956
18	324	57	3249	96	9216	135	18225
19	361	58	3364	97	9409	136	18496
20	400	59	3481	98	9604	137	18769
21	441	60	3600	99	9801	138	19044
22	484	61	3721	100	10000	139	19321
23	529	62	3844	101	10201	140	19600
24	576	63	3969	102	10404	141	19881
25	625	64	4096	103	10609	142	20164
26	676	65	4225	104	10816	143	20449
27	729	66	4356	105	11025	144	20736
28	784	67	4489	106	11239	145	21025
29	841	68	4624	107	11449	146	21316
30	900	69	4761	108	11664	147	21609
31	961	70	4900	109	11881	148	21904
32	1024	71	5041	110	12100	149	22201
33	1089	72	5184	111	12321	150	22500
34	1156	73	5329	112	12544	151	22801
35	1225	74	5476	113	12769	152	23104
36	1296	75	5625	114	12956	153	23409
37	1369	76	5776	115	13225	154	23716
38	1444	77	5929	116	13456	155	24025
39	1521	78	6084	117	13689	156	24336

Root	Square	Root	Square	Root	Square	Root	Square
157	24649	200	40000	243	59049	286	81796
158	24964	201	40401	244	59536	287	82369
159	25281	202	40804	245	60025	288	82944
160	25600	203	41209	246	60516	289	83521
161	25921	204	41616	247	61009	290	84100
162	26244	205	42025	248	61504	291	84681
163	26569	206	42436	249	62001	292	85464
164	26896	207	42849	250	62500	293	85849
165	27225	208	43264	251	63001	294	86436
166	27556	209	43681	252	63504	295	87025
167	27889	210	44100	253	64009	296	87616
168	28224	211	44521	254	64516	297	88209
169	28561	212	44944	255	65025	298	88804
170	28900	213	45369	256	65536	299	89401
171	29241	214	45796	257	66049	300	90000
172	29584	215	46225	258	66564	301	90601
173	29929	216	46656	259	67081	302	91204
174	30276	217	47089	260	67600	303	91809
175	30625	218	47524	261	68121	304	92416
176	30976	219	47961	262	68644	305	93025
177	31329	220	48400	263	69169	306	93636
178	31684	221	48841	264	69696	307	94249
179	32041	222	49284	265	80225	308	94864
180	32400	223	49729	266	70756	309	95481
181	32761	224	50176	267	71289	310	96100
182	33124	225	50625	268	71824	311	96721
183	33489	226	51076	269	72361	312	97344
184	33856	227	51521	270	72900	313	97969
185	34225	228	51984	271	73441	314	98596
186	34596	229	52441	272	73984	315	99225
187	34969	230	52900	273	74529	316	99856
188	35344	231	53361	274	75076	317	100489
189	35721	232	53824	275	75625	318	101124
190	36100	233	54289	276	76176	319	101761
191	36481	234	54756	277	76729	320	102400
192	36864	235	55225	278	77284	321	103041
193	37249	236	55696	279	77841	322	103684
194	37636	237	56169	280	78400	323	104329
195	38025	238	56644	281	78961	324	104976
196	38446	239	57121	282	79524	325	105625
197	38809	240	57600	283	80089	326	106276
198	39204	241	58081	284	80656	327	106929
199	39601	242	58564	285	81225	328	107584

Root	Square	Root	Square	Root	Square	Root	Square
329	108241	372	138384	415	172225	458	209764
330	108900	373	139129	416	173056	459	210681
331	109561	374	139876	417	173889	460	211600
332	110224	375	140625	418	174724	461	212521
333	110889	376	141376	419	175561	462	213444
334	111556	377	142129	420	176400	463	214369
335	112225	378	142884	421	177241	464	215296
336	112896	379	143641	422	178084	465	216225
337	113569	380	144400	423	178929	466	217156
338	114244	381	145161	424	179776	467	218089
339	114921	382	145924	425	180625	468	219024
340	115600	383	146689	426	181476	469	219961
341	116281	384	147456	427	182329	470	220900
342	116964	385	148225	428	183184	471	221841
343	117649	386	148996	429	184041	472	222784
344	118336	387	149769	430	184900	473	223729
345	119025	388	150544	431	185761	474	224676
346	119716	389	151321	432	186624	475	225625
347	120409	390	152100	433	187489	476	226576
348	121104	391	152881	434	188356	477	227529
349	121801	392	153664	435	1892	478	228484
350	122500	393	154449	436	190096	479	229441
351	123201	394	155236	437	190969	480	230400
352	123904	395	156025	438	191844	481	231361
353	124609	396	156816	439	192721	482	232324
354	125316	397	157609	440	193600	483	233289
355	126025	398	158404	441	194481	484	234256
356	126736	399	159201	442	195364	485	235225
357	127449	400	160000	443	196249	486	236196
358	128164	401	160801	444	197136	487	237169
359	128881	402	161604	445	198025	488	238144
360	129600	403	162409	446	198916	489	239121
361	130321	404	163216	447	199809	490	240100
362	131044	405	164025	448	200704	491	241081
363	131769	406	164836	449	201601	492	242064
364	132496	407	165649	450	202500	493	243049
365	133225	408	166464	451	203401	494	244036
366	133956	409	167281	452	204304	495	245025
367	134689	410	168100	453	205209	496	246016
368	135424	411	168921	454	206116	497	247009
369	136161	412	169744	455	207025	498	248004
370	136900	413	170569	456	207936	499	249001
371	137641	414	171396	457	208849	500	250000

Root	Square	Root	Square	Root	Square	Root	Square
501	251001	544	295936	587	344569	630	396900
502	252004	545	297025	588	345744	631	398161
503	253009	546	298116	589	346921	632	399424
504	254016	547	299209	590	348100	633	400689
505	255025	548	300304	591	349281	634	401956
506	256036	549	301401	592	350464	635	403225
507	257049	550	302500	593	351649	636	404496
508	258064	551	303601	594	352836	637	405769
509	259081	552	304704	595	354025	638	407044
510	260100	553	305809	596	355216	639	408321
511	261121	554	306916	597	356409	640	409600
512	262144	555	308025	598	357604	641	410881
513	263169	556	309136	595	358801	642	412164
514	264196	557	310249	600	360000	643	413449
515	265225	558	311364	601	361201	644	414736
516	266256	559	312481	602	362404	645	416025
517	267289	560	313600	603	363609	646	417316
518	268324	561	314721	604	364816	647	418609
519	269361	562	315844	605	366025	648	419904
520	270400	563	316569	606	367236	649	421201
521	271441	564	318096	607	368449	650	422500
522	272484	565	319225	608	369664	651	423801
523	273529	566	320356	605	370881	652	425104
524	274576	567	321489	610	372100	653	426409
525	275625	568	322624	611	373321	654	427716
526	276676	569	323761	612	374544	655	439025
527	277729	570	324900	613	375769	656	430336
528	278784	571	326041	614	376996	657	431649
529	279841	572	327184	615	378225	658	432564
530	280900	573	328329	616	379456	659	434281
531	281961	574	329476	617	380689	660	435600
532	283024	575	330625	618	381924	661	436921
533	284089	576	331776	619	383161	662	438244
534	285156	577	332929	620	384400	663	439569
535	286225	578	334084	621	385641	664	440896
536	287296	579	335241	622	386884	665	442225
537	288369	580	336400	623	388129	666	443556
538	289444	581	337561	624	389376	667	444889
539	290521	582	338724	625	390625	668	446224
540	291600	583	339889	626	391876	669	447561
541	292681	584	341056	627	393125	670	448900
542	293764	585	342225	628	394384	671	450241
543	294849	586	343396	629	395641	672	451584

Root	Square	Root	Square	Root	Square	Root	Square
673	452929	716	512656	759	576081	802	643204
674	454276	717	514089	760	577600	803	644809
675	455625	718	515524	761	579121	804	646416
676	456976	719	516961	762	580644	805	648025
677	458329	720	518400	763	582169	806	649636
678	459684	721	519841	764	583696	807	651249
679	461041	722	521284	765	585225	808	652864
680	462400	723	523729	766	586756	809	654481
681	463761	724	524176	767	588289	810	656100
682	465124	725	525625	768	589824	811	657721
683	466489	726	527076	769	591361	812	659344
684	467856	727	528529	770	592900	813	660969
685	469225	728	529984	771	594441	814	662596
686	470596	729	531441	772	595984	815	664225
687	471969	730	532900	773	597529	816	665856
688	473344	731	534361	774	599076	817	667489
689	474721	732	535824	775	600625	818	669124
690	476100	733	537289	776	602176	819	670761
691	477481	734	538756	777	603729	820	672400
692	478864	735	540225	778	605284	821	674041
693	480249	736	541696	779	606841	822	675684
694	481636	737	543169	780	608400	823	677329
695	483025	738	544644	781	609961	824	678976
696	484416	739	546121	782	611524	825	680625
697	485809	740	547600	783	613089	826	682276
698	487204	741	549081	784	614656	827	683929
699	488601	742	550564	785	616224	828	685584
700	490000	743	552049	786	617795	829	687241
701	491401	744	553536	787	619368	830	688900
702	492804	745	555025	788	620943	831	690561
703	494209	746	556516	789	622520	832	692224
704	495616	747	558009	790	624100	833	693889
705	497025	748	559504	791	625681	834	695556
706	498436	749	561001	792	627264	835	697225
707	499849	750	562500	793	628849	836	698896
708	501264	751	564001	794	630436	837	700569
705	502681	752	565504	795	632025	838	702244
710	504100	753	567009	796	633616	839	703921
711	505521	754	568516	797	635209	840	705600
712	506944	755	570025	798	636804	841	707281
713	508369	756	571536	799	638401	842	708964
714	509796	757	573049	800	640000	843	710649
715	511225	758	574564	801	641601	844	712336

Root	Square
845	714025
846	715716
847	717409
848	719104
849	720801
850	722500
851	724201
852	725904
853	727609
854	729316
855	731025
856	732736
857	734449
858	736164
859	737881
860	739600
861	741321
862	743044
863	744769
864	746496
865	748225
866	749956
867	751689
868	753424
869	755161
870	756900
871	758641
872	760384
873	762129
874	763876
875	765625
876	767376
877	769129
878	770884
879	772641
880	774400
881	776161
882	777924
883	779689
884	781456
885	783225
886	784996
887	786769
888	788544
889	790321
890	792100
891	793881
892	795664
893	797449
894	799236
895	801025
896	802816
897	804609
898	806404
899	808201
900	810000
901	811801
902	813604
903	815409
904	817216
905	819025
906	820836
907	822649
908	824464
909	826281
910	828100
911	829921
912	831744
913	833569
914	835396
915	837225
916	839056
917	840889
918	842724
919	844561
920	846400
921	848241
922	850084
923	851929
924	853776
925	855625
926	857476
927	859329
928	861184
929	863041
930	864900
931	866761
932	868624
933	870489
934	872356
935	874225
936	876096
937	877696
938	878444
939	881721
940	883600
941	885481
942	887364
943	889249
944	891136
945	893025
946	894916
947	896809
948	898704
949	900601
950	902500
951	904401
952	906304
953	908209
954	910116
955	912025
956	913939
957	915849
958	917764
959	919681
960	921600
961	923521
962	925444
963	927369
964	929296
965	931225
966	933156
967	935089
968	937024
969	938961
970	940900
971	942841
972	944784
973	946729
974	948676
975	950625
976	952576
977	954529
978	956484
979	958441
980	960400
981	962361
982	964324
983	966289
984	968256
985	970225
986	972196
987	974169
988	976144
989	978121
990	980100
991	982081
992	984064
993	986049
994	988036
995	990025
996	992016
997	994009
998	996004
999	998001
1000	1000000

A TABLE

A TABLE shewing the Cube of all Numbers from 1 to 200.

Root	Cube	Root	Cube	Root	Cube	Root	Cube
1	1	38	54874	75	421875	112	1404928
2	8	39	59319	76	438976	113	1442897
3	27	40	64000	77	456533	114	1481544
4	64	41	68921	78	474552	115	1520875
5	125	42	74088	79	493039	116	1560896
6	216	43	79507	80	512000	117	1601613
7	343	44	85184	81	531441	118	1643032
8	512	45	91125	82	551368	119	1685159
9	729	46	97336	83	571787	120	1728000
10	1000	47	103823	84	592704	121	1771561
11	1331	48	110592	85	614125	122	1815848
12	1728	49	117649	86	636056	123	1860867
13	2197	50	125000	87	658503	124	1906624
14	2744	51	132651	88	681472	125	1953125
15	3375	52	140608	89	704969	126	2000376
16	4096	53	148877	90	729000	127	2048383
17	4913	54	157464	91	753571	128	2097152
18	5832	55	166375	92	778688	129	2146689
19	6859	56	175616	93	804357	130	2197000
20	8000	57	185193	94	830584	131	2248091
21	9261	58	195112	95	857375	132	2299968
22	10648	59	205379	96	884736	133	2352637
23	12167	60	216000	97	912673	134	2406104
24	13824	61	226981	98	941192	135	2460375
25	15625	62	238328	99	970299	136	2515456
26	17576	63	250047	100	1000000	137	2571353
27	19683	64	262144	101	1030301	138	2628072
28	21952	65	274625	102	1061208	139	2685619
29	24389	66	287496	103	1092727	140	2744000
30	27000	67	300763	104	1124864	141	2803221
31	29791	68	314432	105	1157625	142	2863288
32	32770	69	328509	106	1191016	143	2924207
33	35939	70	343000	107	1225043	144	2985984
34	39306	71	357911	108	1259712	145	3048625
35	42877	72	373248	109	1295029	146	3112136
36	46658	73	389017	110	1331000	147	3176523
37	50655	74	405224	111	1367631	148	3241792

Root	Cube	Root	Cube	Root	Cube	Root	Cube
149	3307949	162	4251528	175	5359375	188	6644672
150	3375000	163	4330747	176	5451776	189	6751269
151	3442951	164	4410944	177	5545233	190	6859000
152	3511808	165	4492125	178	5639752	191	6967871
153	3581577	166	4574296	179	5735339	192	7077888
154	3652264	167	4657463	180	5832000	193	7189057
155	3724875	168	4741632	181	5929741	194	7301384
156	3796416	169	4826809	182	6028568	195	7414875
157	3869893	170	4913000	183	6128487	196	7529536
158	3944312	171	5000211	184	6229304	197	7645373
159	4019679	172	5088448	185	6331625	198	7762392
160	4096000	173	5177717	186	6434856	199	7880599
161	4173281	174	5268024	187	6539203	200	8000000

CHAP

CHAP. II.

The Explication and Uſe of the Tables of LOGARITHMS.

SECT. I.

The Explication of the Tables of the Logarithms, *and of Parts proportional.* § 1.

Defin. THE *Logarithms* were firſt invented, found out and framed, by that never to be forgotten and thrice Honourable Lord, the Lord *Nepeir:* Which Numbers, ſo found out and framed by his diligent Induſtry, he was pleaſed to call *Logarithms*; which in the Greek ſignifies the Speech of Numbers. I ſhall not here trouble you with the manner of the Conſtruction of thoſe Tables of *Logarithms*, but ſhall firſt lay down ſome brief and general Rules, that thereby the better you may underſtand thoſe Tables, and then I ſhall explain their manifold Uſes, in ſundry Examples Arithmetical, *&c.*

PROP. I.

Any Number given under 10000, *or* 100000, *to find the* Logarithm *correſponding thereunto.*

1. If the Number propounded conſiſt of one place, whoſe *Logarithm* is required to be found,

§ 1. as ſuppoſe (5,) look for 5 ~~in the top of~~ the left hand Column under the Letter * *N*, and right againſt 5, and in the next Column under *LOG.* † you will find this Number or Rank of Figures, 0698970, which is the *Logarithm* of the Number 5 required.

* *Signifies the Number, or Figure ſought.*
† *Signifies the* Logarithm *anſwering to the Number oppoſite.*

2. If the Number conſiſteth of two places, as if it were 57, look 57 under *N*, and oppoſite to it, and under *LOG.* you will find this Number 1. 755875, which is the *Logarithm* of 57, the Number propounded.

3. If the Number propounded conſiſt of three places, as 972, look for 972, under *N*, and oppoſite to 972, and under (0) the Column, you ſhall find this Number 2. 987666, which is the *Logarithm* of 972, the Number which was propounded.

4. But if the Number conſiſts of four places, as 1685, look the three firſt Figures 168, under the Column *N*; and oppoſite to that, and under 5 at the top of the Page, you will find this Number 3. 226599, which is the *Logarithm* of 1685, the Number propounded.

☞ *Note this Rule well; for this explains the Uſe of the Table of proportional Parts, printed at the end of this Book.*

5. But if the Number given be above 10000, and under 100000, you may find its *Logarithm* by the Table of Parts proportional, printed at the latter end of this Book. Thus, if the *Logarithm* of 35786 be ſought, firſt ſeek the *Log.* of 3578, which will be 553649, and the common Difference under D is 121; with this Difference 121, Enter the Table of Parts proportional,

tional, and finding 121 in the first Column under D, you may then lineally under 6, find the Number 72, which add to the *Log.* of 3578, that is 553649, it produceth 553712, which is the *Log.* of 35786 the Number propounded: Now because the Number propounded 35786, ariseth to the place of *X. M.* therefore there must be the Figure 4 prefixed before its *Logarithm*, and then it will be thus 4. 553712, which 4 is called the Index, as shall be hereafter shewed. §. 11.

Now before we proceed to find Numbers corresponding to *Logarithms*, it will be necessary to explain the meaning of the first Figure to the left hand of any *Logarithm* placed. Mr. *Briggs* calleth it a Characteristick or Index, which doth represent the Distance of any the first Figure of any whole Number from Unity, whose Index is 0, a Cypher; so the Index of 10 is 1, and so to 100 whose Index is 2, and so to 1000 whose Index is 3, and so to 10000 whose Index is 4; and so if you persist farther the Characteristick is always one less in Dignity than the Places or Figures of the Number propounded. *Definition.*

The Rule to find the Characteristick or Index appertaining to any Logarithm.

PROP. II.

To find the Logarithm *of a Vulgar Fraction, and a Mixt Number.*

First, as is before shewed, if it be a Vulgar Fraction, find the *Log.* of the Numerator, and the *Log.* of the Denominator, then subtract the

§ 1. *Log.* of the Numerator from the *Log.* of the Denominator, the remainder is the *Log.* of the Fraction propounded: Now if you would find the *Logarithm* of $\frac{5}{7}$, do as is prescribed, whose *Log.* I find to be 0. 146121. Now to find the *Log.* of a Mixt Number, reduce it into an Improper Fraction, and then do as before, so the *Log.* of $15\frac{1}{7}$, Improper $\frac{77}{7}$, is 1, 187, 52, and so do for any other Mixt Number.

The *Log.* 7 is ——— 0. 84509
The *Log.* 5 is ——— 0. 69897
The *Log.* $\frac{5}{7}$ is ——— 0. 14612

PROP. III.

A Logarithm *propounded to find the Whole, or Mixt Number, corresponding thereunto.*

For the more speedy finding the Number, answering unto the *Logarithm* propounded observe, that if the Index be 0, then the Number sought may be found between 1 and 10; If 1, between 10 and 100; if 2, between 100 and 1000; if 3, between 1000 and 10000, and so on, still observing the Rules of the Characteristick, or Index; therefore look in the Table until you find the *Logarithm* proposed, and against it in the Margin, according to the aforegoing Directions under *N*, you shall find the Number belonging thereunto. This Rule holds in force in Mixt Numbers also.

Thus.	0. 845098	Are the Logarithms of,	7
	1. 556302		36
	2. 130334		135
	3. 980276		9556

NOTE.

§ 1.

NOTE.

But if you cannot find the *Logarithm* exactly in the Table, as in many Operations it ſo happeneth, you muſt then take the neareſt *Logarithm* Number to the *Logarithm* propounded, and ſo take the Number belonging thereto for the deſired Number.

SECT. II.

Of the Admirable Uſe of the Logarithms *in* Arithmetick. § 2.

PROP. I.

To multiply one Number by another.

ADmit 90 to be multiplied by 42, what is the Product? Firſt find the *Log.* of the Multiplicand 90, Then the *Log.* of the Multiplier 42. Add theſe two *Log.* together, it gives 57748, which is the *Log.* of 3780, the Product of 90, and 42.

The *Log.* of 90 is —	1.	95424
The *Log.* of 42 is —	1.	62324
The *Log.* of 3780 is —	3.	57748

§ 2.

PROP. II.

To divide one Number by another.

Admit the Dividend (or Number to be divided) be 648, and the Diviſor 72, what is the Number that the Quotient ſhall conſiſt of? To find which, firſt write down the *Logarithm* of the Dividend 648, and alſo write down the *Logarithm* of the Diviſor 72. Then ſubſtract the *Log.* of the Diviſor, out of the *Log.* of the Dividend, the remainder is 0. 95424, the *Logarithm* of 9; ſo I conclude that the Diviſor 72. is contained in the Dividend 648, 9 times, and ſo do for any other.

The *Log.* of 648 is	2. 81157
The *Log.* of 72 is	1. 85733
The *Log.* of 9 is	0. 95424

PROP. III.

To find the Square-Root of a Number.

Admit it be required to extract the Square-Root of the Number 144; to perform which, firſt write down the *Log.* of 144, which is 2. 15836. Then take the half thereof, which is 1. 07918, it is the *Log.* of 12, the Root of 144 propounded, and ſo do for any other.

NOTE on the contrary, by doubling the *Log.* of any Number, you have the Geometrical Square thereof: And by tripling the *Log.* of any Number you have the Cube of it.

PROP.

PROP. IV.

To find the Cube Root of any Number. § 2.

Admit it be required to extract the Cube-Root of 1728; to perform which, firſt write down the *Log.* of 1728, which is 3. 23754, then take the third part thereof, which is 1. 07918, the *Log.* of 12, the Cube-root of 1728 the Number propoſed, and ſo for any other. *Note* on the contrary, if you multiply the *Log.* of any Number propounded by 3, it produceth the *Log.* of the Cube thereof.

PROP. V.

A Summ of Money being forborn for any Number of Years, to find how much it will amount unto, reckoning Intereſt on Intereſt, according to any Rate propounded.

Admit 300 Pounds Sterling be put out for 4 Years, for Compound Intereſt at 6 *l. per Cent.* what will it amount to when the four Years are expired? To find which ſubſtract the *Log.* of 300 *l.* the Principal, whoſe *Log.* is 2. 477121, out of the *Log.* of 318 *l.* Principal and Intereſt for a Year, whoſe *Log.* is 2. 502427, the remainder is 0. 025306, which being multiplied by 4, the Number of Years of its Continuance, produceth 0. 101224, which added to the *Log.* of the Principal 300 *l.* to wit, to 2. 477121, makes 2, 578345, which is the *Log.* of 378 *l.* 14 *s.* 10 *d.* 2 *q.* very near, and ſo much will 300 *l.* amount to.

§ 2.

PROP. VI.

A Summ of Money being to be paid hereafter, to find what it is worth in ready Money.

Admit 100 Pounds Sterling to be paid at 30 Years end; I demand how much it is worth in ready Money? After the rate of Intereſt of 6 *l. per Cent.* To find which, ſubſtract the *Logarithm* of 100 the Principal, whoſe *Log.* is 2. 000000 from the *Log.* of 106 Principal and Intereſt, whoſe *Log.* is 2. 025306, the remainder is 0. 025306, which multiplied by 30 the Number of Years to ſucceed, produceth 0. 759180, which ſubſtracted out of 2. 000000, leaveth 1. 240820, which is the *Log.* of $17\frac{411}{1000}$, which ſheweth the ſaid 100 *l.* is worth but 17 *l.* 8 *s.* 2 *d.* 3 *q. fere.*

PROP. VII.

A Yearly Rent, or Annuity, to continue any Number of Years, to find what it is worth in ready Money, at any Rate of Intereſt propounded.

What is 100 Pound *per Annum* to continue 30 Years, worth in ready Money at 6 *l. per Cent.* To find which, firſt ſubſtract the *Log.* of 100 *l.* the Principal, which is 2. 000000 from the *Log.* of 106 *l.* Principal and Intereſt for a Year, whoſe *Log.* is 2. 025306 the remainder is 0. 025306: Then multiply 0. 025306 by 30 the Number of Years of its Continuance, it produceth the Number 0. 759180; Then divide 100 *l.* by 6 the rate of Intereſt, and the Quotient is $16\frac{666}{1000}$, *&c.* which $16\frac{666}{1000}$ is the proportional

tional Parts of 100*l.* the Principal; then add the *Log.* thereof, which is 1. 221829 to the former *Log.* 0. 759180 it produceth 1. 981009, which is the *Log.* of 95 $\frac{7115}{10000}$ parts the Arrearages with the ſaid Summ for that time; then from thoſe Arrearages 95 $\frac{7115}{10000}$ ſubſtract the parts proportional of 100, to wit, 16 $\frac{6667}{10000}$, the remainder is 79 $\frac{0548}{10000}$, which is the bare Arrearages for that proportional part; Then take the *Log.* of 79 $\frac{0548}{10000}$, which is 1. 897929, out of the which take the *Log.* found by Multiplication of Years, to wit, 0. 759180, there remains 1. 138749, which is the *Log.* of the Value of the Arrearages in ready Money; then to the *Log.* 1. 138749, add the *Log.* of 100*l.* Principal, 2. 000000, it produceth this Number 3. 138749; the *Log.* of 137 6 $\frac{48}{100}$ reduced is 1376*l.* 9*ſh.* 7*d.* $\frac{80}{100}$ or $\frac{4}{5}$ *fere:* And ſo much is the ſaid Annuity worth in ready Money. § 2.

CHAP.

CHAP. III.

The Explication of the SINES, TANGENTS *and* SECANTS.

SECT. I.

§ 1. *Of Right Sines, Tangents, Secants, Co-Sines, Tangents, and Secants: Of any Arch, or Angle of a Triangle.*

PROP. I.

To find the right Sine, or Tangent, of any Arch or Angle of a Triangle, containing any Number of Degrees and Minutes.

☞ *And here 'tis neceſſary to underſtand, that every Circle is ſuppoſed to be divided into* 360 *equal parts, which are called Degrees, and every of thoſe Degrees into* 60 *Minutes, and every Minute into* 60 *Seconds, and every Second into* 60 *Thirds, &c. ſo that a Semi-Circle contains* 180 *Degrees, and a Quadrant* 90 *Degrees; Now an Arch or Angle of a Triangle, is the Interſection of its two ſides, and the Meaſure thereof, is an Arch of a Circle, which cutteth each of the two ſides equidiſtant from the Angular Point, (which is the Center.) Now the Logarithm, Sine, or Tangent of any ſuch Arch of a Triangle, containing any Number of Degrees or Minutes of the Quadrant, may be found in the Tables, printed at the End of this Book, where they are plainly expreſſed, and are found as directed in the precedent Rules.*

IF the Angle or Arch of the Triangle propounded be leſs than 45 Deg. the Sine, or Tangent, belonging thereunto, is found in the Column under the Title SINE, or TANGENT, at the top of the Table; and if there be any Minutes annexed unto the Degrees, you muſt find them out in the firſt Column under M. ſignifying Minutes, and oppoſite to thoſe Minutes, and under the Title aforeſaid, you ſhall have the *Logarithm* of the Sine or Tangent, of the Arch or Angle required.

But

But if the Arch or Angle of a Triangle exceed 45 Degrees, you muſt then look for the Sine or Tangent belonging thereunto, in the bottom of the ſaid Table; and if thereunto are Minutes annexed, you muſt look for them in the firſt Column to the Right-hand under M, and ſo oppoſite to thoſe Minutes in the Column above the Title, Sine, or Tangent; there have you the Log. of the Sine, or Tangent, of the Arch or Angle, of the Triangle propounded. § 1.

Examp. Suppoſe it were required to find the Log-Sine or Log-Tangent of an Angle of 25 D. 37 M. whoſe Log-Sine, whereof according to the former Directions, I find to be 9. 635833, and Tangent thereof to be 9. 680768, and ſo for any other under 45 Degrees.

Again, ſuppoſe it were required to find the Log-Sine or Log-Tangent of an Angle of 64 D. 23 M. the Sine whereof I find to be this Number 9. 955065, and the Tangent thereof 10. 319231, and ſo for any other Arch, or Angle of a Triangle, above 45 Degrees.

PROP. II.

To find the Co-Sine or Co-Tangent of any Arch, or Angle propounded.

Defin. The Co-Sine or Co-Tangent of an Angle or Arch, is the remaining part of the Angle propounded to a Quadrant or 90 Degrees; and is by ſome called the Complement of an Angle. Thus the Arch or Angle of 64 D. 23 M. taken out of 90 D. leaves 25 D. 37 M. for its Complement; on the contrary, if 25 D. 37 M. were taken out of 90 Degrees, there would remain

§ 1. main 64 D. 23 M. for its Complement. So you ſee that theſe two Angles are the Complements of each other, becauſe they two are equal to a Quadrant or 90 Degrees.

Now the Logarithm of the Complement may be exactly found with eaſe; for the Sines and Tangents of every Degree, and Minute of the Quadrant in one Column is joyned with his Complement in the next Column; ſo that without ſubſtracting the Angle from 90 D. you may readily find the Complement thereof either the Arch in Degrees and Minutes, or the Log-Sine, or Tangent thereof, as you have occaſion: Thus the Log. of the Sines Complement before mentioned, to wit, 64 D, 23 M. Compl. is 25 D. 37 M. is 9. 635833, Tang. is 9. 680768; ſo 64 D. 23 M. is the others Compl. whoſe Sine is 9. 955065, and his Tang. is 10. 319231; ſo for any other.

PROP. III.

To find the Secant of any Arch or Angle propounded.

In this little Book I have not room to ſet down the Tables of Artificial Secants at large, as I have done with the Sines and Tangents: Nevertheleſs I will not here omit to ſhew how they may be eaſily found out, by the Tables of Sines. The Method is thus, ſubſtract the Logarithm Sine, of the Sines Compl. of an Angle, from the double *Radius* of the Tables, and the remainder ſhall be the Secant required: As if I deſire the Secant of 25 D. 37 M. I find the Logarithm-Sine of his Complement to be 9. 955065, which ſubſtracted from the double *Radius*, that

is

is 20. 000000, there remains 10. 044935, which is the Secant of it, and ſo the Secant of 64 D. 23 M. is 9. 955065, which is the Complement of the former; becauſe they both are equal to 20. 000000, the double *Radius*, and ſo may any other be found out. § 1.

CHAP. IV.

Of GEOMETRY.

Defin. THE End and Scope of Geometry is to meaſure well: For as Number or Multitude is the Subject of Arithmetick, ſo is Magnitude that of Geometry: To meaſure well, is therefore to conſider the Nature of every thing that is to be meaſured; to compare ſuch like things one with another, and to underſtand their Reaſon and Proportion, and alſo their Similitude: And this is the End and Scope of Geometry *.

* *Quod quæritur cognoſcendi illius gratia, quod ſemper eſt, non & ejus quod oritur, quandoque & interit. Geometria, ejus quod eſt ſemper, Cognitio eſt. Ac tollet igitur (ô Generoſe vir) ad veritatem, animum: atque ita, ad Philoſophandum præparavit cogitationem, ut ad ſupera convertamus: quæ nunc, contra quàm decet, ad inferiora dejicimus*, &c. Plato lib. 7. de Rep.

I ſhall not trouble you with the Definitions of Geometry, they being largely handled by many, and herein every one meanly converſant in the Study of the Mathematicks is acquainted, but ſhall immediately fall in hand with the principal Propoſitions which chiefly concern the other following parts of this Treatiſe.

SECT.

SECT. I.

§ 1. *The Explication of ſome Geometrical Propoſitions.*

PROP. I.

To erect a Perpendicular on any part of a Line aſſigned.

Fig. 1. LET the Line be A, B, and on the Point D, 'tis required to raiſe a *Perpendicular* to A, B: To operate which, firſt open your Compaſſes to any convenient diſtance, and placing one Foot thereof in D, with the other, make the two Marks C and E equidiſtant from D; then open the Compaſſes to ſome other convenient diſtance, and ſet one Foot in E, and deſcribe the *Arch* FF; then likewiſe in C, deſcribe the *Arch* GG, then through the Interſections of theſe two *Arches*, and to the Point D, draw HD *perpendicular* to AB, as was required.

PROP. II.

To erect a Perpendicular on the End of a Line.

Fig. 2. Let the given Line be AB, and on the End thereof at B, 'tis required to raiſe a *Perpendicular* Line: To perform which, open your Compaſſes to any diſtance as BD, then on B as a Center, deſcribe the Arch D, E, F, then from D to E place BD; then placing one Foot in E, deſcribe the Arch CF, then remove your Compaſſes to F, and draw the Arch CE.

Laſtly,

Laſtly, through their Interſection draw CB, which is a *Perpendicular* to AB, on the end B, as required. § 1.

PROP. III.

From a Point above to let fall a Perpendicular on a Line.

Let the Line given be BA, and 'tis required from the Point above at C, to let fall a *Perpendicular* to the ſaid Line: To perform which, place one Foot of your Compaſſes in C, and open them beyond the given Line AB, and deſcribe the Arch EF; divide EF into two parts in D; Laſtly, draw CD, which ſhall be perpendicular unto AB, falling from the Point above at C, as was ſo required. Fig. 3.

PROP. IV.

To draw a right Line Parallel to a right Line, at any diſtance aſſigned.

Let the diſtance aſſigned be OE, and the Line given be AB, and 'tis required to draw CD *Parallel* to AB, at the diſtance OE: To perform which, take in your Compaſſes the diſtance OE, and on A deſcribe the *Arch* H, and on B the *Arch* K; then draw CD, ſo as it may juſtly touch the two *Arches*, but cut them not; ſo ſhall CD be parallel to AB, at the aſſigned diſtance OE, as was required. Fig. 4.

PROP.

§ 1.

PROP. V.

To protract an Angle of any Quantity of Degrees propounded.

Fig. 5. Let it be required to *protract*, or lay down an *Angle* of 40 Degrees: To perform which, firſt draw a right Line as AB, then open your Compaſſes to 60 Degrees, in your Line of *Chords*: And with that Diſtance on A, deſcribe the *Arch* EF; then take 40 Degrees in your Compaſſes out of your Line of *Chords*, and place it on the *Arch*, from F to H. Laſtly, through the Point H, and from A, draw AC; ſo ſhall the *Angle* CAB contain 40 Degrees, as required.

PROP. VI.

To meaſure an Angle already protracted.

Fig. 5. Let the *Angle* given be CAB, and 'tis required to find the Quantity thereof: To perform which, take in your Compaſſes 60 Degrees from your Line of *Chords*; and on A deſcribe the Arch EF, then take in your Compaſſes the Diſtance FH, and apply it to your Line of *Chords*, and you will find the *Angle* CAB to contain 40 Degrees.

PROP. VII.

To divide an Angle into two equal parts.

Fig. 6. Let the *Angle* given be BAC, and 'tis required to divide it into two equal parts: To perform which, do thus; firſt take in your Compaſſes any convenient Diſtance, and placing one

one Foot in A, deſcribe the *Arch* FKHE; § 1.
then on H, deſcribe the *Arch* KK, and on K, the *Arch* HH. Laſtly, through the *Interſections* of theſe *two Arches*, draw the Line AD, to the *Angular Point* A; ſo ſhall the *Angle* BAC, be divided into two equal parts, *viz.* BAD, and DAC, as required.

PROP. VIII.

To divide a right Line into any Number of equal or unequal parts; or like to any divided Line propounded.

Let the Line AB be given to be divided into *Fig.* 7.
5 equal parts, as the Line CD. To perform which, do thus; firſt on the Point C, draw out a Line, making an *Angle* with CD at pleaſure, then make CF equal to AB, and join their Extremities FD; then draw *Parallel Lines* to FD, through all the 5 Points of CD (by the 4 Prop. aforegoing) which ſhall divide AB into 5 equal parts, as required: This way is to be obſerved, when the Line given to be divided is greater than the divided Line propounded.

CASE II.

But if AB be ſhorter than the given divided *Fig.* 8.
Line CD, take the Line AB in your Compaſſes, and on D ſtrike the *Arch* F; then draw the *Tangent* CF, then take the neareſt diſtance from the firſt Diviſion of CD to the *Tangent-Line* CF, which diſtance ſhall divide AB into 5 equal parts, as the given divided Line CD, as required.

Theſe two Propoſitions, well underſtood, do demonſtrate many other Propoſitions; and thereon is grounded the Uſe of the Sector, which is the moſt univerſal Inſtrument for Mathematical Operations.

§ 1.

PROP. IX.

How to protract or lay down any of the Regular Figures, called Polygons.

To perform which, divide 360 Degrees, (the Number of Degrees in a Circle) by the Number of the *Poligon* his ſides: As if it be a *Pentagon* by 5, if a *Hexagon* by 6, *&c.* the Quotient is the Angle of the *Center*; its *Complement* to 180 D. (or a Semi-circle) is the Angle at the *Figure*, half whereof is the Angle of the *Triangle* at the *Figure:* Now I will ſhew how to delineate any *Polygon* three ways, *viz.* 1 by *the Angle at the Center*, 2 *by the Angle at the Figure*, 3 *by the Angle of the Triangle at the Figure:* I have hereunto annexed a Table, which gives at the firſt Sight, (without the trouble of Diviſion) 1. the Quantity of the Angle at the Center; 2. the Quantity of the Angle at the Figure; and 3. the Quantity of the Angle at the Triangle of the Figure, from a Triangle to a Decagon.

Names of the *Polygons.*	Sides	Angles at the Center D M	Angles at the Figure D M	Angles at the Trian. D M
Triangle	3	120 00	60 00	30 00
Square	4	90 00	90 00	45 00
Pentagon	5	72 00	108 00	54 00
Hexagon	6	60 00	120 00	60 00
Heptagon	7	51 $43\frac{1}{2}$	128 $34\frac{1}{2}$	64 $17\frac{1}{4}$
Octogon	8	45 00	135 00	67 30
Nonagon	9	40 00	140 00	70 00
Decagon	10	36 00	144 00	72 00

CON-

§ 1.

Construction I.

First by *the Angle at the Center*, to delineate a *Hexagon*, whoſe Angle at the Center is 60 Degrees; firſt lay down an Angle of 60 Deg. (by Prop. the 5. aforegoing) making its ſides of a convenient Length at pleaſure; then take ſuch a diſtance from O the Center of the Figure, equally on both ſides, as may make the third ſide equal to the ſide of the *Polygon* given, which here is 100 parts: * Then divide the third ſide equally into two equal parts, and draw a Line through it, from ⊙ the Center: Set each half of the ſide of the *Polygon* 100, to wit, 50; on each from the middle of the third Line: † Thus having placed the ſide of the *Hexagon* PP, 100 parts in order; deſcribe the whole *Hexagon* PPPPPP, as was required. Fig. 9.

* *But this in a Hexagon need not be done, becauſe the 3 ſides of the Triangle are equal; but in all other Polygons it muſt be done.*

† *But if the third Line do exceed or be ſhort of the ſide of the Polygon propounded: then by Parallels on each ſide, cut the ſides of the Triangle, till you have found by thoſe Interſections where to ſet the Line propoſed in any Polygon, &c.*

Construction II.

Now by the *Angle of the Figure*, to delineate any regular *Polygon*, let it be required to protract a *Hexagon*, whoſe ſide as afore is 100 parts; firſt I draw a Line, and make it 100 of thoſe parts: Then I find in the precedent Table the Angle of a *Hexagon* at the Figure to be 120 Degrees. Then on each ſide of the drawn Line I lay down an Angle of 120 Deg. (according to the 5 precedent Propoſitions) and ſo work 6 Fig. 10.

§ 1. times, (or as many times as your *Polygon* hath ſides) making each ſide 100 parts, and each Angle 120 Degrees; ſo ſhall you have enclofed the *Polygon* PPPPPP, as required.

CONSTRUCTION III.

Fig. 11. To protract or lay down a *Hexagon*, or any other regular *Polygon*, *by the Angle of the Triangle*, do thus; firſt draw the ſide of the *Hexagon* PP, make it 100 parts. I find in the precedent Table, that the Angle of the Triangle is 60 Deg. then at each end of the Line PP, I lay down an Angle of 60 Deg. (by Prop. 5. precedent) and continue the two Lines PO and PO until they interſect each other in O: Then on O, as a Center (OP being *Radius*) deſcribe a Circle, and within it deſcribe the *Hexagon* PPPPPP, as you ſee in the Figure: And ſo may you delineate any other *Polygon*, whoſe Angles from a *Triangle* to a *Decagon*, are all ſpecified in the precedent Table.

☞ *Obſerve theſe Rules well; for you will find them of infinite Uſe in Fortification*, &c.

PROP. X.

To divide a Line according to any aſſigned Proportion.

Fig. 12. Admit the right Line given to be AB, and 'tis required to divide the ſame into two parts, bearing Proportion the one to the other as the Lines E, and F doth: To perform which, firſt draw the Line CD equal to the given Line AB; then draw the Line HC from C, to contain an Angle at pleaſure. Then from C to G place the

the Line F, and from G to H place the Line E: § 1.
Then draw the Line HD. And laſtly, draw GK parallel to HD, (by the 4 Prop. precedent) ſo is the Line DC equal to AB, and divided into two parts, bearing ſuch Proportion to each other, as the two given Lines E and F, as was required.

PROP. XI.

To two Lines given, to find a third proportional to each of them.

Admit the two given Lines be A and B, and Fig. 13.
'tis required to find a third Proportional to A, as A to B: Firſt make an Angle at pleaſure, as HIK; then place the Line B from I unto P, and the Line A from I unto L, and draw PL; then alſo place the Line A from I unto M, and draw QM, parallel unto LP, (by 4 Prop.) ſo ſhall the Line IQ be a third Proportional unto the two given Lines A and B, as was required. For as B is to A, ſo is A unto the Proportional found IQ.

PROP. XII.

To three Lines given to find out a fourth Proportional unto them.

Admit the three given Lines to be A, B, and Fig. 14.
C; and 'tis required to find a third proportional to them, which ſhall have ſuch Proportion unto A as B hath unto C. To perform which, firſt make an Angle at pleaſure as DKG, now ſeeing the Line C, hath ſuch Proportion to B, as the Line A, unto the Line ſought: There-

§ 1. fore place the Line C from K unto H and B, from K to F, and draw FH. Again, place the Line A from K to I, and draw IE parallel unto FH, (by 4 Prop.) until it cutteth DK in E; ſo have you the Line KE, a fourth Proportional, as was required: For as C is unto B, ſo is A unto the found Line KE.

PROP. XIII.

To find a mean proportional Line between any two right Lines given.

Fig. 15. Let the two given Lines be A and B, between which it is required to find a mean proportional Line. To perform which, firſt joyn the two Lines A and B together, ſo as they make the right Line CED: Then deſcribe thereon a *Semicircle* CFD; then on the Point E erect the *Perpendicular* EF, (by 1 Prop.) to cut the Limb of the *Semicircle* in F, ſo ſhall EF be a mean proportional Line between the two given Lines A and B, as required.

PROP. XIV.

To find two mean proportional Lines between any two right Lines given.

Fig. 16. Let the two given Lines be A and B, between which 'tis required to find two mean Proportionals. To perform which, firſt make an Angle containing 90 Deg. making the ſides CD, and CE of a convenient Length: Then from C place the Line B unto F, and the Line A from C unto G, and draw FG; which divide

And this of all other the Inventions of Plato, Apollonius, Sporus, Architas, Diocles, Nicomedus, *and many other famous Geometricians and Philoſophers, I like beſt for the ready Performance of this Concluſion, whoſe ſeveral Methods I could here deſcribe, but for Brevity ſake do omit them.*

divide equally in H, and deſcribe the *Semicircle* FKG. Then take the Line B in your Compaſſes, and placing one Foot in G, with the other make a Mark in the Limb of the *Semicircle* in K, then draw ST in ſuch ſort that it may juſtly touch the *Semicircle* in K, and may cut through the two ſides of the Angle, equidiſtant from the Center of the *Semicircle* H; ſo ſhall SF and TG be two *mean Proportionals* betwixt the two given Lines A and B, as required. § 1.

PROP. XV.

To make a Geometrical Square equal to divers Geometrical Squares.

Let there be given the 5 ſides of five *Geometrical Squares*, viz. A, B, C, D, E; and 'tis required to make one *Geometrical Square* equal to the ſaid five Squares: To perform which, firſt make a Right Angle as ABC, making its contained ſides of a convenient Length. Then from B place A to D, and from B place B to E, and draw ED; then place ED from B to F, and C from B to G, and draw GF; then place GF from B to H, and D from B to I, and draw IH. Laſtly, from B unto K place IH, and from B unto L, place the Line E, and draw LK. So ſhall LK be the ſide of a Square, equal to the five Squares propounded. *Fig.* 17.

§ 1.

PROP. XVI.

To make a Circle equal to divers Circles propounded.

Fig. 18. Let the two Circles propounded be A and B, and 'tis required to make a third Circle, equal to the ſaid Circles propounded. To perform which, firſt take the Diameter of the leſſer Circle A, and place it as a *Tangent* on the *Diameter* of the greater Circle B, at *right Angles*, as ECD. Then draw the *Diagonal* ED, which divide equally in F, on which as a Center deſcribe the Circle K, making ED the *Diameter*, of which Circle K ſhall be equal unto the two given Circles A and B, as required *.

* *So after the ſame manner, may divers Circles be added into one by the help of the former Propoſition well underſtood.*

SECT. II.

Of Planometry, *or the way to meaſure any plain Superfice.*

PLanometry is that part of the *Mathematicks* derived from that noble Science *Geometry*, by which the *Superficies* or *Planes* of things are meaſured, and by which their *Superficial Content* is found, which is done moſt commonly by the Squares of ſuch Meaſures, *viz. a Square Inch*, *Square Foot*, *Square Yard*, *Square Pace*, *Square Perch*, &c. That is, whoſe ſide is an *Inch*, *Foot*, *Yard*, *Pace*, or *Perch Square*. So that the Content of any Figure is ſaid to be found, when you

you know how many ſuch *Inches*, *Feet*, *Yards*, *Paces*, &c. are contained therein. Thus the End and Scope of *Geometry* is to meaſure well. § 2.

PROP. I.

To find the Superficial Content of a Geometrical Square.

Let the ſide of the Square AA be 4 *Perch*, what is the *Area*, or Superficial Content thereof? To find which multiply its ſide 4, by its ſelf, it produceth 16, which is the Content of that Square AAAA propounded. Fig. 19.

PROP. II.

To find the Superficial Content of a Parallelogram, or long Square.

Multiply the Length in Parts by the Breadth in Parts, the Product is the Content thereof. So in the *Parallelogram*, or long Square ABCD, the Length of the ſide AB, or CD, is 20 *Paces*, and the Breadth AC, or BD, is 10 Paces, and his Superficial Content is required. I ſay therefore if according unto the Rule, you multiply the Length 20 by the Breadth 10, it produceth 200 Paces, which is the Content of the *Parallelogram* or long Square ABCD. Fig. 20.

PROP.

§ 2.

PROP. III.

To find the Superficial Content of any Right-lined Triangle.

Fig. 21. Although right-lined Triangles are of ſeveral Kinds and Forms; as firſt in reſpect unto their Angles, they are either *Right-angled*, or *Oblique-angled*, i. e. *Acute-angled*, or *Obtuſe-angled*. Secondly, in reſpect of their ſides, they are either an *Equilateral*, *Iſoſceles*, or *Scalenium Triangle*: But ſeeing they are all meaſured after the ſame manner, I ſhall give one general Rule, which *The Rule.* is this: *Multiply the Length of the Baſe by the Length of the Perpendicular; the half of this Product is the Area or Superficial Content.* So if the Content of the *Triangle* ABC be required: To find which, firſt from the Angle B let fall the *Perpendicular* DB, on the *Baſe* AC, (by Prop. 3. § 1.) let therefore the Length of the *Perpendicular* BD be 24, and the *Baſe* AC 44 parts. Now if the *Baſe* AC 44 were multiplyed by BD 24, the Product is 1056, half whereof is 528, the Content of the *Triangle* ABC, propounded.

PROP. IV.

To find the Superficial Content of a Rhombus.

Fig. 22. Firſt let fall a *Perpendicular* from one of the *Obtuſe-angles* unto its oppoſite ſide, (by Prop. 3. § 1.) and then multiply the Length of the ſide thereof by the Length of the *Perpendicular*, their Product is the Content thereof.

So

So in the *Rhombus* ABCD, the ſide AC, or BD is 16 *Inches*, and the *Perpendicular* KC is 14 *Inches*, which multiplied into the ſide 16, produceth 224 *Inches*; which is the *Area*, or Superficial Content of the *Rhombus* ABCD, propounded. § 2.

PROP. V.

To find the Superficial Content of a Rhomboides.

Firſt let fall a *Perpendicular*, as in the former Propoſition, then the Length thereof multiply by the Length of the *Perpendicular*; the Product is the *Area*, or Superficial Content thereof. For in the *Rhomboides* EDAH, whoſe Length AH, or ED is 32 *Feet*, and the Length of the *Perpendicular* HK is 16 *Feet*, which multiplyed together produceth 512 *Feet*, which is the *Area* or Superficial Content of the *Rhomboides* AHED, propounded. Fig. 23.

PROP. VI.

To find the Superficial Content of any Polygon, or many equal ſided Superficies.

Fig. 94. Firſt, from the *Center* unto the middle of either of the ſides of the *Polygon* let fall a *Perpendicular*, (by 3. Prop. § 1.) then multiply the Length of half the *Periphery* by the *Perpendicular*, the Product ſhall be the *Superficial Content* of the *Polygon*. Fig. 24.

Admit the *Polygon* to be an *Hexagon* AAAAAA, whoſe ſide AA is 22 *Feet*, and the *Perpendicular* BE 19 *Feet*; now, if 66, half the *Periphery*,

§ 2. *riphery* be multiplied by 19, it produceth 1254 *Feet*; which is the *Content* of the *Polygon* AA, *&c.* as required.

PROP. VII.

To find the Area or Superficial Content of an irregular Figure or Polygon.

This and the like Figures are practically meaſur'd by dividing it into Triangles; which done, *meaſure each Triangle ſeverally, and add the ſeveral Contents together, ſo will the Summ be the Content of the ſaid Figure.*

Suppoſe the Dimenſions of an irregular Figure are as in Plate 94 what's the Content?

Work according to the preceding Direction, and you ſhall find 2375 Foot 6 Inches.

PROP. VIII.

To find the Superficial Content of a Circle.

Fig. 25. Multiply half the *Circumference* by one half of the *Diameter*, their Product is the *Superficial Content* thereof.

Admit the *Circumference* of a *Circle* ACBD be 44 *Inches*, what is the *Area* or *Content* thereof? (by the 94 Prop. § 2.) I find the *Diameter* to be 14 *Inches*, therefore I ſay if 22, half the *Circumference*, be multiplied by 7, half the *Diameter*, it ſhall produce 154 *Inches*; which is the *Superficial Content* of the *Circle* ACDB, as required.

PROP.

§ 2.

PROP. IX.

By the Diameter of a Circle given, to find the Circumference.

Suppoſe the *Diameter* be 14, what is the *Circumference?* The *Analogy* or *Proportion* holds thus, as 7 to 22, ſo is 14 unto 44, the *Circumference* required.

PROP. X.

By the Circumference of a Circle given, to find the Diameter.

Suppoſe the *Circumference* of a *Circle* be 44, what is the *Diameter?* The *Analogy* or *Proportion* is as 22 to 7, ſo is 44 unto 14, the *Diameter* required.

But here note, Theſe Proportions of 7 to 22, and the like, are courſe, and fit only for rough Calculations: And therefore where Exactneſs is required, the following Proportions muſt be uſed; which is,

As 113 :: 355, or as 1 : 3. 141592, *&c.* ſo is the *Diameter* to the *Circumference.*

The firſt of theſe is *Adrianus Metius* his Proportion, being the neareſt in feweſt Figures; the ſecond is *Archimedes*'s Numbers, and is very convenient, by reaſon the Labour of Diviſion is ſaved, the firſt Term being Unity.

PROP.

§ 2.

PROP. XI.

By the Content of a Circle given, to find the Circumference.

Suppoſe the Content of a *Circle* be 154; what is the *Circumference*, the *Analogy*, or *Proportion?*

As 7 unto 4 times 22, which is 88, ſo is 154 the Content of the given *Circle*; to the Square of the *Circumference* 1936, whoſe Root being extracted, as is taught (in Prop. 8. § 1. Chap. 1.) gives the *Circumference* 44, as required.

PROP. XII.

By the Content of a Circle given, to find the Diameter.

Suppoſe the *Superficial Content* of a *Circle* be 154 parts, what is the *Diameter* thereof? To find which, this is the *Analogy* or *Proportion.*

As 22,
To 4 *times* 7, *which is* 28,
So is 154, *the given Content*,
To the *Square of the Diameter* 196, whoſe *Root* being extracted (by 8 Prop. chap. 1. § 1.) giveth the *Diameter* 14, as required.

PROP. XIII.

By the Diameter of a Circle given to find the ſide of a Square equal thereto.

To find which, this is the *Analogy* or *Proportion.*

As

As 1. 000000, § 2.
To 0. 886227,
So is the *Diameter of the Circle propounded,*
To the ſide of a Square, whoſe *Superficial Content*, is equal unto the *Superficial Content* of the *Circle* propounded.

PROP. XIV.

By the Circumference of a Circle given, to find the ſide of a Square equal to it.

This is the *Analogy*, or *Proportion.*
As 1. 000000,
To 0. 282093,
So is the Circumference of the Circle propounded, *to the ſide of a Square equal to the Circle.*

PROP. XV.

By the Content of a Circle given, to find the ſide of a Square equal to it.

To do which, extract the *Square-Root* of the Content propounded, (by Prop. 8. Chap. 1. §. 1.) ſo is the *Root* the ſide of a *Geometrical Square*, equal thereunto.

PROP. XVI.

By the Diameter of a Circle given, to find the ſide of an inſcribed Square.

This is the *Analogy*, or *Proportion.*
As 1. 000000,
To 0. 707107,

So

§ 2. *So is the Diameter of the Circle* propounded, *to the ſide of the inſcribed Square.*

PROP. XVII.

By the Circumference of a Circle given, to find the ſide of an inſcribed Square.

This is the *Analogy*, or *Proportion.*
As 1. 000000,
To 0. 225079,
So is the Circumference of the Circle propounded, *to the ſide of the inſcribed Square.*

PROP. XVIII.

To find the Superficial Content of an Oval, or Elleipſis.

Fig. 26. Let the *Oval* given be ABCD, and 'tis required to find the *Area* or *Superficial Content* thereof: To do which, multiply the Length AB 40 *Inches*, by the Breadth CD 30 *Inches*, the *Product* is 1200. Which divide by 1.27324; the *Quotient* is 942 $\frac{48}{100}$ parts. Which is the *Area* or *Superficial Content* of the *Oval* ABCD propounded.

PROP. XIX.

To find the Superficial Content of any Section, cut from a Circle, by Lines drawn from the Center.

Multiply half the *Circuit* of the Section, by the *Semidiameter* of the whole *Circle*, and the Product thence ariſing is the *Area* or *Superficial Content* thereof.

Sup-

Suppoſe there be a *Circle* whoſe *Diameter* is 14 parts, and the *Circuit* of the *Quadrant* ABC is 11 parts, and the Content of the ſaid *Quadrant* is deſired: To find which, multiply $5\frac{1}{2}$ or 5.5 half the *Circuit* of the *Quadrant* by 7 the *Semidiameter*, the Product is $38\frac{1}{2}$, which is the Content of the *Quadrant* ABC propounded. § 2. Fig. 27.

PROP. XX.

To meaſure a Parabola, *which is a Figure bounded by two Lines; one of which is curve, the other ſtrait: It is generated by cutting a Cone with a plain Parallel to the ſide thereof.*

The Rule for meaſuring it is, to multiply the Line AB by the Line DC, and the Product again by 2. This laſt Product divide by 3, the Quote is the Area of the *Parabola:* As

Suppoſe the Breadth of a *Parabola* be 29 Foot, and the Length or Height 45 Foot, what's the Area? Work according to the preceding Direction, and you ſhall find 870 Foot 21 Inches, to be the Area. fig: 95

SECT. III.

Of Stereometry: *Or the way how to meaſure any Regular Solid.* § 3.

Stereometry is that part of the *Mathematicks*, ſpringing from *Geometry*, by which the Content of all *ſolid Bodies* are diſcovered by two *Multiplications* or three *Dimenſions*, and is va-

§ 3. lued by the *Cube* of ſome famous Meaſure; as an *Inch-Cube*, a *Foot-Cube*, a *Yard-Cube*, or a *Perch-Cube*, &c.

PROP. I.

To find the ſolid Content of a Cube.

Multiply the ſide into its ſelf, and that Product by its ſide again; their Product is the ſolid Content thereof.

Fig. 28. Suppoſe there be a *Cube* A, whoſe ſide is 2 Feet, and his ſolid Content is required: I ſay, if his ſide 2 be multiplied by its ſelf, it produceth 4; which again multiplied by 2, it produceth 8 Feet, which is the ſolid Content of the *Cube* propounded.

PROP. II.

To find the ſolid Content of a Parallelopipedon.

Firſt get the *Superficial Content* of the End, (by Prop. 1, or 2, § 2.) which multiply into the Length, the Product is the *ſolid Content*.

Fig. 29. Suppoſe there be a *Parallelopipedon* B, whoſe ſides of the *Baſe* is 40 and 30 *Inches*, and Length 120 *Inches*, and his *ſolid Content* is demanded: I ſay if you multiply 30 by 40, the Product is 1200, which is the *ſuperficial Content* at the *Baſe*; Which multiplied by the Length, 120 Inches produceth 144000 Inches, which is the *ſolid Content* of the *Parallelopipedon* B, propounded.

PROP.

§ 3.

PROP. III.

To find the solid Content of a Cylinder.

First get the *superficial Content* of the Circle at the *Base*, (by Prop. 8. § 2.) and by that multiply its Length, the Product is the *solid Content.*

Suppose there be a *Cylinder*, as D, whose *Diameter* of the *Circle* at the *Base* is 7 parts, and the Length of the *Cylinder* is 14 parts, and 'tis required to find the *solid Content* thereof: First I find the *superficial Content* of the *Base* to be 38.5, which multiplied into 14 the Length, giveth 539 parts, which is the *solid Content* of the *Cylinder* propounded. Fig. 30.

PROP. IV.

To find the solid Content of a Pyramid.

First get the *superficial Content* of the *Base* of the *Pyramid*, (by some of the aforegoing Propositions in *Planometria*) and then multiply that into $\frac{1}{3}$ of his *Altitude*, the Product is the *solid Content* thereof.

Suppose there be a *Pyramid* H, whose side of the *Base* is $4\frac{1}{2}$ parts, or 4.5, and his *Altitude* 12 parts, and his *solid Content* is required: First I find (by Prop. 1. § 2.) the *superficial Content* of the *Base* to be $20\frac{1}{4}$ or 20.25; which multiplied by 4, (which is $\frac{1}{3}$ of the *Altitude* 12) produceth 81 parts, for the *solid Content* of the *Pyramid* propounded. Fig. 31.

§ 3.

PROP. V.

To find the ſolid Content of a Cone.

Firſt find the *ſuperficial Content* of the *Circle* at the *Baſe*, (by Prop. 7. § 2.) then multiply it by $\frac{1}{3}$ of its *Altitude* or Heighth, the Product is the *ſolid Content* thereof.

Fig. 32. Suppoſe there be a *Cone* as B, whoſe *Diameter* of the *Baſe* is 7, and his *Altitude* or Heighth is 15 parts, and his *ſolid Content* is required: Firſt, I find the *ſuperficial Content* of the *Baſe* to be $38\frac{1}{2}$ or 38. 5; which multiplied into 5, ($\frac{1}{3}$ of its *Altitude* or Heighth) produceth 192. 5, or $\frac{5}{1}$, which is the *ſolid Content* of the *Cone* propounded.

PROP. VI.

By the Diameter of a Globe to find his ſolid Content.

This is the *Analogy*, or *Proportion.*

As 6 *times* 7, *which is* 42,

Is to 22;

So is the Cube of the Diameter of the Sphere, or Globe propounded,

To the ſolid Content thereof.

Suppoſe there be a *Sphere* or *Globe*, whoſe *Diameter* is 12 Inches, what is the *ſolid Content* thereof? Say, (ſee the *Globe* R.)

As 42,

Fig. 33. *Is to* 22,

So is 1728, *the Cube of the Diameter,*

To

To the ſolid Content 905 $\frac{6}{11}$, or $\frac{1}{7}$ *of the Globe,* §. 3.
or Sphere propounded; This, and all other ſuch like Propoſitions, are performed by the help of the firſt Propoſition of the firſt Chapter of this Book.

☞ *Note, That every Sphere is equal unto two Cones, whoſe Height and Diameter of the Baſe is the ſame with the Axis of the Sphere: And a Sphere is two thirds of a Cylinder, whoſe Height and Diameter of the Baſe is the ſame with the Axis of the Sphere, according to the 9th Manifeſtation of the firſt Book of* Archimedes *of the Sphere and Cylinder; that is, the Cylinder having the Sphere inſcribed, and the Cone inſcribed are as 3. 2. 1.*

PROP. VII.

By the Circumference of a Sphere, or Globe, to find his ſolid Content.

This is the *Analogy*, or *Proportion.*
As 1. 000000,
To 0. 016887,
So is the Cube of the Circumference of the Globe, or Sphere propounded,
To the ſolid Content thereof.

PROP. VIII.

By the Axis of a Globe, to make a Cube equal thereunto.

This is the *Analogy*, or *Proportion.*
As 1. 00000,
To 0. 80604,
So is the Axis of the Sphere propounded,
To the *Cube-Root*, which ſhall be equal to it.

§. 3.

PROP. IX.

By the Circumference of a Globe, to make a Cube equal thereunto.

This is the *Analogy*, or *Proportion*.
As 1. 000000,
To 0. 256556,
So is the Circumference of the Globe propounded,
To the *Cube-Root*, which shall be equal to the *Sphere* or *Globe* propounded.

PROP. X.

By the solid Content of a Sphere or Globe, to make a Cube equal thereunto.

Extract the *Cube-Root* of the *solid Content* of the *Sphere* or *Globe*, (by Prop. 9. § 1. Chap. 1.) so shall the Root, so found, be the side of a *Cube* equal unto the *Globe* or *Sphere* propounded.

PROP. XI.

A Segment of a Sphere being given to find the solid Content thereof.

To find which, first say, *As* the *Altitude* of the other *Segment* is to the *Altitude* of the *Segment* given, so is that *Altitude* of the other *Segment* increased by half the *Axis* unto a *fourth*: Then say, *As* 1, *to* 1, 0472, *so is the Product of the Quadrant of half the Chord of the Circumference of that Segment, multiplied by that fourth, to the solid Content of the Segment propounded.*

CHAP.

CHAP. V.

Of TRIGONOMETRY: *Or the Doctrine of* Triangles.

SECT. I.

Some General Maxims *belonging to Plain, or Right-lined* Triangles. § 1.

TRIGONOMETRY is neceſſary in moſt parts of the *Mathematicks*, and herein indeed conſiſteth the moſt frequent Uſe of the *Logarithms*, *Sines*, *Tangents*, and *Secants*: It is converſant in the meaſuring of *Triangles*, *Plain* or *Spherical*; comparing their *Sides* and *Angles* together, according unto their known *Analogies*, or *Proportions*: So that any three parts of a *Triangle* being given, the three *Angles* excepted, the other parts may be found out and known. Now in the *Doctrine* of *Right-lined Triangles*, it will be neceſſary to know theſe *Maxims* following.

1. That a *Right-lined Triangle* is a Figure conſtituted by the Conjunction, or Interſection, of the three Right or Streight-lines in their Angles or Meeting-places: So that every *Triangle* hath ſix diſtinct Parts, *viz. Three Sides*, and *Three Angles*.

§ 1. 2. That all *Right-lined Triangles* are either *Right-angled*, that is, which hath one *Right-Angle*, as ABC Fig. 34. *Or Oblique-angled*, whose three *Angles* are all *Acute*; that is, less than a *Quadrant*, or 90 Deg. or else they have *One Angle Obtuse*, or greater than a *Quadrant*: So all *Triangles* that have not one *Right-Angle*, are called *Oblique-Triangles*; as Fig. 36. to wit, the *Triangle* ABC.

Fig. 34.

Fig. 36.

3. That the three *Angles* of any *Right-lined Triangle*, are equal unto two *Right-Angles*; or 180 *Degrees*: So that any two of their *Angles* being known, the third *Angle* is also found, being the *Complement* of the other two, unto 180 *Degrees*; but this is more readily found in a *Rectangled Triangle*; for the *Rectangle* being a *Quadrant*, or 90 Degrees, one of the acute *Angles* therefore being given, the other is readily known, being the *Complement* thereof unto 90 Degrees.

Fig. 35. 4. That the three sides, comprehending the *Triangle*, some call *Legs*, others *Sides*; but in *Rectangled Triangles*, as in the *Triangle* ABC, I call AB the *Base*, BC the *Cathetus*, or *Perpendicular*, and AC the *Hypothenuse*.

5. That the *Sines* of the *Angles* are proportional unto their opposite *Sides*, and their *Sides* to their opposite *Angles*: So that if the *Side* of a *Triangle* were desired, put the *Sine* of the opposite *Angle* in the first place. Also if an Angle be required, put the *Logarithm* of his opposite side in the *first place*.

6. That the *Angles* of any plain *Triangle* are measured by a *Scale* of *Chords*; but the *Sides* of any plain *Triangle* are measured by any

Scale

Scale of equal Parts; as *Inches*, *Feet*, *Yards*, *Poles*, *Miles*, *Leagues*, &c. § 1.

7. That if an *Angle* propounded be greater than 90 Deg. and ſo not to be found in the Tables, take the *Complement* thereof unto 180 Deg. and work by the *Sine* or *Tangent* thereof, and the Work will be the ſame.

And here, for the more ſhort and ſpeedy Performance of theſe Concluſions in *Trigonometry*, I have annexed and uſed theſe following *Symbols*, which I would have you take notice of,

= *Equal, or Equal to.*	X *Difference.*
+ *More.*	S *Sine.*
− *Leſs.*	Sc *Co-ſine.*
x *Multiplied by.*	T *Tangent.*
° *Degrees, as* 15°.	Tc *Co-tangent.*
′ *Minutes, as* 40′.	Se *Secant.*
cr. *A Side.*	Sec *Co-ſecant.*
crs, *Sides.*	Co. Ar. *Comp. Arithmetic.*
V An Angle.	R *A Right-angle.*
VV Angles.	2 R *Two Right-angles.*
Z *Sum.*	Q *Square.*

SECT.

SECT. II.

§ 8. ### *Of Plain Rectangled* Triangles.

PROP. I.

Two Angles and the Baſe of a Rectangled Triangle given, to find the other parts.

Fig. 34. ADmit the *Triangle* given be ABC: Now the *Angle* at B is an *Angle* of 90°, or a *Right-angle*; and the *Angle* at C is 57° 35′, and the *Baſe* AB is 736 parts.

Now firſt I find the *Angle* at A to be 32° 25′, it being the *Complement* of the *Angle* at C unto 90°: Secondly, to find the *Cathetus* or *Perpendicular*; this is the *Analogy*, or *Proportion*.

As S, *V*. C, 57° 35′ ——— 9. 92643

To Log. AB. *Baſe* 736 *parts* ——— 2. 86687
So is S. *V*. A. 32° 25′ ——— 9. 72922

12. 59609

To Log. of the Cathetus, or Perpendicular BC 467 $\frac{4}{10}$ *parts.* ——— 2. 66966

Obſerve this for a general Rule in Trigonometry.

Add the *Log.* of the third and ſecond *Terms* together, and from their Sum deduct the *Log.* of the firſt Number, ſo is the Remainder the *Log.* of the fourth *Term*, or Number ſought, as you ſee in the aforegoing *Example*.

Thirdly, to find the *Hypothenuſe* AC, the *Analogy* or *Proportion* holds thus.

As S. *V*, C 57° 35′,

To

To Log. Base AB 736 *parts*; §. 2.
So Radius or S. 90°, Fig. 34.
To Log. Hypothenuse AC 871$\frac{1}{10}$ *parts required.*
Thus are the three required parts of the given *Triangle* ABC found, *viz.* the *Angle* A to be 32° 25′, the *Cathetus* BC to be 467$\frac{1}{10}$ parts, and the *Hypothenuse* AC to be 871$\frac{1}{10}$ parts, as was so required to be found.

PROP. II.

The Hypothenuse, Base, and one of the Angles of a Rectangled Triangle given, to find the other parts thereof.

In the *Triangle* ABC the *Hypothenuse* AC is 871$\frac{1}{10}$ parts, the *Base* AB is 736 parts, and the *Angle* at B is known to be a *Right-angle*, or 90°: *First*, to find the *Angle* at the *Cathetus* C, the *Analogy* or *Proportion* holds thus.
As Log. Hypothen. AC 871$\frac{1}{10}$ *parts* Fig. 34.
To Radius or S. 90°.
So Log. Base AB 736 *parts*,
To the S. *V. at Cathetus* C 57° 35′.
Secondly, Now having found the *Angle* at the *Cathetus* C, to be 57° 35′, I say the *Angle* of the *Base*, A, is 32° 25′, being the *Compl.* of the *Angle* C unto 90°.
Thirdly, to find the *Cathetus* BC, this is the *Analogy* or *Proportion*.
As Radius or S. 90°,
To Log. Hypothen. AC 871$\frac{1}{10}$ *parts*,
So S. *V. at Base* A 32° 25′. Fig. 34.
To Log. Cathetus BC, 467$\frac{1}{10}$ *parts required.*
It may also be found, as in the former Proposition.

PROP.

§ 2.

PROP. III.

In a Rectangled Triangle, the Baſe, and Cathetus *given to find the other parts thereof.*

In the Triangle ABC *the Baſe* AB is 736 parts, and the *Cathetus* BC is 467 $\frac{4}{10}$ parts, and the Angle B between them is a *Right Angle*, or 90°; and here you may make either ſide of the Triangle, *Radius*: But I ſhall make BC the *Cathetus Radius*, and then to find the Angle at the *Cathetus* C, this is the *Analogy* or *Proportion.*

Fig. 35. *As Log. Cathet.* BC. 467 $\frac{4}{10}$ *parts,*
To Radius or S. 90°.
So Log. Baſe AB 736 *parts*;
To T. *V. Cathet.* C 57° 35′, as *required.*

Secondly, I find the other Angle at A to be 32° 25′, it being the *Complement*, to C 57° 35′, unto 90°.

Thirdly, To find out the *Hypothenuſe* AC, this is the *Analogy* or *Proportion.*

As S. *V. Cathet.* C 57° 35′,
To Log. Baſe AB 736 *parts*;
So Radius or S. 90°,
To Log. Hypothenuſe AC 871 $\frac{8}{10}$ *parts, required*: But making the *Baſe* AB *Radius*, you may find the *Hypothenuſe* AC by this *Analogy* or *Proportion.*

As Radius or S. 90°,
To Log. Baſe AB 736 *parts*;
So Sc. *V. Baſe* A 32° 25′,

Fig. 35. *To Log. Hypothenuſe* AE 871 $\frac{8}{10}$ *parts required*; and thus you have all the parts of the Triangle propounded.

PROP.

§ 2.

PROP. IV.

The Baſe and Hypothenuſe, with the Angle between them given, to find the other parts of a Rect-angled Triangle.

In the Triangle ABC the *Baſe* AB is 736 parts, and the *Hypothenuſe* AC is 871 $\frac{4}{10}$ parts, and the Angle A included between them is 32° 25′. *Firſt*, To find the Angles: And firſt remember, that the Angle B is a right Angle, or 90°. *Secondly*, That the Angle at C is the *Complement* to the Angle at A 32° 25′ unto 90°; and therefore is 57° 35′. Now theſe being known, you may find the *Cathetus* by this *Analogy* or *Proportion*. Fig. 35.

As S. *V. Cathet.* C 57° 35′,
To Log. Baſe AB 736 *parts*;
So S. *V. Baſe* A 32° 25′,
To Log. Cathet. BC 467 $\frac{4}{10}$ *parts required.*

Thus I have ſufficiently explained all the *Caſes* of *Plain Rect-angled Triangles*; for to theſe Rules they may be all reduced.

SECT.

SECT. III.

§ 3. *Of Oblique-Angled Plain Triangles.*

PROP. I.

Two Angles, and a Side opposite, in an Oblique-Angle Triangle given, to find the other parts thereof.

IN the Triangle ABC the Angle at A is 50°, and at C is 97°, and the side AB is 30 parts, and opposite to the Angle C.

First, To find the Angle B, remember that (as 'tis said, in the third *Maxim* aforegoing) 'tis the *Complement* to the Angles A 50°, and C 37° to 180°, and therefore is the Angle at B 93°.

Fig. 36. *Secondly*, Having thus found the Angles, the two unknown sides may be found by the Proportion they bear to their opposite Angles; for that Proportion holds alſo in theſe. Thus to find the side BC, this is the *Analogy* or *Proportion.*

As S. *V.* C 37° 00′,
To Log. side AB 30 *parts*;
So S. *V.* A. 50° 00′,
To Log. side BC 38 $\frac{19}{100}$ *parts required to be found.*

But it may be more readily found, and performed in ſuch Caſe as this, where you have a *Sine* or *Tangent* in the firſt place, by the *Arithmetical Complement* thereof, and ſo ſave the *Subſtraction.*

Now

§ 3.

☞ *The Rule to find the Complement Arithmetical of any Logarithm Number.*

Now the readieſt way to find the *Arithmetical Complement* is that of Mr. *Norwood*, in his *Doctrine of Triangles*; which is thus: Begin with the firſt Figure towards the left hand of any *Number*, and write down the *Complement*, or the Remainder thereof, unto 9. And ſo do with all the reſt of the *Figures*, as you ſee here done; ſaying 9, wants of 9, 0: And again 9, wants 0; 6, wants 3; 2, wants 7; 3, wants 6; 9, wants 0: Only when you come to the laſt Figure to the right hand, take it out of 10; ſo 8 wants 2 of 10. Thus you may readily find the *Co-Ar.* of any *Sine*, almoſt as ſoon as the *Sine* it ſelf.

9.	962398
0.	037602

But if you want the *Complement Arithmetical* of any *Tangent*, you may take the *Co-tang.* which is exactly the *Co-Arith.* of the double *Radius*; ſo that the *Tangent* and *Co-tangent* of an *Arch* makes exactly 20. 000000.

Now if the *Radius* be in the firſt place, then there is no need of taking the *Co-Arith.* of the firſt Number, only you muſt cut off the firſt 1, to the left hand thus 1, and you will have the *Logarithm* of the Number deſired.

Thirdly, Now to find the Side AC by the oppoſite Angle B, which is 93° 00'. (And ſeeing the Angle B exceeds 90°, you muſt work by the *Complement* to 180°) as in the ſeventh Work in page 61 is taught.

§ 3.

Co-Arith.

As S. *V.* C. 37° 00'. ——— 0. 22054.
To Log. cr. AB 30 *parts* ——— 1. 47712.
Fig. 36. *So is* Sc. *V.* B. 87° 00', ——— 9. 99940.

To Log. cr. AC 49 $\frac{78}{100}$ *parts.* ——— 11. 69706.

Thus having found all the parts of the *Triangle* propounded, *viz.* The Angle B to be 93° 00', the ſide AC to be 49 $\frac{78}{100}$ parts, and the ſide BC to be 38 $\frac{12}{100}$ parts, as was required to be found.

NOTE, 'Twill be doubtful ſometime, whether the V ſought be acute or obtuſe; tho' in the Operation there will always come out the Sine of an acute Angle, as in the immediate preceding Operation, the *Log-ſine* that came out was but 87 Deg. where the Angle C was really 93 Deg. Now when it happens that the Angle is within a Degree or leſs of 90 Deg. the Eye cannot diſtinguiſh whether 'tis more or leſs than a right Angle: And therefore, in ſuch Caſe, to know which it is, take this general Rule. *See if the Square of the Side ſubtending the doubtful Angle exceed the Sum of the other two Squares, then 'tis obtuſe: If they be equal, 'tis right; if the Square of the ſaid Side be leſs than the Sum of the Squares of the other two Sides, the Angle is acute.*

PROP.

§ 3.

PROP. II.

Two ſides, and an Angle oppoſite to one of them in an Oblique-angled Triangle given, to find the other parts thereof.

In the Triangle ABC the ſide AB is 30 parts, and the ſide AC is $49\frac{78}{100}$ parts, and the oppoſite Angle C, is 37° 00′.

Firſt, To find the Angle at B, this is the *Analogy* or *Proportion*. *Fig.* 36.

As Log. cr. AB 30 *parts*,
To S. V. at C 37° 00′;
So Log. cr. AC $49\frac{78}{100}$ *parts*;
To Sc. *V.* B 93° 00′, *as was required to be found.*

Now ſeeing that the Angle C is 37° 00′, and the Angle B is 93° 00′, which makes 130° 00′, therefore muſt the Angle A be 50° 00′; the *Complement* to 180°: So having found all the three Angles, you may find the other ſide CB $38\frac{19}{100}$ parts, as afore in the firſt Propoſition, by his oppoſite Angle.

PROP. III.

Two Sides of an Oblique-angled Triangle, with the Angle included between them given, to find the other parts thereof.

In the Triangle ABC, the ſide AC is $49\frac{78}{100}$ parts, the ſide DB is 30 parts, and the Angle A between them is 50° 00′; and 'tis required to find the other parts of the Triangle propounded. To reſolve this Concluſion, let fall

§ 3. a *Perpendicular* DB, from the Angle B, on the ſide AC; (by Prop. 3. § 1. Chap. 4.) and then proceed thus:

Fig. 37. *Firſt*, Seeing the *Oblique-angled Triangle* ABC is divided into two *Rectangled Triangles*, viz. ADB and BDC. Now I will begin with the Triangle ADB, in which is given the Angle A 50° 00′, and the Angle D is a right Angle, or 90°, and the ſide AB 30 parts, and the ſides AD and DB, and the Angle at B, are required.

Firſt to find the Angle at B, remember that it is the *Complement* unto the Angle A 50° 00′, unto 90° 00′, and therefore muſt the Angle B be 40° 00′: Now for to find the *Cathetus* BD, (as in Prop. 1. and 2. § 2. Chap. 5.) by the Rule of Oppoſition, the *Analogy* or *Proportion* holds thus:

As Radius or S. 90°,
To Log. Hypoth. AB 30 *parts*;
So S. *V.* at A 50° 00′,
To Log. Cath. BD $22\frac{98}{100}$ *parts ſought.*

And again ſay,

As Radius or S. 90°,
To Log. Hypoth. AB 30 *parts*;
So S. *V.* at B 40° 00′,
To Log. Baſe AD $19\frac{28}{100}$ *parts ſought.*

Thus in the Triangle ADB, you have found the *Angle* B to be 40° 00′, the *Cathetus* BD to be $22\frac{98}{100}$ parts; and the *Baſe* AD to be $19\frac{28}{100}$ parts, as was required.

Now for the other Triangle, which is BDC, in which there is given the ſide BD, $22\frac{98}{100}$ parts, and

and the Angle at D is a *Right-angle*, or 90°, § 3.
and the sides DC and CB, and the Angles B and C, are required.

Firſt to find the ſide DC ſubſtract AD 19$\frac{11}{100}$ Fig. 37.
parts, out of AC 49$\frac{71}{100}$ parts; there remains the *Baſe* DC, 30$\frac{16}{100}$ parts: Thus have you the two ſides of the Triangle, to wit, the *Baſe* DC, 30$\frac{16}{100}$ parts, and the *Cathetus* BD 22$\frac{92}{100}$ parts, and the Angle D between them is a *Right-angle* or 90°. Now you may find the Angle at B by the *Tangent* (as in Prop. 3. § 2. Chap. 5.) thus

As Log. Cath. BD 22$\frac{92}{100}$ parts,
To Radius or S. 90°;
So Log. Baſe CD 30$\frac{16}{100}$ parts,
To T. V. B. 53° 00′.

Secondly, For the Angle C, remember 'tis the *Complement* of the Angle B, 53° to 90°; and therefore is the Angle C 37° 00′, required.

Thirdly, To find the *Hypoth.* BC, this is the *Analogy* or *Proportion.*

As S. *V.* B 53° 00′,
To Log. Baſe DC 30$\frac{16}{100}$ parts;
So Radius or S. 90°,
To Log. Hypoth. BC 38$\frac{12}{100}$ parts: Thus have you found all the required parts of the Triangle ABC propounded, *viz.* the Angle C to be 37° 00′, the Angle B to be 93° 00′, * and the *Side* BC 38$\frac{12}{100}$ parts, as required to be found.

* *That is equal unto the two Angles B 40° and B 53° as afore in the former Propoſition.*

§ 3. *Another way to perform the same.*

Side. AB $30\frac{98}{100}$
Side. AC $49\frac{78}{100}$
Sum. $79\frac{78}{100}$
Difference. $19\frac{78}{100}$

Take the Sum of the two sides, and the difference of the two sides; and work as followeth.

Now to find the two Angles B and C, this is the Manner, and by this *Analogy* or *Proportion*, they are found out and known.

Fig. 37.

As Log. Z. cr'. AB and CA, $79\frac{78}{100}$ *parts*,
To Log. X. cr'. AB and CA, $19\frac{78}{100}$ *parts*;
So T. *of* $\frac{1}{2}$ *VV unknown*, 65° 00',
To T. $\frac{1}{2}$ X. of *VV*, 28° 00'.

This Difference of Angles 28° 00', add unto 65° 00', (half the [illegible] of the unknown Angles) and it shall produce 93° 00', which is the greater Angle, and subtracted from it, leaves 37° 00', which is the lesser Angle C, so have you the required Angles.

Summ

PROP. IV.

The three sides of an Oblique-angled Triangle given, to find the Angles.

In the Triangle ABC, the side AC is $49\frac{78}{100}$ parts, the side AB is 20 parts, and the side BC, is $38\frac{19}{100}$ parts; and the three Angles of the Triangle are required.

Fig. 37.

Side. AB $20\frac{00}{00}$
Side. BC $38\frac{19}{100}$
Sum. $68\frac{19}{100}$
Diff. $08\frac{19}{100}$

The Resolution of this Conclusion is thus: take the *Summ* and *Differ.* of the two sides AB and BC; and then work as follows: To find a *Segment* of the *Base* AC, to wit, CE; say, *As*

As Log. Base AC 49$\frac{78}{100}$ *parts,* § 3.
To Z. cr. AB and BC 68$\frac{19}{100}$ *parts;*
So X. cr. AB and BC 8$\frac{19}{100}$ *parts,*
To Log. of a Segment of the Base AC, *to wit,* CE 11$\frac{22}{100}$ *parts.*

This *Segment* of the *Base* CE 11$\frac{22}{100}$ parts, being subſtracted from the whole *Base* AC 49$\frac{78}{100}$ parts, the remainder is EA 38$\frac{56}{100}$ parts, in the middle of which as at D, the Perpendicular DB will fall from the Angle B; and ſo divide it into two *Rectangled Triangles*, to wit, ADB and CDB, whoſe Baſe DA is 19$\frac{28}{100}$ parts, which taken from AC 49$\frac{78}{100}$ parts, leaves the Baſe of the greater Triangle CD 30$\frac{50}{100}$ parts. *Fig.* 37.

Now having the two *Baſes* of theſe two *Triangles*, and their *Hypothenuſes*; to wit, CD 30$\frac{50}{100}$ parts, DA 19$\frac{28}{100}$ parts, CB 38$\frac{19}{100}$ parts, and BA 30 parts, you may find all their Angles by the Rule of oppoſite ſides, to their Angles as afore.

I. *In the Triangle* CDB.

To find the Angles, this is the *Analogy* or *Proportion.*
As Log. BC 38$\frac{19}{100}$ parts,
To Radius or S. 90°,
So Log. DC 30$\frac{50}{100}$ parts,
To S. *V.* B. 53° 00′: *whoſe Complement* is the Angle at C 37° 00′ unto 90: or a *Quadrant.*

II. *In the Triangle* ADB.

To find the Angles, this is the *Analogy* or *Proportion.* *Fig.* 37.

§ 3. *As Log.* AB 30 parts,
To Radius or S. 90°:
So Log. AD 19$\frac{1}{2}$ parts,
To S. *V.* B 40° 00′.

The *Complement* whereof, unto 90° 00′, is the Angle at A 50° 00′.

Now in the firſt Triangle CDB, there is found the Angle C to be 37° 00′, and the Angle B to be 53° 00′.

In the ſecond Triangle ADB, there is found the Angle A, to be 50° 00′, and the Angle B to be 40° 00′.

Now the two Angles at B, to wit, 53° 00′; and 40° 00′ makes 93° 00′, which is the Angle of the *Oblique-angled Triangle* ABC at B: Thus the three Angles of the ſaid given *Triangle* ABC are found as was required, *viz.* the Angle A to be 50° 00′, the Angle B to be 93° 00′, and the Angle C to be 37° 00′, as ſought.

Thus I have ſufficiently, fully and plainly explained all the Caſes of *Plain, Right-lined Triangles*, both *Right* and *Oblique-angled:* I ſhall now fall in hand with *Spherical Triangles*, both *Right* and *Oblique-angled.*

SECT.

SECT. IV.

Of Spherical Rectangled Triangles. § 4.

And here first it will be necessary also to understand those few general Maxims or Rules, that are of special Moment, in the Doctrine of Spherical Triangles.

1. THat a *Spherical Triangle* is comprehended and formed by the Conjunction and Intersection of three *Arches* of a *Circle*, described on the *Surface* of the *Sphere* or *Globe*.

2. That those *Spherical Triangles* consist of six distinct parts, *viz.* three Sides and three Angles, any of which being known, the other is also found out and known.

3. That the three Sides of a *Spherical Triangle*, are Parts or *Arches* of three great Circles of a *Sphere*, mutually intersecting each other: And as plain or Right-lined Triangles are measured by a Measure, or *Scale* of equal parts: So these are measured by a *Scale* or *Arch* of equal *Degrees*.

4. That a *Great Circle* is such a Circle that doth bisect the *Sphere*, dividing it into two equal parts; as the *Equinoctial*, the *Ecliptick*, the *Meridians*, the *Horizon*, &c.

5. That in a *Right-angled Spherical Triangle*, the Side subtending the Right-angle we call the *Hypothenuse*, the other two containing the Right-angle we may simply call the *Sides*, and for Distinction either of them may be called the *Base* or *Perpendicular*.

§ 4. 6. That the Summ of the Sides of a *Spherical Triangle* are leſs than two *Semicirc'es* or 360°.

7. That if two Sides of a *Spherical Triangle* be equal to a *Semicircle*; then the two Angles at the *Baſe* ſhall be equal to two Right-angles; but if they be leſs, then the two Angles ſhall be leſs; but if greater, then ſhall the two Angles be greater than a *Semicircle*.

8. That the Summ of the Angles of a *Spherical Triangle* is greater than two *Right-angles*, and leſs than ſix.

9. *That every Spherical Triangle* is either a *Right*, or *Oblique-angled Triangle*.

10. That the *Sines* of the Angles are in Proportion unto the *Sines* of their oppoſite Sides; and the *Sines* of their oppoſite Sides are in Proportion unto the *Sines* of their oppoſite Angles.

11. That in a *Right-angled Spherical Triangle*, either of the *Oblique-angles* is greater than the *Complement* of the other, but leſs than the *Difference* of the ſame *Complement* unto a *Semicircle*.

12. That a *Perpendicular* is part of the *Arch* of a *Great Circle*, which being let fall from any Angle of a *Spherical Triangle*, cutteth the oppoſite Side of the *Triangle* at *Right-angles*, and ſo divideth the *Triangle* into two *Right-angled Triangles*, and theſe two parts (either of the Sides or Angles) ſo divided muſt be ſometimes added together, and ſometimes ſubſtracted from each other, according as the *Perpendicular* falls within or without the Triangle.

PROP.

§ 4.

PROP. I. Case 1.

A Side and an Angle adjacent thereunto being given, to find the other Side.

In the Triangle ABC, there is given the Side AB 27° 54′; and the Angle A 23° 30′, and the Side BC is required; to find which this is the *Analogy* or *Proportion.*

As the Radius or S. 90° 00′, ——— 10. 00000

To S. of cr. AB 27, 54 ——— 9. 67018
So is T. *V.* A 23, 30 ——— 9. 63830 Fig. 38.

To cr. BC 11, 30 ——— 9. 30848

NOTE, If in the right angled Spheric Triangle Figure 38 you allow the Base AB to be part of the Equinoctial, AC part of the Ecliptick, and CB part of a Meridian passing through the Sun's place, then doth the preceding Case find (CB) which is the Sun's present Declination, by having his greatest Declination and right Ascension given.

PROP. II. Case 2.

A Side and an Angle adjacent thereunto being given, to find the other Oblique-angle.

In the Triangle ABC, there is given the Side AB 27° 54′, and the Angle A 23° 30′, and the Angle at C is required; to find which, say by this *Analogy* or *Proportion,* Fig. 38.

As

§ 4. *As the Radius* or S. 90° 00′,
To Sc. *of* cr. AB 27, 54.
So is S. *V.* at A 23, 30,
To Sc. *V.* at C 69, 22 required.

This Propoſition finds the V of the Sun's Poſition, that is the V the Meridian makes with the Ecliptick, from the forementioned *Data*.

PROP. III. Caſe 3.

A Side and an Angle adjacent thereunto being given, to find the Hypothenuſe.

Fig. 38. In the Triangle ABC, there is given the Side AB 27° 54′, and the Angle at A 23° 30′, and the *Hypothenuſe* AC is required, which may be found by this *Analogy* or *Proportion*.

As the Radius or S. 90° 00′,
To Sc. of *V.* at A 23, 30;
So is Tc. cr. AB 27, 54.
To Tc. *Hypothenuſe* AC 30, 00 required.

NOTE, This Caſe finds the Sun's place in the *Ecliptick* from the ſame *Data*.

PROP. IV. Caſe 4.

A Side and an Angle oppoſite thereunto being given, to find the other Oblique-angle.

Fig. 38. In the Triangle ABC, there is given the Side BC 11° 30′, and the Angle A 23° 30′, and the Angle C is required; to find which, ſay by this *Analogy* or *Proportion*,

As Sc. cr. BC 11° 30′,
To Radius or S. 90, 00;
So is Sc. *V.* at A 23, 30,
To S. *V.* at C 69, 22, as required. This

This Caſe finds the Angle of the Sun's Poſition, by having the greateſt and preſent Declination given. § 47.

PROP. V. Caſe 5.

A Side and the oppoſite Angle given, to find the Hypothenuſe.

In the Triangle ABC, there is given the Side BC 11° 30′, and the Angle at A 23° 30′, and the *Hypothenuſe* AC is required, which may be found by this *Analogy* or *Proportion.* Fig. 38.

As S. *V.* at A 23° 30′,
To Radius or S. 90, 00;
So is Ser. BC 11. 30,
To S. *Hypothenuſe* AC 30, 00, as required.

This Caſe from the ſame things given, as in the laſt finds the Sun's place.

PROP. VI. Caſe 6.

A ſide and the oppoſite Angle given, to find the other ſide.

In the Triangle ABC, there is given the Side BC 11° 30′, and the Angle at A 23° 30′, and the ſide AB is required; to find which, this is the *Analogy* or *Proportion.* Fig. 38.

As Radius or S. 90° 00′,
To Tc. of *V.* at A 23, 30;
So is T. cr. BC 11, 30,
To S. of cr. AB 27, 54, as was required.

This Caſe from the laſt *Data* finds the Sun's right Aſcenſion.

PROP.

§ 4.

PROP. VII. Case 7.

The Hypothenuse, and an Oblique-angle given, to find the Side adjacent thereunto.

In the Triangle ABC, there is given the *Hypothenuse* AC 30° 00′, and the Angle A 23° 30′, and the Side AB is required, which is found by this *Analogy* or *Proportion.*

Fig. 38. *As the Radius* or S. 90° 00′,
To Sc. *V.* at A 23, 30;
So is T. *Hypoth.* AC 30, 00,
To T. cr. AB 27, 54, as was required.

This Case supposes the Sun's place and greatest Declination given, to find his right Ascension.

PROP. VIII. Case 8.

The Hypothenuse, and an Oblique-angle given, to find the opposite Side.

In the Triangle ABC, there is given the *Hypothenuse* AC 30° 00′, and the Angle at A 23° 30′, and the Side BC is required, which is found by this *Analogy* or *Proportion.*

Fig. 38. *As the Radius* or S. 90° 00′,
To S. *Hypoth.* AC 30, 00;
So is S. *V* at A 23, 30,
To the S. cr. BC 11, 30, which was required.

Having the same things given as in the last, to find the Sun's present Declination.

PROP.

§ 4.

PROP. IX. Case 9.

The Hypothenuse, and an Oblique-angle given, to find the other Oblique-angle.

In the Triangle ABC, there is given the *Hypothenuse* AC 30° 00′, and the Angle A 23° 30′, now the Angle at C is required, which may be found by this *Analogy* or *Proportion.*

As the Radius or S. 90° 00′,
To Sc. *Hypoth.* AC 30, 00;
So is T. of *V.* at A 23, 30,
To Tc. of *V.* at C 69, 22, as was required. Fig. 38.

From the last *Data* is found the Angle of the Sun's Position.

PROP. X. Case 10.

The Sides given, to find the Hypothenuse.

In the Triangle ABC, there is given the Side AB 27° 54′, and the Side BC 11° 30′, and the *Hypothenuse* AC is required; to find which, say by this *Analogy* or *Proportion.*

As the Radius or S. 90° 00′,
To Sc. cr. BC 11, 30; Fig. 38.
So is Sc. cr. AB 27, 54,
To Sc. *Hypothenuse* AC 30, 00, required.

Having the Sun's present Declination and right Ascension given, by this Case you find the Sun's Place; and by the following Case, having the same *Data*, is found the Sun's greatest Declination.

PROP.

§ 4.

PROP. XI. Case 11.

The Sides given, to find an Angle.

In the Triangle ABC, there is given the Side AB 27° 54′, and the Side BC 11° 30′, and the Angle at A is required, which may be found by this *Analogy* or *Proportion*.

Fig. 38. *As the Radius* or S. 90° 00′,
To S. cr. AB 27, 54;
So is Tc. cr. BC 11, 30,
To Tc. of *V.* at A 23, 30, as required.

PROP. XII. Case 12.

The Hypothenuse, and a Side given, to find the other Side.

Fig. 38. In the Triangle ABC, there is given the *Hypothenuse* AC 30° 00′, and the Side AB 27° 54′, and the Side BC is required, which may be found by this *Analogy* or *Proportion*.

As Sc. cr. AB 27° 54′,
To Radius or S. 90, 00;
So is Sc. *Hypothenuse* AC 30° 00′,
To Sc. cr. BC 11° 30′, as required.

Having the Sun's Place, and right Aſcenſion given, by this Caſe you find the Sun's preſent Declination.

PROP.

§ 4.

PROP. XIII. Case 13.

The Hypothenuse, and a Side given, to find the contained Angle.

In the Triangle ABC, there is given the *Hypothenuse* AC 30° 00′, and the Side AB 27° 54′, and the Angle at A is required, which may be found by this *Analogy* or *Proportion.* Fig. 38.

As the Radius or S. 90° 00′,
To T. cr. AB 27° 54′;
So is Tc. *Hypoth.* AC 30° 00′,
To Sc. of *V.* at A 23° 30′, as required.

By this Case, from the last mentioned *Data*, may the Sun's greatest Declination be found.

PROP. XIV. Case 14.

The Hypothenuse, and a Side given, to find the opposite Angle.

In the Triangle ABC, there is given the *Hypothenuse* AC 30° 00′, and the Side AB 27° 54′, now the Angle C is required, which may be found by this *Analogy* or *Proportion.* Fig. 38.

As the S. *Hypoth.* AC 30° 00′,
To Radius or S. 90° 00′;
So is S. of cr. AB 27° 54′,
To S. of *V.* at C 69, 22, as required.

This Case finds the Angle of the Sun's Position, from the foregoing *Data.*

PROP.

§ 4.

PROP. XV. Case 15.

The Oblique-angles given, to find either Side.

In the Triangle ABC, there is given the Angle A 23° 30′, and the Angle at C 69° 22′, and the Side BC is required, which may be found by this *Analogy* or *Proportion.*

Fig. 38. *As the* S. of *V.* at C 69° 22′,
To the Radius or S. 90° 00′;
So is the Sc. of *V.* at A 23° 30′,
To the Sc. of cr. BC 11° 30′, as required.

Having the Sun's greateſt Declination, and Angle of his Poſition given, by this Caſe you may find the preſent Declination.

PROP. XVI. Case 16.

The Oblique-angles given, to find the Hypothenuſe.

In the Triangle ABC, there is given the Angle A 23° 30′, the Angle C 69° 22′, and the *Hypothenuſe* AC is required, which may be found by this *Analogy* or *Proportion,*

Fig. 38. *As the Radius* or S. 90° 00′,
To Tc. of *V.* at C 69°, 22′;
So is Tc. of *V.* at A 23, 30,
To Sc. *Hypoth.* AC 30, 00, as required.

This Caſe, by the ſame things given as in the laſt, finds the Sun's Place.

Theſe USES *at the end of each* Propoſition, *will make the Calculations more pleaſant; and when you come to apply them to Aſtronomy, you will be much more ready at them.*

SECT.

§ 5.

SECT. V.

Of Oblique-angled Spherical Triangles.

PROP. I. Case 1.

Two Sides, and an Angle oppoſite to one of them given, to find the other oppoſite Angle.

IN the *Triangle* ADE, there is given the Side AE 70° 00′, the Side ED 38° 30′, and the Angle A 30° 28′: Now the Angle at D is required, to find which, this is the *Analogy* or *Proportion.* Fig. 39.

As S. cr. DE 38° 30′,
To S. *V.* at A 30 28;
So is S. cr. AE 70 00,
To S. *V.* at D 130 03,
required.

☞ Note, *That in theſe Operations, for the more Facility of the Learner, I omit* Seconds, *which doth belong unto the* Angles, *&c.*

PROP. II. Case 2.

Two Angles and a Side oppoſite to one of them given, to find the Side oppoſite to the other.

In the *Triangle* ADE, there is given the Angle at D 130° 03′, the Angle E 31° 34′, and the Side AE 70° 00′: Now the Side AD is required, which may be found by this *Analogy* or *Proportion.* Fig. 39.

As S. *V.* at D 130° 03′,
To S. cr. AE 70, 00;
So is S. *V.* at E 31, 34,
To S. cr. AD 40, 00, required.

§ 5.

PROP. III. Case 3.

Two Sides and an Angle included between them being known, to find the other Angles.

Fig. 39. In the *Triangle* ADE, there is given the Side AE 70° 00′, the Side AD 40° 00′, and the Angle A 30° 28′: Now the Angles D and E are required, which is thus found; take the *Sum* and *Difference* of the two *Sides*, and work as followeth, saying,

Side. AE 70° 00′,
Side. AD 40 00

Sum.	110 00	½ Z. 55° 00′.
Diff.	30 00	½ X. 15 00.
Angle. A	30 28	½ *V.* 15 14.

As S. ½ Z. cr′. AE and AD, 55° 00′,
To S. ½ X. cr′. AE and AD, 15 00;
So is Tc. ½ *V.* at A, 15 14,
To T. ½ X. *VV.* D and E. 49 14 30″.

☞ *And here observe, that if the Sum of the two contained Sides exceed a* Semicircle, *then subſtract each Side ſeverally from* 180°, *and proceed with thoſe* Complements, *as with the ſides given, the Operation produceth the* Complements *of the* Angles *ſought, unto a* Semicircle *or* 180 Degrees.

AGAIN.

As Sc. ½ Z. cr′. AE and AD, 55° 00′,
To Sc. ½ X. cr′. AE and AD, 15° 00′;
So is Tc. ½ *V.* at A, 15° 14′,
To T ½ Z. *VV.* D and E, 80 48′ 30″.

This

This Difference of the Angles unknown D and E 49° 14′ 30″, being added unto the half Sum of the Angles 80° 48′ 30″, (unknown) produceth the greater Angle D 130° 03′, and ſubſtracted from it, leaves the leſſer Angle E, to wit, 31° 34′. § 5.

PROP. IV. Caſe 4.

Two Angles, and their Interjacent Side being known, to find the other Sides.

In the *Triangle* ADE, there is given the Angle at A 30° 28′, and the Angle at D 130° 03′, and their *Interjacent-ſide* AD 40° 00′, and the Sides DE and EA are required: Which is thus found. Fig. 39.

Take the *Sum* and *Difference* of the two *Angles*, and work as followeth, ſaying,

☞ *And here obſerve, That if the Sum of the two given Angles exceed a* Semicircle, *or* 180°, *ſubſtract them from a* Semicircle, *and proceed with the Reſidues, the Operation will produce each Side's* Complement *to a* Semicircle, *or* 180 Degrees.

Fig. 39.

Angle. D 130° 03′.
Angle. A 30 28.

Sum. 160 31 } ½ Z. is 80° 15′ 30″.
Diff. 99 35 } ½ X. is 49 47 30.
Side. AD 40 00 } ½ cr. is 20 00 00.

As S. ½ Z. of *VV.* A and D, 80° 15′ 30″.
To S. ½ X. of *VV.* A and D, 49 47 30
So is T. ½ cr. AD, 20 00 00,
To T. ½ X cr. DE and EA, 15 45 00.

§ 5.

AGAIN Say,

As Sc. ½ Z. of *VV.* A and D,	80° 15′ 30″,
To Sc. ½ X. of *VV.* A and D,	49 47 30;
So is T. ½ cr. AD,	20 00 00,
To T. ½ cr'. Z. DE and AE.	54 15 00.

Add the half Difference of the Sides DE and AE 15° 45′, unto half the Sum of the Sides DE and AE 54° 15′. It produceth the greater Side, the Side AE 70° 00′, but if deducted from it, leaves the leſſer Side ED, which is 38° 30′, as was required.

PROP. V. Caſe 5.

Two Sides and an Angle oppoſite to one of them given, to find the third Side.

Fig. 39. In the *Triangle* ADE, there is given the Side AE 70° 00′, the Side DE 38° 30′, and the Angle A 30° 28′, the Side AD is required.

Firſt by *Caſe* 1. *Prop.* 1. I find the Angle at D to be 130° 03′, and then proceed thus:

Firſt take the *Sum* and *Difference* of the two *Angles*; then alſo find the *Difference* of the two Sides given, and then work as followeth.

Angle D 130° 03′,
Angle A, 30 28.

Sum.	160 31	½ Z. 80° 15′ 30″.
Differ.	99 35	½ X. 49 47 30.

Side.

Side. AE 70 00. § 5.
Side. ED 38 30.

Diff. 31 30. } ½ X. 15° 45'.

Now ſay,
As S. ½ X. *VV.* D and A, 49° 47' 30",
To S. ½ Z. *VV.* D and A, 80 15 30;
So is T. ½ X. cr'. AE and ED, 15 45 00,
To T. ½ cr. AD. 20° 00' 00"; which doubled giveth the Side AD, 40 00 00, as was required.

PROP. VI. Caſe 6.

Two Angles and a Side oppoſite to one of them given, to find the third Angle.

In the *Triangle* ADE, there is given the Angle A 30° 28', the Angle D 130° 03', and his oppoſite Side AE 70° 00', and 'tis required to find the Angle at E. Fig. 39.

Firſt by *Prop.* 2. *Caſe* 2. I find the Side DE, oppoſed to the Angle A, to be 38° 30', then proceed thus:

Firſt find the *Sum* and *Difference* of the *Sides*. Then find the *Difference* of the *Angles*.

Side. DE 38° 30',
Side. AE 70 00.

Sum. 108 30. } ½ Z. 54° 15'.
Differ. 31 30 } ½ X. 15 45.

§ 5. *Angle* D 130 03
Angle A 30 28
Differ. 99 35 } ½ X. 49° 47′ 30″.

Now ſay,
As S. ½ X. cr'. DE and AE 15° 45′,
To S. ½ Z. cr'. EA and DE 54 15;
So is T. ½ X. *VV.* D and A 49 47 30″,
To Tc. ½ *V.* at E 15° 47′ 00″, which doubled giveth the Angle at E 31, 34, as required.

PROP. VII. Caſe 4.

Two Sides and an Angle oppoſite to one of them given, to find the included Angle.

Fig. 39. In the *Triangle* ADE, there is given the Side AE 70° 00′, the Side ED 38° 30′, and the Angle oppoſite thereunto at A 30° 28′, and the Angle E is required.

Firſt by *Prop.* 1. *Caſe* 1. I find the Angle D, oppoſite to AE, to be 130° 03′; then proceed thus;

Firſt find the *Difference* of the *Angles*, then find the *Sum* and *Difference* of the *Sides*.

Angle D 130° 03′.
Angle A 30 28.
Differ. 99 35 } ½ X. 49° 47′ 30″.

Side

Side. AE 70° 00′ § 5.
Side. ED 38 30.

Sum. 180 30 ½ Z. 54° 15′.
Differ. 31 30 ½ X. 15 45.

Now ſay,
As S. ½ X. cr'. AE and ED 15° 45′.
To S. ½ Z. cr'. AE and ED 54 15;
So is T. ½ X. of *VV.* D and A 49 47 30″, *Fig.* 39.
To Tc. ½ *V.* at E 15° 47′: Which doubled is the Angle at E 31° 34′, as was required.

PROP. VIII. Caſe 8.

Two Angles and a Side oppoſite to one of them being known, to find the Interjacent Side.

In the *Triangle* ADE, there is given the Angle E 31° 34′, the Angle D 130° 03′, and his oppoſite Side AE 70° 00′: Now the Side ED is required.

Firſt by *Prop.* 2. *Caſe* 2. I find AD oppoſed to E, to be 40° 00′, and then work thus.

Take the *Sum* and *Difference* of the *Angles*, then alſo find the *Difference* of the two *Sides*.

Angle D 130° 03′
Angle E 31 34 *Fig.* 39.

Sum 161 37 ½ Z. 80° 48′ 30′.
Differ. 98 29 ½ X. 49° 14′ 30″.

§ 5 *Side* AD 40° 00′
Side AE 70 00
Differ. 30 00 } ½ X. 15° 00′.

Now ſay,
As S. ½ X. *VV.* D and E 49°. 14′ 30″,
To S. ½ Z. *VV.* D and E 80 48 30;
So is T ½ X. cr⁴. AD and AE 15 00 00″,
To T. ½ cr⁴. ED 19° 15′ 00″, which being doubled, is the Side ED 38° 30′, as required.

PROP. IX. Caſe 9.

Two Sides and their included Angle being known, to find the third Side.

Fig. 40. In the *Triangle* APZ, there is given the Side ZP 38° 30′, the Side PA 70°, and the Angle P, let be 31° 34, and the Side AZ is required.

The Reſolution of this Caſe depends on the *Catholick Propoſition* of the *Lord of Marchiſton*, by ſuppoſing the *Oblique-Triangle* to be divided (by a ſuppoſed *Perpendicular* falling either within or without the *Triangle*) into two *Rectangulars*.

Now in the *Triangle* AZP, let fall the *Perpendicular* ZR; ſo is the *Triangle* AZP divided into two *Rectangulars* ARZ and ZRP. Now the Side AZ may be found at two *Operations* thus: Say,

As the Radius or S. of 90° 00′,
To Sc. *of the included V*, P. 31 34;
So is T. *of the leſſer Side* PZ. 38 30,
To T. *of a fourth Arch.* 34 08.

If

If the *contained Angle* be leſs than 90°, take this *fourth Arch* from the *greater Side*; but if it be greater than 90°, from its *Complement* unto 180°, the Remainder is the *Reſidual Arch*: Now again ſay, § 5.

As Sc. *of the fourth Arch,*	34°	08′,
To Sc. *Reſidual Arch,*	35	52;
So Sc. *of the leſſer Side* PZ,	38	30,
To Sc. AZ *the Side required,*	40	00.

☞ But note, that many times the *Perpendicular* will fall without the *Triangle*, as it doth now within; in ſuch caſe the Sides of the *Triangle* muſt be continued, ſo will there be two *Rectangulars*, the one included within the other: As in the *Triangle* HIK, the *Perpendicular* let fall is KM, falling on the Side HE, and ſo the two *Rectangulars* found thereby will be IMK, and KMH, and ſo by the Directions in the former Propoſition find out the Side IK, if required to be found. Fig. 41.

☞ Note, *That if the Angles at the* Baſe *be both of one kind, that then the* Perpendicular *falls within the* Triangle: *If of divers kinds, without the* Triangle.

PROP. X. Caſe 10.

Two Angles and their Interjacent Side known, to find the third Angle.

In the *Triangle* AZP, there is given the Side ZP 38° 30′, the Angle P 31° 34′, and the Angle Z 130° 03′, and the Angle at A is required. Fig. 40.

Firſt the *Oblique-Triangle* AZP, being reduced into two *Rectangulars* ARZ, and ZRP, by *Caſe*

§ 5. *Case* 9. *aforegoing*, I find the Angle RZP, to be 64° 19′, (in the *Triangle* ZRP.) which taken out of Angle AZP 130° 03′, leaves the Angle AZR 65° 44′: Now the Angle A is found by this *Analogy* or *Proportion*.

As S. *V.* PZR 64° 19′,
To S. *V.* AZR 65 44;
So is Sc. *V.* at P 31 34,
To Sc. *V. at* A 30 28: Which was required to be found out and known.

PROP. XI. Case 11.

Three Sides given, to find an Angle.

In the *Triangle* APZ, the Side AZ is 40° 00′, the Side ZP is 38° 30′, the Side AP is 70° 00′, and the Angle Z is required. To find which do thus.

Fig. 40. Add the three Sides together, and from half their Sum, deduct the Side oppoſite to the required Angle; and then proceed as you ſee in the Operation following.

Side

§ 5.

Side	AP 70° 00′	*Co-Arith.*
Side	AZ 40 00	0, 191933
Side	ZP 38 30	0, 205850
Sum	148 30	Sines.
½ *Sum*	74 15	9, 983380
Side	AP 70 00	
Rem.	04 15	8, 869868
Sum is		19, 251031

Fig. 40.

½ *Sum* is 65° 07′ 30″, the Sc. ½ *V.* at Z, 9,625515 which doubled is 130° 03′ 12″; the Angle at Z required.

This 11th Propoſition or Caſe is of great Uſe in *Aſtronomy*, *Navigation*, and *Dyalling*.

For in *Aſtronomy* 'tis by this Propoſition we find the time of Twilight's Beginning and End. In *Navigation* it finds the Sun's Azimuth, in order to find the Variation of the Needle; and in *Dyalling* it gets the Azimuth, by help of which, we find the Declination of any Wall or Plain.

PROP. XII. Caſe 12.

Three Angles given, to find a Side.

In the *Triangle* AZP, the Angle A is 30° 28′ 11″, the Angle Z 130° 03′ 12″, the Angle P is 31° 34′ 26″, and the Side AZ, oppoſite to P, is required. *Fig.* 40.

This

§ 5. This *Case* is likewiſe performed as the former Caſe or Propoſition, the Angles being converted into Sides, and the Sides into Angles, by taking the *Complement* of the greateſt Angle unto 180°: See the Work.

Angle E	31° 34′ 26″		*Co-Arith.*
Angle A	30 28 11	———	0, 294920
Compl. V. Z	49 56 49	———	0, 116084
Sum is	111 59 26		Sines
½ *Sum is*	55 59 43	———	9, 918549
Fig. 40. *Angle* P	31 34 26		
Differ.	24 25 17	———	9, 616417
Sum total		———	19, 945970

The ½ *Sum is* Sc. *of* 20° 00′ *half the Side* AZ, ——— 9, 972985

which being doubled, gives the Side AZ 40° 00′, required to be found out and known.

☞ But if the greater Side AP were required, the Operation would produce the *Complement* thereof unto a *Semicircle* or 180°; therefore ſubſtract it from 180°, it leaves the remaining required Side ſought.

Thus have I laid down all the *Caſes* of *Triangles*, both *Right-lined* and *Spherical*; either *Right*, or *Oblique-angled*; I might hereunto have annexed many Varieties unto each *Caſe*, and ſome fundamental *Axioms*, which ſomewhat more would have illuſtrated and demonſtrated thoſe *Caſes* and *Proportions*; but becauſe of the

the Smallneſs of this *Treatiſe*, which is intended more for *Practice* than *Theory*, I have for Brevity ſake omitted them, and refer you for thoſe things to larger Authors, who have largely diſcourſed thereon to good purpoſe. § 5.

CHAP. VI.

Of ASTRONOMY.

Defin. ASTRONOMY *is an Art Mathematical, which meaſureth the diſtinct Courſe of Times, Days, Years,* &c. *It ſheweth the Diſtance, Magnitude, Natural Motions, Appearances and Paſſions, proper unto the Planets and fixed Stars, for any time paſt, preſent and to come. By this we are certified of the Diſtance of the ſtarry Sky, and of each Planet, from the Center of the Earth, and the Magnitude of any fixed Star or Planet, in reſpect of the Earth's Magnitude; what is the Cauſe of Eclipſes, and the Method of finding the Quantity of the Digits eclipſed.*

SECT. I.

Of Aſtronomical Definitions. § 1.

Defin. 1. A *Sphere* or *Globe* is a ſolid Body, contained under one *Superficies*, in whoſe middle there is a Point (called the *Center*,) from which all right or ſtreight Lines drawn

§ 1. drawn unto the *Circumference* or *Superficies*, are Equal.

Fig. 43. 2. The *Poles* of the World are two fixed Points in the *Heavens Diametrically* opposite the one to the other; the one called the *Artick* or *North-Pole*, noted in the *Scheme* by P. The other is called the *Antartick* or *South-Pole*; as S. and is not to be seen of us, being in the lower *Hemisphere*.

Fig. 43. 3. The *Axis* of the World is an imaginary Line drawn from the *North-Pole*, through the *Center* of the *Earth*, unto the *South-Pole*, about which the *Diurnal* Motion is performed, from the *East* to the *West*; as the Line PS.

Fig. 43. 4. The *Meridians* are great *Circles*, concurring and intersecting one another in the *Poles* of the World; as PES and Pc. S.

5. The *Equinoctial* or *Equator* is a great Circle, 90 Deg. distant from the *Poles* of the World, cutting the *Meridians* at *Right-angles*,

Fig. 43. and divideth the World into two Equal Parts, called the *Northern* and *Southern Hemispheres*, as E ♎ Q. in *Scheme* 42.

6. The *Ecliptick* is a great Circle crossing the *Equinoctial*, in the two opposite Points *Aries* and *Libra*, and maketh an Angle therewith (called, its *Obliquity*) of 23° 30′, represented by ♑ ♎ ♈. This *Circle* is divided into 12

Fig. 43. *Sines*, each containing 30° 00′: As *Aries* ♈, *Taurus* ♉, *Gemini* ♊, *Cancer* ♋, *Leo* ♌, *Virgo* ♍, (which are called *Northern Sines*) *Libra* ♎, *Scorpio* ♏, *Sagittarius* ♐, *Capricornus* ♑, *Aquarius* ♒, and *Pisces* ♓; these are called *Southern Sines*.

7. The

7. The *Zodiack* is a *Zone* or *Girdle*, having 8 Deg. of *Latitude* on either ſide the *Ecliptick*, that is, the *Ecliptick* is juſt in the middle of the *Zodiack*, in which ſpace the *Planets* make their Revolution. This *Circle* is a Circle which regulates the *Years*, *Months*, and *Seaſons*, * and is diſtinguiſhed in the *Scheme* by the 12 *Signs*. § 1. *Fig.* 43.

* *Which is by the Greeks called* Zodiakos, *i. e. bring Life; becauſe the Life of all Creatures depend on the Cauſe of that Circle; for the Sun aſcending in it, and moving towards us, brings the Generation of things, and in deſcending the Corruption of all things ſenſible, and inſenſible, which are below the Concavity of the Moon, &c.*

8. The *Colures* are two *Meridians*, dividing the *Ecliptick* and the *Equinoctial* into four equal parts; one of which paſſeth by the *Equinoctial* Points *Aries* and *Libra*, and is called the *Equinoctial Colure*, as P ♎ S. The other by the beginning of *Cancer* and *Capricorn*, and is called the *Solſtitial Colure*, as P ♋ S ♑. *Fig.* 43.

9. The *Poles* of the *Ecliptick* are two Points, 23° 30′ diſtant from the *Poles* of the World, as I and K. *Fig.* 43.

10. The *Tropicks* are two *ſmall Circles Parallel* unto the *Equinoctial*, and diſtant therefrom 23° 30′, limiting the *Sun's greateſt Declination*. The *Northern Tropick* paſſeth by the beginning of *Cancer*, and is therefore called the *Tropick of Cancer*, as ♋ D. The *Southern Tropick* paſſeth by the beginning of *Capricorn*, and is therefore called the *Tropick* of *Capricorn*; as B b ♑. *Fig.* 43.

11. The *Polar Circles* are two ſmall Circles *parallel* to the *Equinoctial*, and diſtant therefrom 66° 30′; and from the *Poles* of the World

23°

§. 1. 23° 30′. That which is *adjacent* unto the *North-Pole*, is called the *Artick Circle*, as G d I,
Fig. 43. and the other the *Antartick Circle*, as K d M.

12. The *Zenith* and the *Nadir* are two Points *Diametrically* oppoſite the one to the other: The *Zenith* is the *Vertical Point*, or the Point over our Heads, as Z. The *Nadir* is oppoſite thereto, as the Point N.

13. The *Azimuths* or *Vertical Circles* are great Circles of the *Sphere*, concurring and interſecting each other in the *Zenith* and *Nadir*; as Z f N.

Fig. 43. 14. The *Horizon* is a great Circle, 90 Deg. diſtant from the *Zenith* and *Nadir*; cutting all the *Azimuths* at Right-angles, and dividing the World into two equal parts, the upper and *viſible Hemiſphere*, and the lower and *inviſible Hemiſphere*, repreſented by H ♎ R.

15. The *Meridian* of a *Place* is that *Meridian*
Fig. 43. which paſſeth by the *Zenith* and *Nadir* of the Place, as P Z S N.

16. The *Almicanthars*, or *Parallels of Altitude*, are ſmall Circles parallel unto the *Horizon*, imagined to paſs through every Degree and Minute of the *Meridian*, between the *Zenith* and *Horizon* B a F.

17. *Parallels of Latitude*, or *Declination*, are
Fig. 43. ſmall Circles parallel unto the *Equinoctial*; they are called *Parallels* of *Latitude*, in reſpect to any place on the *Earth*, and *Parallels* of *Declination*, in reſpect of the *Sun* or *Stars* in the *Heavens*. Hence the Declination of the *Sun*, *Moon*, or *Stars*, is no more than their Diſtance from the *Equator*.

18. The

18. The *Latitude of a Place* is the Height of the *Pole* above the *Horizon*, or the Diſtance between the *Zenith* and the *Equinoctial*; or 'tis the Diſtance of a Place from the *Equator*. § 1.

19. The *Latitude of a Star* is the *Arch* of a *Circle*, contained betwixt the *Center of a Star*, and the *Ecliptick Line*: This Circle making *Right-angles*, with the *Ecliptick*, is accounted either *Northward* or *Southward*, according to the Situation of the *Star*.

20. *Longitude* on *Earth* is meaſured by an *Arch* of the *Equinoctial*, contained between the *Primary Meridian*, (or *Meridian* of that place, where *Longitude* is aſſigned to begin) and the *Meridian* of any other place, counted always *Eaſterly*.

21. The *Longitude* of a *Star* is that part of the *Ecliptick*, which is contained between the *Star*'s place in the *Ecliptick* and the beginning of *Aries*, counting them according unto the Succeſſion of *Signs*.

22. The *Altitude* of the *Sun* or *Stars* is the *Arch* of an *Azimuth*, contained betwixt the *Center* of the *Sun*, or *Star*, and the *Horizon*.

23. *Aſcenſion* is the riſing of any *Star*, or part of the *Equinoctial*, to any Degree above the *Horizon*; and *Deſcenſion* is the ſetting of it.

24. *Right Aſcenſion* is the Number of *Degrees* and *Minutes* of the *Equinoctial*; (i. e. from the beginning of *Aries*) which cometh unto the *Meridian*, with the *Sun* or *Stars*, or with any Portion of the *Ecliptick*.

25. *Oblique-Aſcenſion* is an *Arch* of the *Equinoctial*, between the beginning of *Aries* and that part of the *Equinoctial*, which riſeth with

§ 1. the *Center* of a *Star*; or with any *Portion* of the *Ecliptick* in an *Oblique Sphere*: And *Oblique Deſcenſion* is that part of the *Equinoctial*, that ſetteth therewith.

26. The *Aſcenſional Difference* is an *Arch* of the *Equinoctial*, being the Difference betwixt the *Right* and *Oblique-Aſcenſion*.

27. The *Amplitude* of the *Sun* or *Stars* is the Diſtance of the riſing or ſetting thereof, from the *Eaſt* or *Weſt* Point of the *Horizon*.

28. The *Parallax* is the Difference between the *true* and *apparent Place* of the *Sun* or *Star*; ſo the *true Place* in reſpect of *Altitude*, is in the Line ACE, or ADG, the *Sun* or *Star* being at C or D, and the *apparent Place* in the Line BCF and BDH; ſo likewiſe the Angles of the *Parallax* are ACB or ECF, and ADB or GDH: Alſo in the ſaid *Scheme* ABK repreſenteth a *Quadrant* (of the *Globe* or *Earth*) on the *Earth*'s *Superficies*: A the *Center* of the *Earth*, and B any Point of the *Earth*'s *Surface*. Fig. 42.

29. The *Refraction* of a *Star* is cauſed by the *Atmoſphere*, or *Vaporous* Thickneſs of the *Air* near the *Earth*'s *Superficies*, whereby the *Sun* and *Stars* ſeem always to riſe ſooner, and ſet later than really they do; that is, they appear above the *Horizon* when really they are under it.

SECT.

SECT. II.

Of Aſtronomical Propoſitions. § 2.

PROP. I.

The Diſtance of the Sun from the next Equinoctial Point (either Aries *or* Libra*) being known, to find his* Declination.

THE *Analogy* or *Proportion.*

As Radius or S. 90°,

To S. *of the Sun's diſtance from the next Equinoctial Point*;

So is S. *of the Sun's greateſt* Declination,

To the S. *of the Sun's preſent* Declination *ſought.*

PROP. II.

The Sun's Place given, to find his Right-Aſcenſion.

This is the *Analogy* or *Proportion.*

As Radius or S. 90°,

To T. *of the Sun's Longitude from the next Equinoctial Point*;

So is the Sc. *of his greateſt* Declination,

To T. *of his Right-Aſcenſion from the next Equinoctial Point.*

§ 2.

PROP. III.

To find the Sun's Place or Longitude from Aries, *his Declination being given.*

This is the *Analogy* or *Proportion.*
As S. *of the Sun's greateſt Declination,*
To Radius or S. 90° 00′;
So is S. *of his preſent Declination,*
To S. *of the Sun's Place or Longitude from* Aries *.

* *If the* Sun's Declination *be* North, *and increaſing, this Proportion finds the* Sun's *Diſtance from* ♈; *but if decreaſing from* ♎ *in the Northern* Sines. *But if the* Sun's Declination *be* South, *and increaſing from* ♎; *if decreaſing from* ♈ *among the* Southern Sines.

PROP. IV.

By knowing the Sun's Declination, to find his Right Aſcenſion.

This is the *Analogy* or *Proportion.*
As Radius or S. 90°,
To Tc. *of the Sun's greateſt Declination;*
So is T. *of the Declination given,*
To S. *of the Sun's right Aſcenſion* required †.

† *From the next* Equinoctial *Point, either* ♈ *or* ♎.

PROP. V.

By knowing the Latitude of a Place, and the Sun's Declination, to find the Aſcenſional Difference.

This is the *Analogy* or *Proportion.*
As Radius or S. 90°,

To

To Tc. *of the Latitude given*; § 2.
So is T. *of the Sun's Declination given*,
To the S. *of the Aſcenſional Difference* required.

☞ NOTE, That if you reduce the Degrees, *&c.* of the *Aſcenſional Difference* into Hours, it will ſhew you how much the *Sun* riſeth or ſetteth before, or after ſix a Clock, in that Latitude.

PROP. VI.

To find the Sun's Oblique Aſcenſion or Deſcenſion.

Firſt find the *Aſcenſional Difference* by the 5th Propoſition, and his *Right Aſcenſion* by the fourth: Now if the *Sun's Declination* be Northerly, deduct the *Aſcenſional Difference* out of his *Right Aſcenſion*, from the beginning of ♈, (for the ſix Northern Signs ♈ ♉ ♊ ♋ ♌ ♍) it leaves the *Oblique Aſcenſion*, and added unto the *Right Aſcenſion*, giveth the *Oblique Deſcenſion*.

But if the *Sun*'s *Declination* be Southerly, the *Aſcenſional Difference*, added to the *Right Aſcenſion*, (for the ſix Southern Signs ♎ ♏ ♐ ♑ ♒ ♓) giveth the *Right Aſcenſion*, and ſuſtracted therefrom, leaves the *Oblique Deſcenſion*.

PROP. VII.

By knowing the Sun's Declination, and the Latitude of a Place, to find the Sun's Amplitude.

This is the *Analogy* or *Proportion*.
As Sc. *of the Latitude*,

§ 2. *To the Radius or* S. 90°;
So is the S. *of the Sun's Declination,*
To the S. *of the Amplitude* from the *Eaſt* or *Weſt* Points of the *Horizon.*

This Propoſition is of great Uſe at Sea, in finding the Variation of the Needle: For the Summ of Difference of the *Sun*'s Amplitude and Magnetick Amplitude, according as it is, is the Variation of the Needle.

PROP. VIII.

By knowing the Sun's Declination and Amplitude, from the North part of the Horizon, to find the Latitude.

This is the *Analogy* or *Proportion.*
As Sc. *of the Amplitude* from the *North,*
To Radius or S. 90° 00′;
So is S. of his *Declination* given,
To Sc. of the required *Latitude.*

PROP. IX.

By knowing the Latitude of a Place, and the Sun's Declination, to find at what time the Sun will be on the true Eaſt or Weſt Points.

The *Analogy* or *Proportion* is,
As T. of the given *Latitude,*
To T. of the *Sun*'s *Declination* propounded;
So is Radius or S. 90° 00′,
To Sc. of the *Hour* from *Noon.*

PROP.

§ 2.

PROP. X.

By knowing the Sun's Declination, and Latitude of a Place, to find his Altitude at ſix a Clock.

This is the *Analogy* or *Proportion.*
As Radius or S. 90° 00′,
To S. of the *Sun's Declination*;
So is S. of the *Latitude of the Place*,
To S. of the *Sun*'s *Altitude* at ſix a Clock.

PROP. XI.

By knowing the Latitude of a Place, and the Sun's Declination, to find the Azimuth at ſix.

This is the *Analogy* or *Proportion.*
As Radius or S. 90° 00′,
To the T. of the *Sun's Declination*;
So is Sc. of the *Latitude of the Place*,
To the T. of the *Azimuth* ſought.

PROP. XII.

By knowing the Latitude of a Place, and the Sun's Declination, to find the Sun's Altitude when he is on the true Eaſt or Weſt Points.

This is the *Analogy* or *Proportion.*
As S. *of the Latitude*,
To the Radius or S. 90° 00′;
So is the S. of the *Declination*,
To the S. of the *Sun*'s *Altitude* being due *Eaſt* or *Weſt.*

§ 2.

PROP. XIII.

To find the Sun's Altitude at any time of the Day.

The *Analogy* or *Proportion* is,
As Radius or S. 90° 00′,
To Tc. of the *Poles Height*;
So is S. of the *Sun's Diſtance*,
From the Hour of Six,
To the T. of an *Arch:* Which being ſubſtracted from the *Sun's Diſtance* from the *Pole*, ſay,
As Sc. of the *Arch found*,
To Sc. of the *remaining Arch* of the *Sun's Diſtance* from the *Pole*;
So is S. of the *Pole's Height*,
To the S. of the *Sun's Altitude* at the Hour required.

PROP. XIV.

By knowing the Latitude of a Place, with the Sun's Declination, and Altitude, to find the Hour of the Day.

To ſolve this Concluſion do thus: Get the Sum of the *Complements* of the *Latitude*, *Declination* and *Altitude* given *, Then find the Difference betwixt their half Sum, and the *Complement* of the *Altitude*; then ſay,

* *As in Caſe* 11 *of Oblique Spherical Triangles.*

As Radius or S. 90° 00′,
To Sc. of the *Sun's Altitude*;
So is Sc. of the *Latitude* of the Place,
To a fourth Sine: Then again ſay,
As the fourth S.
To the S. of ½ Z. of the *Lat. Declin.* and *Alt.*

So

So is the S. of X. of the *Altitude* to the ⁞ Z, § 2.

To a fifth of S. unto which *Sine*, if you add the *Radius* or 90° 00′, half that Sum ſhall be the *Sine* of an *Arch*, whoſe double *Complement* is the Hour from the *Meridian*.

This is a very uſeful Propoſition (tho' one of the moſt troubleſome to calculate it being done by the 11th Caſe of Oblique Spheric Triangles:) For by it you find the Hour of the Day, having the Sun's Altitude given; and conſequently may know how to ſet any Clock or Watch, or Sun-Dyal, if the Sun ſhines.

The Sun's Altitude, if he ſhines, is gotten thus: Set a ſtrait Pin or Wyer exactly perpendicular on a true Horizontal Plain; then meaſure the Length of the Wyer, and the Length of the Shade of ſuch Wyer, and ſay,

As the Length of the Shade to the Length of the Wyer, ſo is Radius to the Tangent of the Angle of the Sun's Height.

Alſo by having the ſame things, *viz. Latitude*, *Declination*, and Sun's *Altitude* given, 'tis eaſy to find the Sun's Azimuth from the North, and by Conſequence from the South; which having gotten, is of great uſe in finding the *Variation* of the *Needle*.

This *Azimuth* is alſo always to be found, in order to find the Declination of any Wall or Plain on which any Dyal is to be delineated.

PROP.

§ 2.

PROP. XV.

To find the Time of the Sun's Riſing or Setting, and conſequently the Length of the Day or Night.

To reſolve this *Concluſion*, firſt by Prop. the 5. find the *Aſcenſional Difference*, which reduced into *Hours* and *Minutes* of Time, by allowing for every 15 *Deg.* one *Hour*, and for every *Deg.* leſs than 15°, 4′ of Time, and for every 15 *Min.* one *Minute* of Time.

Secondly, If the *Sun*'s *Declination* be Northerly, the *Aſcenſional Difference* added unto 6 Hours, gives the Time of *Sun-ſetting*, and ſubſtracted therefrom, leaves the Time of *Sun-Riſing*: On the contrary, if the *Sun*'s *Declination* be *Southerly*, the *Aſcenſional Difference* added unto 6 Hours, gives the Time of *Sun-Riſing*, and deducted therefrom the Time of *Sun-ſetting*.

Thirdly, If you double the Time of *Sun-Riſing*, it gives you the Length of the *Night*; and the Time of *Sun-ſetting*, the Length of the *Day*.

PROP. XVI.

The Sun's Declination, Altitude and Azimuth known, to find the Hour of the Day.

The *Analogy* or *Proportion* is,
As the Sc. of the *Sun*'s *Declination*,
To the S. of the *Azimuth*;
So is the Sc. of the *Altitude*,

To

To the S. of the Hour from Noon: Which converted into Time, will ſhew the Hour of the Day. § 2.

PROP. XVII.

By knowing the Sun's Declination, Altitude, and Hour from Noon, to find the Azimuth.

The *Analogy* or *Proportion* is,
As Sc. of the *Sun's Altitude*,
To S. of the *Hour from Noon*;
So is Sc. of the *Sun's Declination*,
To the S. of the *Azimuth* required.

PROP. XVIII.

By knowing the Latitude of a Place, the Altitude of the Sun, and the Hour from Noon, to find the Angle of the Sun's Poſition.

This is the *Analogy* or *Proportion*.
As Sc. of the *Sun's Altitude*,
To S. of the *Hour from Noon*;
So is Sc. of the *Latitude*,
To S. of the *V.* of the *Sun's Poſition*, at the time of the *Queſtion*.

PROP. XIX.

By knowing the Sun's Altitude, Declination, and Azimuth, to find the Latitude.

The *Analogy* or *Proportion* is,
As S. of the *Sun's Azimuth*,
To S. of his *Diſtance* from the *North-pole*;
So is S. of *V.* of the *Sun's Poſition*,
To Sc. of the *Latitude* required. PROP.

§ 2.

PROP. XX.

To find the Length of the Crepuſculum, or Twilight.

The *Crepuſculum* or *Twilight*, is nothing elſe but the *Refraction* of the *Sun*'s *Beams* in the *Denſity* of the *Air*: Which the Learned *Pet. Nennius* found the Length of the *Crepuſculum* (by his many ſtrict Obſervations *) to continue from the time of the *Sun*'s paſſing below the *Horizon* of a Place, until the *Sun* had run below the ſaid *Horizon* 18° 00′, and then followed the ſhutting in of the *Twilight*, and until the *Sun* was departed ſo low the *Twilight* continued. — To find which obſerve this *Analogy* or *Proportion*.

* *Watched the Time after* Sunſetting, *when the* Twilight *in the* Weſt *was ſhut in; ſo that no more* Twilight *than in any other part of the* Sky *near the* Horizon *appeared there: Then by one of the known* fixed Stars, *having found the true* Hour *of the* Night, *he found the Length of the* Twilight, *to be as in the Rule is mentioned.*

As Radius or S. 90°,
To Sc. of the *Sun*'s *Declination*;
So is Sc. of the *Pole*'s Height,
To a fourth Sine, which keep.

Then out of the *Sun*'s *Diſtance* from the *South Pole*, ſubduct the *Complement* of the *Pole*; and of that Remains and the Degrees 62, being added to it, their *Sum* and *Difference* found, ſay again;

As the fourth Sine found,
To S. $\frac{1}{2}$ Z. of the *Remainder* and 62° 00′;
So is S. $\frac{1}{2}$ X. of the *Remainder* and 62° 00′,
To a Number, which being multiplied by the *Radius* is equal unto the *Quadrant* of the *Sine* of the

the ½ *Angle* of the *Sun*'s *Diſtance* at the ending of the *Twilight*, from *Noon* next enſuing. § 2.

Then from the Sum of the whole *Angle* converted into *Hours*, ſubtract the *Hour* of the *Sun's ſetting* *, it gives you the Length of the *Crepuſculum* or *Twilight*.

* *Or* ½ *Diurnal Arch.*

But the *Sun* being in the *Winter Tropick*, makes the *Twilight* longeſt of any *Twilight*, the whole *Winter* half Year: Now in a certain *Parallel*, betwixt that *Tropick* and the *Equinoctial*, is the ſhorteſt *Crepuſculum*: The *Declination* of which *Parallel* is thus found.

To find the Length of the leaſt Crepuſculum *or* Twilight.

As the Tc. of the *Pole*,
To the S. of the *Pole*;
So is the T. of 09° 00′,
To S. of the *Declination* of the *Parallel*, in which the *Sun* maketh the ſhorteſt *Crepuſculum* of the Year.

But before the *Crepuſculum* come to be ſhorteſt, there is another *Parallel*, in which the *Crepuſculum* is equal to that of the *Equinoctial*: the *Declination* of which is found thus.

As the Radius or S. 90° 00′,
To S. of the *Pole*'s *Elevation* or *Altitude*;
So is S. of 18° 00′,
To S. of the *Declination* of the *Parallel*, in which the *Sun* maketh the *Crepuſculum* equal to that in the *Equinoctial*.

PROP.

§ 2.

PROP. XXI.

To find the Quantity of the Angles, which the Circles of the 12 *Houſes make with the Meridian.*

This is the *Analogy* or *Proportion.*

As the Radius or S. 90°,

To T. *of* 60°: *for the* 11th, 9th, 5th, and 3d *Houſe*; or to the T. 30° *for the* 12th, 8th, 6th, and 2d *Houſe*;

So is the Sc. of the *Pole*,

To the Tc. of any *Houſe*, with the *Meridian.*

PROP. XXII.

To find the Right Aſcenſion of the Point in the Equinoctial: And alſo the Point in the Ecliptick, called Medium Cœli, or *Cor Cœli.*

Firſt, To find the *Right Aſcenſion* of the Point of the *Equinoctial*, called *Medium Cœli*, *vel Cor Cœli*, find out the *Sun*'s *Right Aſcenſion*, by Prop. 2. Then reduce the whole Time from Noon laſt paſt into *Degrees*, which add unto the *Right Aſcenſion* of the *Sun*, ſo ſhall their *Aggregate* be the *Right Aſcenſion* of the Point, which in the *Equinoctial* is called *Medium Cœli*, *vel Cor Cœli*, required to be found.

Secondly, By the 2 Propoſitions aforegoing, you may find the *Right Aſcenſion* of the Point in the *Ecliptick Culminant* in the *Meridian*, called *Medium Cœli*, vel *Cor Cœli*; which is the *Cuſpis of the Tenth Houſe*, and his *Declination* by Prop. the firſt.

PROP.

§ 2.

PROP. XXIII.

To find the Angle of the Ecliptick with the Meridian.

The *Analogy* or *Proportion* is,
As the Radius or S. 90°,
To S. of the *Sun's Greateſt Declination*;
So is Sc. of the *Sun's Right Aſcenſion*, from the next *Equinoctial Point*,
To Sc. of the *V.* of the *Ecliptick*, with the *Meridian.*

PROP. XXIV.

To find the Angle of the Ecliptick with the Horizon.

The *Analogy* or *Proportion* is,
As Radius or S. 90°,
To Sc. of the *Altitude* of *Cor Cœli*;
So is S. of the *V. Ecliptick* with the *Meridian*,
To Sc. of the *V.* of the *Ecliptick* and *Horizon* ſought.

PROP. XXV.

To find the Amplitude Ortive of the Aſcendant, or Horoſcopus.

This is the *Analogy* or *Proportion*,
As Radius or S. 90°,
To S. of *Altitude* of *Med. Cœli*;
So is T. of *V. Ecliptick* with the *Meridian*,
To Tc. of the *Amplitude Ortive of the Aſcendant, or the Diſtance of the* Azimuth *from the Meridian.*

PROP.

§ 2.

PROP. XXVI.

To find the Aſcendant Degree of the Ecliptick, or the Cuſpis of the firſt Houſe.

The *Amplitude Ortive* of the *Aſcendant*, is equal to the Diſtance of the *Azimuth* of 90°, from the *Meridian*, wherefore the *Cuſpis* of the firſt *Houſe*, or *Aſcendant Degree* of the *Ecliptick*, may be found thus:

As Radius or S. 90°,
To Sc. of the *V. Ecliptick* with the *Meridian*;
So is Tc. of the *Altitude* of *Med. Cœli*,
To T. of the Diſtance of *Med. Cœli*, from the *Aſcendant Degrees*.

PROP. XXVII.

To find the Diſtance of the Cuſpis of any Houſe, from Med. Cœli.

This is the *Analogy* or *Proportion*,
As Sc. of the remaining part of *V.* of the *Ecliptick* with the *Meridian*, (found by Prop. 28.)
To Sc. of the former part of the *V*;
So is T. of the *Altitude* of *Med. Cœli*,
To T. of the Diſtance of the *Cuſpis* of that *Houſe* ſought, from *Med. Cœli*.

PROP. XXVIII.

To find the parts of the Angle of the Ecliptick with the Meridian, cut with an Arch perpendicular to the Circle of any of the Houſes.

The *Analogy* or *Proportion* is,
As Radius or S. 90°,

To

To Sc. *Altitude* of *Med. Cœli*, § 2.
So is T. of the Circle of any Houſe with the *Meridian*,
To Tc. of that part of that Angle which is next the *Meridian*:
Then ſubſtract that part found, out of the whole Angle, for the remaining or latter part.

PROP. XXIX.

To find the Pole's Altitude above any of the Circles of the Houſes.

The *Analogy* or *Proportion* is,
As the Radius or S. 90°,
To S. of *V.* of the Circle of the Houſe with the *Meridian*: (found by the 21 Prop.)
So is the S. of the *Pole's Elevation*, above the *Horizon* of the Place,
To S. of the *Altitude* of the *Pole*, above the *Circle of Poſition*.

PROP. XXX.

By knowing the Latitude and Longitude of any fixed Star, to find his Right Aſcenſion and Declination.

The *Analogy* or *Proportion* is,
1. *As Radius* or S. 90°,
To S. of the Stars *Longitude* from the next *Equinoctial Point*;
So is Tc. of the *Stars Latitude*,
To T. of a *fourth Arch*.
Which compared with the *Arch of Diſtance* betwixt the *Poles of the World* and the *Ecliptick*

§ 2. 23°, 30′: And if the *Latitude* and *Longitude* of the *Star* be both of one Dignity, *i. e.* when the Star hath *North Latitude* in the ſix *Northern Signes*, ♈, ♉, ♊, ♋, ♌, ♍, or *South Latitude* in the ſix *Southern Signes*, ♎, ♏, ♐, ♑, ♒, ♓: Then ſhall the Difference between this *found Arch*, and the Diſtance of the *Poles* be your *fifth Arch*: But if the *Latitude* and *Longitude* of the *Star* be of contrary Qualities, *i. e.* the one *Northern* and the other *Southern*; then add this *fourth Arch* to the Diſtance of the *Poles* 23° 30′, and the Sum thereof ſhall be your *fifth Arch*; with which,

AGAIN ſay,

2. *As* S. of the *fourth Arch*,
To S. of the *fifth Arch*;
So is T. of the *Stars Longitude*,
To T. of the *Stars Right Aſcenſion* from the next *Equinoctial Point*.

3. *As* Sc. of the *fourth Arch*,
To Sc. of the *fifth Arch*;
So is S. of the *Stars Latitude*,
To S. of the *Stars Declination*.

☞ I might alſo ſhew how by having the *Latitude* and *Longitude* of any two *fixed Stars*, to find their Diſtance: But becauſe 'tis the very ſame with finding the Diſtance of any two Places on Earth, I refer you to the Directions of *Prop.* 1, 2, and 3, of *Chap.* 7. enſuing where you will ſee the plain Demonſtration thereof.

PROP.

PROP. XXXI.

By knowing the Pole's Altitude, to find when any fixed Star ſhall be due Eaſt or Weſt.

This is the *Analogy* or *Proportion*,
As Radius or S. 90°,
To T. of the *Stars Declination*;
So is Tc. of the *Pole*,
To Sc. of the *Stars Horary Diſtance* from the *Meridian*.

PROP. XXXII.

By knowing the Pole's Altitude, to find the Elevation of any fixed Star above the Horizon, being due Eaſt or Weſt.

This is the *Analogy* or *Proportion*,
As S. of the *Pole's Altitude*,
To Radius or S. 90°;
So is S. of the *Stars Declination*,
To S. of the *Stars Elevation*, above the *Horizon*, at due *Eaſt* or *Weſt*.

PROP. XXXIII.

To find out the Horizontal Parallax of the Moon.

The Analogy or *Proportion*.
As the Moon's Diſtance from the *Center* of the *Earth*,
To the Earth's Semidiameter;
So is Radius or S. 90°,
To S. of the *Moon's Horizontal Parallax* in that Diſtance.

§. 2.

PROP. XXXIV.

The Horizontal Parallax of the Moon being known, to find her Parallax in any apparent Latitude.

This is the *Analogy* or *Proportion.*
As Radius or S. 90°,
To S. of the *Moon's Altitude*;
So is S. of the *Moon's Horizontal Parallax*,
To S. of the *Parallax* in that *Altitude.*

PROP. XXXV.

By knowing the Moon's Place in the Ecliptick, (having little or no Latitude) and her Parallax of Altitude, to find the Parallaxes of her Longitude and Latitude.

Firſt, If the *Moon* be in the 90° of the *Ecliptick*, ſhe hath then no *Parallax* of *Longitude*, and the *Parallax* of the *Latitude* is the very *Parallax* in that *Altitude.*

Secondly, But if the *Moon* be not in the 90th Degree of the *Ecliptick*, to find the *Parallaxes* of the *Latitude* and *Longitude*, the *Analogy* or *Proportion* is,

1. *As Radius* or S. 90°,
To T. of the *V.* of the *Ecliptick* and *Horizon*;
So is Sc. of the *Moon's Diſtance* from the *Aſcendant*, or *Deſcendant deg.* of the *Ecliptick*,
To Tc. of the *Ecliptick's V*, with the *Azimuth* of the *Moon.*

AGAIN

§ 1.

AGAIN *ſay*,

2. *As the Radius* or S. 90°,
To S. of that *V.* found;
So is the *Parallax* of the *Moon's Altitude*,
To the *Parallax* of her *Latitude* ſought.

LASTLY *ſay*,

3. *As the Radius* or S. 90° 00′,
To Sc. *of the former V. found*;
So is the Parallax of the Moon's Altitude,
To the Parallax of his Longitude ſought, which being added to the true Motion of the *Moon*, if ſhe be on the *Eaſt* part of the 90° of the *Ecliptick*. Or from it to be deducted, if ſhe be on the *Weſt* part of the 90° of the *Ecliptick*.

PROP. XXXVI.

How by knowing the Refraction of a Star, to find his true Altitude.

He that hath any Knowledge in the *Dioptricks* or the Doctrine of refracted Rays, muſt needs know that becauſe of the *Atmoſphere* or thick *vaporous Air* that ſurrounds the Earth, all the Cœleſtial Bodies, but particularly the Stars, muſt needs appear higher than really they are: And therefore to find their true Height, I have annexed this Table of Refractions which ſerve for the Stars only, calculated by *Tycho Brahe*, a Nobleman of *Denmark*, and a moſt famous Aſtronomer.

§ 2.

A Table of the Refraction of the Stars obſerved by *Tycho Brahe.*

Altitude.	*Refraction.*	
0°	30′	00″
1	21	30
2	15	30
3	12	30
4	11	00
5	10	00
6	9	00
7	8	15
8	6	45
9	6	00
10	5	30
11	5	00
12	4	30
13	4	00
14	3	30
15	3	00
16	2	30
17	2	00
18	1	15
19	0	30
20	0	00

The USE of which TABLE is thus.

EXAMPLE:

Suppoſe the *Altitude* of a *Star* were found by Obſervation to be 13°; the correſpondent *Refraction* is 4′ 00″, which ſubſtracted from 13° leaves 12°, 56′, which is the true Altitude.

CHAP.

CHAP. VII.

Of COSMOGRAPHY.

SECT. I.

COSMOGRAPHY, *according to its Etymology, is a Deſcription of the viſible World, being a Science that teaches the Knowledge of the Structure or Frame of the Univerſe; examining into its Figure, Magnitude and Diſpoſition, with their reſpective Diſtances, Motions, and other Properties. It hath two Branches, Aſtronomical and Geographical.*

The *Aſtronomical* part is that which informs us of the *Diſtinction* and *Diviſion* of the *Heavens*, and of the Laws and Motions of the Stars and Planets, without ſome previous Knowledge of which, *Geography* cannot be well underſtood.

Now the beſt *Idea* we can have of the Figure of the World is, that it is a *Sphere* or *Globe* compoſed of the four Elements, included within the Heaven, as an Egg within its Shell; in which Sphere each Element hath a Situation moſt agreeable to its Nature.

The *Water* leſs ſolid, and ſo more liquid is above the Earth; but by the Decree of the *Omnipotent*, leaving certain Places open and undiſcovered for the Habitation of *Man* and other *Animals*.

§ 1. The *Air* more rare and clear is above both, and encompassing them on all parts.

The *Fire* more subtil, light and agil, is above the *Air*, and encompassing it every where, and is neareſt to the Heaven.

The *Heaven* or *Firmament* enviroſſing the whole Body of Matter unknown to us, tranſparent, exquiſite, incorruptible and unchangeable; which by reaſon of its Perfection, perpetual Motion, and by the Ornament of the Stars there placed, may be termed Divine Matter.

Now each particular Element hath a Quality predominant in it, and peculiarly natural to it; as the Earth is dry, Water-cold, Air humid, Fire-hot.

From all which, how ought we to admire the infinite Wiſdom of the Great *God*, who to preſerve a Concord among Contraries, hath placed a Moderator between two different Qualities, of whoſe Nature both the one and the other ſhould participate: For that between the dry and the moiſt the Cold is placed, and between the Cold and the Hot the Humid?

So that each Element favours of the Quality of its Neighbouring Element; the Earth cold and dry, the Water cold and moiſt, the Air hot and moiſt, the Fire hot and dry.

But leaving this, which is more the Buſineſs of a *Philoſopher*, I proceed more ſtrictly into the Aſtronomical part; which informs us of the Deſcription of the Heavens and Laws of the Stars, without ſome previous Knowledge of which (as I ſaid before) the right Underſtanding of *Geography* cannot be attain'd: For we cannot comprehend why in one place the Days

are

are more long or ſhort than in another, and in ſome Countries equal to the Night; and which is more to be admir'd, at certain Seaſons equal throughout the World, and why the Qualities of the Winds are different according to the various Points of the Compaſs from whence they proceed. § 1.

And firſt of the Heaven: It is found by Experience there are many Heavens, one including another, and by Conſequence one larger than another, in Number no leſs than *Eight*; of which, more in the following Diſcourſe. Some are of Opinion there are *Nine*, others *Ten*, beginning at the loweſt, according to the Order of the Seven Planets or wandring Stars.

The Heaven of the *Moon*, of *Mercury*, *Venus*, the *Sun*, of *Mars*, *Jupiter* and *Saturn*; the Heaven where the fixed Stars, the Twelve Signs, and the Poles are: And laſtly, that which is called the *Primum Mobile* and *Emperique* Heaven, the Principal Habitation of God; for it is ſaid in the *Scripture*, That *Heaven is my Seat*, &c.

The Heaven which environs the others beneath and within it, carried by the Motion of the *Primum Mobile*, moves according to the apparent Motion of the Sun, Moon and Stars, which is from Eaſt to Weſt, as is very evident from a few Hours Obſervation in the Day and Night: But for the more plain Deſcription and Diviſion of this Heaven into certain parts, imagine a *Globe* or *Ball* between two Fingers, and that I cauſe it to turn without removing my Fingers, would you not allow that the two Minute Points under the Fingers are immoveable, that

§ 1. that is to ſay, they do not change their Place? And if you make a Line round this Ball equally diſtant from thoſe two immoveable Points, will not the Ball then be divided into two equal Parts? Further, if when that is done you make two other Lines round the Ball, the one on the one ſide, the other on the other Side of the firſt Line equally diſtant, is it not reaſonable to ſay, that the Space included between the two laſt Lines may be compared to a Girdle encompaſſing ſuch Globe?

Juſt ſo is the Conception to be made of the Heavens: The two immoveable Points are the Poles, the one called the *Artick* or North, the other the *Antartick* or South; the Line dividing the Globe into two equal Parts, may be called the *Æquator*, and the two other Lines drawn on each ſide the Æquator may be termed the *Tropicks*.

And the Space on the Globe or Ball between thoſe two laſt Lines, is what we call the *Torrid Zone*, in the Limits whereof the Sun is evermore confin'd, every Day making his *Tour* round the World from the Eaſt unto the Weſt in the time of twenty four Hours; notwithſtanding which Motion, it hath another Motion from one Tropick to the other, and is contrary to the former Motion, and therefore from Weſt to Eaſt. By this Motion the Sun finiſheth his Courſe in one Year; hence proceeds the Four Seaſons, as ſhall be hereafter mentioned. In the whole Region of the Sun's Courſe rarely or never any Winds but the North and Eaſt blow, whereby frequent Showers and conſtant Breezes; and ſo renders the

the Air more temperate than its generally thought to be. This done, let there be drawn upon the Globe or Ball two Lines round, the one between one of the Poles and one of the *Tropicks*, and the other between the other Pole and the other *Tropick* of the like Distance as the first, which is 23 Deg. $\frac{1}{2}$ from each Pole; in such manner, that besides the *Torrid Zone*, the Globe shall be divided into four, which with the *Torrid Zone* makes up five *Zones*: Whereof the two next the Poles are *frozen* and extreamly cold, the Sun never going beyond the *Tropicks*, that is to say, out of the *Torrid Zone* towards the Poles. § 1.

And those two *Zones* which are between the frozen, on the one side, and the other of the *Torrid*, are the *temperate Zones*; under which those parts of the *Earth* are most commodious for the Habitation of living Creatures: For the same Division is conceived to be made upon the Earth as was in the Heaven.

Therefore granting there are five like *Zones* upon the *Earth*, those which are under the Cold shall be cold, those under the Temperate temperate, and those under the Torrid Zone, extreamly hot.

It is also observable, that the half of the Globe from the Æquator to the North Pole is called the North *Hemisphere*, and the other Moiety towards the South is called the Southern *Hemisphere*.

Having divided the *Sphere* into five Parts, as naturally it is, it remains that the other necessary Circles be described; and there is a principal one in the eighth Sphere, which is more

§ 1. more demonſtrable upon a Cœleſtial Globe, being ſomewhat difficult to be comprehended only by the Imagination. However, it is a Circle as large as one of the *Zones*, but obliquely ſet upon the Torrid Zone, cutting it in certain Places, exceeding the Limits of the Tropicks, and croſſing in *Byas* (that is, obliquely) the *Æquator*: And this is what we call the *Zodiack*, where the Twelve Signs are; ſo called, by reaſon thoſe Signs by the Poſition of the Stars make certain Figures of *Animals*, and likewiſe of the Temper of thoſe *Animals* whoſe Names they bear. This Circle is divided into twelve Parts, the firſt whereof is *Aries* ♈, the next *Taurus* ♉, *Gemini* ♊, *Cancer* ♋, *Leo* ♌, *Virgo* ♍, *Libra* ♎, *Scorpio* ♏, *Sagittarius* ♐, *Capricornus* ♑, *Aquarius* ♒, *Piſces* ♓.

Now as to our Imagination, a Line paſſeth thro' the Torrid Zone called the *Æquator*, ſo through the middle of the Zodiack paſſeth a Line called the *Ecliptick*: For that when the Moon is in Oppoſition to the Sun in this Line, it cauſeth an *Eclipſe*; of which, more in the following Diſcourſe. And this Line never exceeds the Bounds of the *Tropicks*, there are other neceſſary Circles, as the *Horizon*, *Meridian*, &c. which ſhall be deſcribed before the end of this Treatiſe. But to proceed to the Planetary Orbs or Heavens.

The whole Heaven of the Planets turn under the Zodiack from the Eaſt to the Weſt, making the *Tour* of the World in the Space of twenty four Hours; by reaſon of their being carried by the Motion of the *Primum Mobile*, and this Motion is called a *rapid* Motion: For they

they have a contrary Motion, which is natural to them, which is from the Weſt to the Eaſt, which they continue notwithſtanding the other Motion, as we have already ſaid the Sun hath, and as Experience ſhews us; and as we ſhall make more clear by the Example of the two moſt apparent Planets, *viz.* the Sun and the Moon. § 1.

Manifeſt it is, when it is new Moon, ſhe being then near the Sun, that ſhe paſſeth with him towards the Weſt, according to the rapid Motion, making her Tour in twenty four Hours; and proportionable to her Increaſe, removes from the Sun towards the Eaſt, and thus by little and little gets ſo far, that the fourteenth Day when it is full Moon, ſhe is in the Eaſt at the time of Sun-ſet, which is made by the Motion proper and natural to her. By this Motion the Planets paſs through all the Signs of the *Zodiack* one after another; and tho' by Experience we know this Motion to be contrary to the firſt, it is no wonder that for divers Cauſes they may at the ſame time have divers Motions, as on divers other Occaſions: For if I am upon the Water in the Forecaſtle of a Ship ſailing to the Weſt, though I am carried with the Ship from the Eaſt, it does not hinder me to walk to the Stern towards the Eaſt: So that the Planets have a Motion proper, which is from the Weſt to the Eaſt, while they are making the Tour of the World; and departing from a certain Point in the Zodiack, in a certain time they return to the ſame Point again: But the one ſooner the other later, according to the Largeneſs of the Orbs they move in, or by

§ 1. by the Will of the *God* of Wonders, it is not therefore without Cauſe that *David* ſaid, *The Heavens declare the Glory of God, and the Firmament ſheweth his handy Work.*

It is alſo found by Experience, that the *Moon* which is in the loweſt Sphere, and conſequently hath a leſs Circle to make, finiſheth her Courſe in twenty eight Days; *Mercury* which is above her in eighty eight Days, *Venus* in two hundred and twenty five Days, the *Sun* in three hundred ſixty five Days and one Quarter, *Mars* in two Years, *Jupiter* in twelve, *Saturn* in thirty, and the *Eighth Heaven* in thirty ſix thouſand Years; that is to ſay, every hundred Years one Degree.

And this Difference of Motion among the Planets, one advancing faſter than another, is the Cauſe of the Conjunctions, Oppoſitions and Aſpects of the Planets, which vary the Diſpoſition of the Air: And from this Diverſity of Motion, and from their Paſſage through the twelve Signs in the *Zodiack*, *Prognoſticators* derive their *Predictions* of the Diſpoſitions of Seaſons of *Heat*, *Cold*, *Rain*, *Storms*, *Dearth*, or *Sickneſs*; all which would be tolerable, did they not intermeddle with what depends not upon the Influences of the Stars. But as to that part which is called *Judiciary*, in regard it requires an exact Supputation of Degrees, Hours, and Minutes, being not neceſſary to ſuch as are employ'd in Matters of great Moment, we leave it to the Learned, who profeſs it, and content our ſelves with their *Ephemerides*, a Work almoſt *Divine*: And it is ſufficient Occaſion for us to admire the wonderful Works of

of our *Creator*; how that by the Course of the Sun in the *Zodiack* is caused the four Seasons in the Year, *viz. Spring*, *Summer*, *Autumn*, *Winter*; and how that by the same Course the Moon increaseth and decreaseth, and how the *Eclipses* both of the one and the other doth happen. *Moses* in his Account of the Creation of the World, mentions only those two *Luminaries* which he calls the greater and the lesser Light. § 1.

So that the Sun by its proper Motion finisheth its Course in the Zodiack in one Year, or three hundred sixty five Days and one Quarter, and when he enters into the Sign *Aries*, which is part of the *Æquator*, then it is the beginning of *Spring*; and passing through *Taurus* and *Gemini*, draws near to the Point which is the highest, from us called *Cancer*, which is that Point or Part through which passeth the *Summer Tropick*: And being there the highest, and consequently the strongest, it causeth great Heat and long Days; of which more when I come to the Description of the *Horizon*. And continuing from *Cancer* through *Leo* and *Virgo*, he passeth into *Libra*, which is the other Sign answerable to *Aries* in the *Æquator*; and then begins *Autumn*, being near about the middle of *September*, and from *Libra* through *Scorpio* and *Sagittarius* enters into *Capricorn*, through the beginning whereof passeth the Northern *Tropick*: And then begins *Winter*, which continues till he be got through *Aquarius* and *Pisces*, and then he comes again into *Aries*; from whence he sets out, continuing constantly the same Course, which he finisheth, as hath been

+ Southern

§ 1. been already ſaid, in one Year: And Aſtronomers have found by long Experience, that in one Year there are three hundred ſixty five Revolutions, and one Quarter of the rapid Motion of the firſt Heaven, which are ſo many Days: For we call that Space of time a natural Day, which is between the riſing of the Sun and the ſucceeding riſing of the Sun, or from one Noon to the Noon following; and from this Quarter, which is ſix Hours, is made one Day every fourth Year, that the Account of the Year might agree with the Courſe of the Sun. This is attributed to *Cæſar* the *Dictator*; but in Reality, a Tropical Year is not three hundred ſixty five Days and ſix Hours, but five Hours and forty nine Minutes: So that every Year there is wanting eleven Minutes to make up the ſix Hours.

NOTE, The Obſervation of *Cæleſtial Bodies* are the ſureſt *Marks* and *Meaſures* of Time.

The Moon alſo goes on in the ſame Courſe through the twelve Signs in the Zodiack, without making Diſtinction of Seaſons; but in making that Courſe, ſhe ſhews her ſelf to us in four different Shapes or Forms: For being under the Sun in the ſame Sign, which Sign ever it is, ſhe is called New; and making her Progreſs towards the Eaſt in the Space of eight and twenty Days, in ſeven Days time ſhe paſſeth through three Signs of the *Zodiack*, and therefore is removed from the Sun the fourth part of the *Zodiack*, and then it is increaſing: And when removed from the Sun ſix Signs, that is to ſay half the *Zodiack*, ſhe is in the full, and is in the Eaſt at the very Hour the Sun ſets; and

and paſſing farther, when ſhe hath gone through three parts of the *Zodiack*, which is Nine Signs, then ſhe appears to be in her Decreaſe, or Laſt Quarter: And when ſhe hath paſſed through the Twelve Signs, and comes again under the Sun, ſhe is then again at New: And this is her perpetual and conſtant Courſe. § 1.

To the more plain underſtanding how that, in her Courſe, her Clearneſs Encreaſeth and Decreaſeth; firſt conſider, That the *Moon* is a Body *Round*, *Opaque* and *Globical*; that is to ſay, not Tranſparent, borrowing her Light from the *Sun*, which, by reaſon of the Roundneſs and *Opacity* of the *Moon*, cannot Illuminate her on every Side; becauſe all Rays of Light proceed in a direct Line: Inſomuch, as hath been already ſhewn, when ſhe is in Conjunction with the *Sun*, we ſee nothing of her Clearneſs; in regard the *Sun* being higher than her, and he Illuminating the Upper-part only: For her Body being Round, and not Tranſparent, the *Sun* can Illuminate no more than one half thereof, or ſomething more; and therefore then we ſee not any thing of her Clearneſs: But removing from the *Sun*, that half which was illuminated begins to appear on the Weſt-part thereof, the *Moon* tending towards the Eaſt: And this is what we call *Encreaſing*, with regard to us, in proportion to her Diſtance from the *Sun*; and when removed the Diſtance of the half of Heaven, we ſee the whole part which the *Sun* illuminates; and thence approaching the *Sun* on the other ſide, ſhe ſeems to Diminiſh, becauſe the Shining-part ſeems to

§ 1. turn its Countenance, and, in proportion to her drawing near to the *Sun*, we are deprived of the ſight of that Clearneſs. And, as hath been already ſaid, in divers times all the other Planets do the ſame; ſave only, that we cannot ſo ſenſibly obſerve either the *Diminution* or *Augmentation* of their Light: And, by means of the Diverſity of their Motions, with the Sign wherein they are, it happens that their Influences, to which inferiour Bodies are ſubject, are diverſe, and at the time of our Birth hath impreſſed in us various *Qualities* and *Temperaments*.

Now it ſometimes happens that thoſe two Great *Luminaries* are deprived of their Light; the *Moon* ſimply, the *Sun* only in appearance to us, which is call'd an *Eclipſe*.

As to the *Sun*, his Clearneſs is always after the ſame manner, not ſubject to any Viciſſitude, Alteration, or Change. The Eclipſe of the Sun is when the Moon is Diametrically conjoin'd to the Sun, both being in the *Ecliptick-Line*, which cannot happen but when ſhe is at New: And by reaſon of the Opacity and Obſcurity of the Body of the *Moon*, which cannot be penetrated by the Light of the *Sun*, it obſtructs our view of the Whole or Part of the Body of the *Sun*.

An *Eclipſe* of the *Moon*, may be either Total or Univerſal, and that for divers Reaſons: To the more eaſie underſtanding whereof, let us preſuppoſe, that the *Sun* making its ordinary Tour with the Heavens round the World, drives the Shadow of the Earth into the contrary Part: So that the Night is nothing elſe but the

the Shadow of the Earth; and this Shadow thus turning, growing narrow towards the Heaven, not in the Form of a *Column*, but in a Round *Pyramid*, and riſes up to a part of the Heavens, above the *Lunar*-Heaven: And, as hath already been affirm'd, that there is never any Eclipſe of the *Sun*, but when the *Moon* is at New; ſo there can be no Eclipſe of the *Moon*, but when ſhe is in the Full. For while the *Moon* doth traverſe the Point of the Shadow of the Earth, to us ſhe is not viſible during ſome ſpace of time, ſometimes more, ſometimes leſs, according as by her Courſe ſhe paſſeth through a broader or narrower quantity of the Shadow of the Earth; yet this doth not always happen, although every Fourteen Days ſhe is at Full, and in Oppoſition to the *Sun*, being then half the Heavens diſtance from Him; yet ſhe is not exactly Diametrically oppoſite, that being only when both are in Oppoſition to each other in the *Ecliptick-Line*, or very near: For ſhe is not in the *Ecliptick* more than twice every Revolution, as the *Sun* is not in the *Æquator* more than twice in its *Annual* Courſe. § 1.

The Body of the *Sun* is much greater than the *Earth*, as is manifeſt from the following Reaſon. If an Opaque or Dark Body, oppoſite to an Illuminated Body, be greater than the Illuminated Body, it makes an infinite Shadow; that is to ſay, which terminates nowhere, but the more remote, the greater: As the Vulgar Experience of thoſe who take Birds in the Night doth plainly ſhew, when they carry a Candle behind an Object much bigger than that Candle. And on the contrary, when

§ 1. an Illuminated Body is greater than the Dark Body which is oppoſite to it, the Shadow will terminate in a certain Point, and ſhall go on until ſuch time as the Rays, which proceed from the Illuminated Body, aſſemble and ſet bounds to the Shadow. This is ſo agreeable to Reaſon, that it needs no other Demonſtration.

It hath been here affirmed, That the Shadow of the Earth ſhuns the Sun, and extends it ſelf on the ſide contrary to him: If the Earth were greater than the Body of the Sun, the Shadow of the Earth (which would be much larger) every Full-Moon, would *Eclipſe* the Moon, which wou'd endure ſome Days both before and after; and this Shadow, by the ſame reaſon, wou'd darken all the Fixed and Wandring-Stars which ſhou'd be oppoſite to the Sun. And yet we ſee, that the Shadow does not obſcure any of the Stars, and but ſometimes cauſeth an *Eclipſe* of the Moon. It may therefore be neceſſarily concluded, That although this Shadow reacheth the Heaven of the Moon, it reaches no farther, and moreover, that it terminates in a *Pyramid*; the Rays of the Sun, who is greater than the Earth, aſſembling round the Shadow.

Briefly to Conclude concerning the Magnitude of the Sun. Since the Shadow of the Earth terminates in ſome certain Place, and that it riſes growing narrow at the end; it muſt be clear to a Rational Creature, That the Sun, which is an Illuminated Body, is larger than the Earth: For otherwiſe the Rays could not re-unite, and terminate the Shade of the Body of the Earth.

Hitherto

Hitherto we have only deſcribed the *Sphere*, § 1.
that is to ſay, the Number of the Heavens, and ſhewn their Motions, and on what Side they turn, and in how long Time: We have likewiſe divided them into certain Parts, which are call'd *Zones*, ſpeaking of their Qualities and Temperaments, and by which *Zones* the *Planets* move, and the reaſon of certain Accidents to them. And now that from thence ſome fruit may be drawn, to the better underſtanding of the Deſcription of the *Earth*, for that all this Diviſion of the Heaven ſhould have place in the Deſcription of the Earth, we muſt imagine, that the ſame Circles have place upon the *Terreſtrial Globe*, that it may be better underſtood what Part of the Earth correſponds to each Zone. By which means, with the help of *Maps*, it may eaſily be known, who thoſe are that inhabit the *Torrid*, *Temperate*, or *Frozen Zones*, and the reaſon why it is more Cold or Hot in one Place than another, and what is nearer, or more remote, from the *Eaſt*, *Weſt*, *South*, or *North*: Which could not be well underſtood, without a precedent Deſcription of the *Heavens*.

To the better underſtanding why the Days in one Climate are Longer than in another, and why under the *Æquator* they are always Equal to the Nights, and that in *March* and *September*, they are Equal throughout the World, we muſt neceſſarily ſpeak of the *Horizon*.

We muſt imagin the *Horizon*, or Limiter of our Sight, to be a Circle in the Sphere of the World: The *Latins* call it *Circulus Dividens*, becauſe it Divides the Sphere into Two Equal

§ 1. Parts, but not like the others before-mention'd; And you are to preſuppoſe, that in whatever Place you are, eſpecially upon a Hill or Mountain, caſting your Eye round, you ſee the Half of the Heaven. So that the *Horizon* is an Imaginary Circle, which Divides the Whole Heaven into Two Equal Parts; that is to ſay, that which we do ſee, from what is not ſeen by us.

Conſidering then a *Right-Sphere*; Suppoſe we are in ſome Part of the World under the *Torrid-Zone*, and directly under the *Æquator*, ſo that the *Æquator* being equally diſtant from the *Poles*, the South and North-Poles may be ſeen by us, and conſequently within our Horizon; that is to ſay, at that proper Place that ſeparates that Part of the Heaven which we ſee, and is above us, from that Part of the Heavens which is below us, and which is not ſeen by us, in that manner which we behold the Sun, which in the Morning, at its Riſing, enters into our Horizon: And the Line of Circumference which we may imagine to be in that Separation of the two Moieties of Heaven, is that we call the *Horizon*.

We may therefore ſay, That being under the *Æquator*, the Poles would be under our *Horizon*, and that the Sun continues there the ſpace of Twelve Hours, for that it makes the whole Circle of the World in Twenty four Hours: And thence it is manifeſt, that the Days are Equal to the Night; and that at Noon-tide, the Sun being in the *Æquator*, Bodies perpendicular to the *Horizon* have no Shadow: And ſo it is with thoſe Perſons inhabiting

biting that part of the Globe, having the Sun directly over their Heads, that is, in their *Zenith.* § 1.

Now removing out of the *Æquator*, this *Horizon* will ſhew us, by the Elevation of the Pole, how much our *Zenith* is diſtant from the *Æquator:* And, on the contrary, by the Diſtance from the *Æquator* to our *Zenith*, we ſhall know the Height of the Pole above our *Horizon.* And in regard more than Half the Heaven cannot be ſeen by us, the Bulk of the Earth obſtructing our View, and that being under the *Æquator*, our *Zenith* is in the *Æquator*; and by this means we ſee the Poles on one ſide and the other of the *Horizon*; it cannot be otherwiſe, but that departing from the *Horizon* ſome remarkable ſpace Northward, but that our *Zenith* changeth equally; and ſo much our Sight gaineth towards the North, and loſeth towards the South; and hereby we loſe the ſight of the South-Pole, and elevate the North-Pole above our *Horizon:* And this is call'd an *Oblique-Sphere*, in regard it ſeems to lean on one ſide.

Put the caſe that we are in the Middle of the *Temperate-Zone*, we ſhall find, by the Way which the Sun ſhall make in our Horizon, that Matters will ſoon be changed: For the Days there will be Equal to the Nights but twice in the Year, *viz.* in *March* and *September*; and the remaining part they will be always Unequal, as in the beginning of *Winter* they are Short, in the beginning of *Summer* very long; and that Bodies at Mid-day ſhall always caſt a Shadow. Which Things cannot be under-

§ 1. ſtood, without underſtanding the Nature of the *Horizon*; which cannot well be comprehended without ſome Inſtruction upon the Globe.

Take therefore a Terreſtrial-Globe whereon there is a moveable Circle for the *Horizon*, and I will declare the Reaſon of what ſhall happen, placing the Circle where it ought to be for thoſe who inhabit in the Middle of the *North-Temperate-Climate*, as thoſe who inhabit in *France*, you will find, that as much as the *Pole* ſhall be elevated above the *Horizon*, ſo much their *Zenith* ſhall be diſtant from the *Æquator*. And that all the Lines which the Sun deſcribes above the Horizon, by his Diurnal Motion, ſhall be Unequal, except it be when the Sun is in the *Æquator*, and that the Shorteſt Line of all is the *Southern-Tropick*, which we have already ſaid to be the entrance into *Capricorn*, and the beginning of Winter; and the Longeſt Line of the *Northern-Tropick*, which is the entrance of the Sun into *Cancer*, and the beginning of Summer.

And when the Days are Equal to the Nights, in the Middle of the Temperate-Zone, the Sun is in the *Æquator*; which is wonderful: The reaſon is, for that although the *Horizon* riſes or falls with regard to the Poles, it notwithſtanding cuts the *Æquator* in Equal Parts; for all the Circles which divide a Globe into Equal Parts, whether they cut them Obliquely, or at Right-Angles, they can no other way interſect but in Equal Parts. Now the knowing how much our *Zenith* is diſtant from the *Æquator*, is the knowing how much our Sphere

is

is Oblique; which the *Aſtrolabe* ſhews us in two manners, addreſſing our ſelves to the Pole elevated above the Horizon, or to the Sun at the Hour of Noon, adding or diminiſhing according to its Declination. To the underſtanding whereof, there will be ſome occaſion of Experience; which will not be difficult, after ſo ſhort and eaſie a Declaration. § 1.

The Reaſon is, The Inhabitants under the Pole have the *Æquator* for their *Horizon*; and hence it is that the half of the *Zodiack* always appears to them above the *Horizon*: And therefore ſuch as inhabit under the *Artick-Pole* ſee the Sun from the Month of *March* to the Month of *September*, and the other Six Months it is out of their ſight; as more evidently may be demonſtrated either by a *Cæleſtial* or *Terreſtrial Globe*. For you will find, if the *Æquator* be the *Horizon*, we cannot ſee the Sun but when it is above the *Horizon*. The like in the other Hemiſphere; whence thoſe that inhabit under either *Pole*, their Whole Year is but One Day and Night.

The *Antipodes* are thoſe whoſe Feet (the Earth interpoſing) are oppoſite to ours: The Inhabitants under the *Antartick-Pole*, are the *Antipodes* to thoſe under the *Artick*.

In Anſwer to a Vulgar Scruple. There is no more Danger of Falling Down, than that the Earth ſhould be deprived of its Natural Situation: The Heavens encompaſs the Earth; and our Bodies being Terreſtrial, they adhere to the Earth; for all things of Ponderoſity tend downwards; as a Bullet forc'd up perpendicularly, which, notwithſtanding the Violence of the

§ 1. the Powder, at length tends to the Centre of the Earth. And though the Air, by reaſon of its Rarity, yields to any violent Motion, it is not a Conſequence that it ſhould give place to the Globe of the Earth; for each Element preſerves its Station entirely, and can be in no other Place; ſo that it yields to no other Element, either in general or particular, without ſome Force; as may be proved by the Air, which though it be moveable, doth always more or leſs reſiſt any other Body: Experimented by any who run againſt the Wind; and likewiſe by that of an empty Veſſel's being plung'd in Water, which being full of Air, (for there is no Vacuity in Nature) and having a narrow Mouth, the Water, the Element of more gravity, tending downwards, to poſſeſs its natural ſtation, the Air ſallies out, and the Oppoſition between them, puts the Water into a violent Motion, with ſome Noiſe.

The Natural Cauſe of the Earth's continuing to be the Centre of the World, is its Gravity; from which ſtation it naturally cannot be removed; for then, contrary to its Nature, it ſhould aſcend.

All Ponderous things tend to the Centre of the Earth: And it is demonſtrable, that if a Hole were made in the Earth from one part to the other, and ſome weighty thing, as a Leaden-Bullet, ſhould fall into that perforated part, it would have its Motion no farther than the Centre, not beyond it, for then it would aſcend.

But to return to the *Antipodes*. The *Antipodes* are thoſe who are diametrically oppoſite to

to us, and distant from us one half of the Terrestrial-Globe. Imagine then a Line passing from our *Zenith* through our Head, and through the Centre of the Earth to the other side: The Point where this Line goes out at is the *Antipodes*. So that to find this upon the Terrestrial-Globe, first observe in what *Parallel*, that is to say, what Distance from the *Æquator* we are, and then observing where the *Meridian* doth cross the *Parallel*, which answers to ours in the other *Hemisphere*, under our *Horizon*, and in that Part shall be the *Antipodes*. But this is common. And thus you will find that they are contrary to us; when it is Day with them, it will be Night with us; Winter with them, Summer with us; when the Days are Short with them, they are Long with us: And still they have their Seasons after the same manner as we have ours; because being in the same *Parallel* with us, they are as much distant from the *Æquator* as we are. § 1.

Thus much of COSMOGRAPHY, so necessary to the due understanding of GEOGRAPHY.

SECT. II.

Of GEOGRAPHY, with its Definitions, *&c.*

GEOGRAPHY *is an Art Mathematical, which sheweth how the Situation of* Kingdoms, Provinces, Cities, Towns, Villages, Forts, Castles, Mountains, Woods, Havens, Rivers, § 2.

§ 2. Rivers, Creeks, *&c. being on the Surface of the Terreſtrial-Globe, may be Deſcribed, and Deſigned, in Commenſuration analogical to Nature and Verity, and moſt aptly to our View may be Repreſented: That is, It is a Deſcription of all the known Earth, imitated by Writing and Delineation; with all other things belonging thereunto.*

I ſhall not here inſiſt on the Names and Deſcription of *Kingdoms, Provinces, Cities, Villages*, &c. nor of their *Manners, Cuſtoms, Laws, Commodities*, &c. but ſhall only give you ſome general *Definitions* of the Diſtinction and Denomination of ſeveral Parts of the Earth and Water, with ſome *Problems* for finding the Diſtance of any two Places on the Terreſtrial-Globe.

DEFINITIONS.

1. The *Globe of the Earth*, is a Spherical Body, compos'd of Earth and Water, and is divided into *Continents, Iſlands* and *Seas*.

2. A *Continent*, is a great quantity of Land not ſeparated, interlac'd or divided by the Sea, wherein are *Kingdoms, Principalities* and *Nations*; as EUROPE, ASIA and AFRICA, are one *Continent*; and AMERICA is another.

3. An *Iſland*, is ſuch a Part of the Earth that is environ'd round with Water on every ſide; as the *Iſle* of *Great-Britain, Java, Wight*, &c.

4. A *Peninſula*, is ſuch a Tract of Land which being almoſt cut off from the Main-Land, and encompaſſed round with Water, yet nevertheleſs is joined unto the firm Land, by ſome little *Iſthmus*; as *Peloponneſus, Peruviana, Taurica, Cymtrica*, and *Morea* in the *Levant*.

5. An

5. An *Iſthmus*, is a little narrow Neck of Land which joineth the *Peninſula* unto the *Continent*. § 2.

6. A *Promontory*, is ſome high Mountain, which ſhooteth it ſelf into the Sea, the utmoſt end of which is call'd a *Cape*; as *Cape-Boon*, *Eſperance*, *Cape de Verde*, and *Cape de Coquibocao*.

7. The *Ocean*, is a general Collection of Waters, which environeth the World on every ſide, and produceth *Seas*, *Straits*, *Bays*, *Lakes* and *Rivers*: Of which, and other Waters, *Ovid* thus ſpeaks in his *Metamorphoſis*:

Tum Freta diffudit, rapidiſque tumeſcere ventis
Juſſit, & ambitæ circundare littora terræ.

"He ſpread the Seas, which then he did command
"To ſwell with Winds, and compaſs round the Land.

8. The *Sea*, is part of the *Ocean*, to which we cannot come but through ſome *Strait*; as the *Mediterranean*, or *Baltick-Sea*.

9. A *Strait*, is a part of the *Ocean* reſtrained within narrow bounds, yet openeth a way to the *Sea*; as, the *Straits* of *Gibralter*, *Helleſpont*, &c.

10. A *Creek*, is a crooked Shore, thruſting, as it were, two Arms forth to hold the *Sea*; as, the *Adriatick*, *Perſian* and *Corinthian-Creeks*; from whence are produced *Rivers*, *Brooks* and *Fountains*; which are engender'd of *Congealed Air* in the *Earth's Concavity*, and ſeconded by *Sea-water* creeping through the hidden Cranies of the Earth.

11. A *Bay*, is a great In-let of Land; as, the *Bays* of *Mexico* and *Biſcay*.

12. A *Gulph*, is a greater In-let of Land, and deeper than a *Bay*; as the *Gulphs* of *Venice* and *Florida*.

13. A

§ 2. 13. A *Climate*, is a certain ſpace of Earth and Sea, included within the ſpace of two *Parallels*; and there have been anciently accounted theſe ſeven, (*viz.*) 1. *Dia Meros*, 2. *Dia Syenes*, 3. *Dia Alexandria*, 4. *Dia Rhodes*, 5. *Dia Rhomes*, 6. *Dia Boriſthenes*, and 7. *Dia Ripheos*.

14. A *Zone*, is a certain ſpace of the Earth contained betwixt certain *Circles* of the *Sphere*, of which there are Five; (*viz.*) the *Torrid* or *Burning-Zone*, *two Temperate*, and *two Frigid* or *Frozen-Zones*.

The *Torrid-Zone*, is that which lieth on each ſide the *Equinoctial*, whoſe bounds are the two *Tropicks* of ♋ and ♑.

The *two Temperate-Zones*, are thoſe which lie betwixt the two *Tropicks* of ♋ and ♑, and the *Polar-Circles*.

The *two Frigid-Zones* lie between the *Artick* and *Antartick-Circles*, and their reſpective *Poles*.

PROP. I.

How to find the Diſtance of any two Cities or Places, which differ only in Latitude.

In this *Propoſition* there are two Varieties, which are theſe:

1. If both the Places lie under one and the ſame *Meridian*, and on one and the ſame ſide the *Equinoctial*, either on the North or South-ſide thereof; then ſubſtract the leſſer *Latitude* from the greater, the Difference multiply by 70, (becauſe 70 *Engliſh* Miles is a Degree) the Product is the Diſtance of the two Places in *Engliſh* Miles. If you multiply by 60, it gives

gives it in *Meridional* Miles, that is, ſuch Miles as are uſed in *Navigation*. § 2.

2. But if the two Places lie under one and the ſame *Meridian*, but the one on the South-ſide of the *Equinoctial*, and the other on the North-ſide; then add both their *Latitudes* together, and multiply their Sum, as before, it gives their Diſtance.

PROP. II.

To find the Diſtance of any two Places, which differ only in Longitude.

There are alſo in this *Propoſition* two Varieties.

1. The two Places may both lie under the *Equinoctial*, and ſo have no *Latitude*: And if ſo, ſubſtract the leſſer *Longitude* out of the greater, and convert the Remainder into Miles, by multiplying their Difference, as before; ſo have you the Diſtance of any two Places ſo poſited. And here *Note*, That if the Difference of *Longitude* be above 180 *deg.* then you muſt uſe the Supplement of that to 360.

2 But if the two Places differ only in *Longitude*, and lie not under the *Equinoctial*, but under ſome other *Intermediate Parallel of Latitude*, between the *Equinoctial* and one of the *Poles*. Then to find their Diſtance, this is the *Analogy* or *Proportion*:

As Radius or S. 90°,
To Sc. *of the Latitude*;
So is S *of* ½ X *of Longitude*,
To S. *of* ½ *their Diſtance*: Which being Doubled, and converted into Miles, gives the required Diſtance.

NOTE,

§ 2. NOTE, We compare the *Sine* of *half the Difference of Longitude*, by reaſon the *whole Difference of Longitude* may be above 90 *Deg.* of which there is no *Sine*.

PROP. III.

To find the Diſtance of any two Places, which differ both in Latitude and Longitude.

In this *Propoſition*, three Varieties do preſent themſelves to our View.

1. One of the Places may lie under the *Equinoctial* and have no *Latitude*, and the other under ſome *Parallel of Latitude* between the *Equinoctial* and one of the *Poles*. In ſuch caſe, obſerve this *Analogy* or *Proportion*:

As Radius or S. *of* 90°,
To Sc. *of their* X. *of Longitude*;
So is Sc. *of the given Latitude*,
To Sc. *of their Diſtance required.*

2. But if both the Places propoſed ſhall be without the *Equinoctial*, but on the one ſide, either both towards the *North*, or both towards the *South*, then their Diſtance may be found, by this *Analogy* or *Proportion*:

As Radius or S. 90°,
To Sc. *of their* X *of Longitude*;
So is Tc. *of the greater Latitude*,
To the T. *of a fourth Arch*: Which ſubſtracted from the *Complement* of the *leſſer Latitude*, the Remainder muſt be the *fifth Arch*. Then ſay,
As Sc. *of the fourth Arch*,
To Sc. *of the fifth Arch*;
So is S. *of the greater Latitudes*,
To Sc. *of the Diſtance of the two propoſed Places.*

3. The

3. The two Places propounded may be ſo ſituated, that one of them may lie on the *North*, and the other on the *South*-ſide the *Equinoctial*; the Diſtance of Places ſo ſituated may be obtained by this *Analogy* or *Proportion*: § 2.

As Radius or S. 90°,
To Sc. *of* X. *of Longitude*;
So is Tc. *of the greater Latitude*,
To T. *of a fourth Arch*: Which being ſubſtracted out of the Sum of the other *Latitude*, and the *Radius* or 90°, the Remainder is a *fifth Arch*. Then ſay,

As Sc. *of the fourth Arch*,
To Sc. *of the fifth Arch*;
So is S. *of the Latitude firſt taken*,
To Sc. *of the Diſtance required.*

Theſe are all the Varieties of the Poſitions of Places on the *Terreſtrial-Globe*: For if the Diſtance of any two Places be required, they muſt fall under one or other of theſe Varieties, and may be obtained by one or other of the *Proportions*, mention'd in the three foregoing *Propoſitions*.

Alſo, if you know the *Latitude* and *Longitude* of any two *Fixed-Stars*, or their *Right Aſcenſion* and *Declination*; then, by theſe Rules, their Diſtance may be found; which is of good Uſe in *Aſtronomy*. It may alſo be applied to *Circular Sailing*; of all other ways, the moſt perfect: Which is treated of in its due Place.

CHAP. VIII.

Of NAVIGATION.

NAVIGATION, *ſo call'd from* Navis *a* Ship, *is an Art Mathematical; which ſheweth how, by the ſhorteſt good Way, by the apteſt Direction, and in the ſhorteſt Time, to conduct a Ship from any one Place unto any other Place aſſigned: It hath been highly eſteemed by the Ancients: It is the Glory, Beauty, Bulwark, Wall and Wealth of the* Engliſh *Nation, and the Bridge that joins it to the Univerſe.* Navigation *is commonly divided into Three ſorts of Sailing; viz.* Plain-Sailing, Mercator's-Sailing, *and* Circular-Sailing: *Of all which Three Parts, I ſhall treat in their Order.*

SECT. I.

Of Plain-Sailing, *or Sailing by the* Plain-Chart.

§ 1. PLain-Sailing, or Sailing by the *Plain-Chart*, is the plaineſt way, and the Foundation of all the reſt: And although the Ground and Projection of the *Plain-Chart* is erroneous; yet is it more facile to the Learner, and may ſerve indifferently near the *Equinoctial*; becauſe there the *Degrees* of *Longitude*, as well as

as the *Degrees* of *Latitude*, are Equal: Each *Degree* being divided into 60 *Minutes* or *Miles*, though they are somewhat more than *English Miles*, each *Minute* or *Mile* containing about 6000 Feet. § 1.

PROP. I.

The Doctrine of Right-lined Triangles, both Right, and Oblique-Angled, applied to Propositions of Plain-Sailing.

The Rhumb, and Distance sailed thereon, being given; to find the Difference of Latitude, and the Departure from the Meridian.

Admit a Ship sails N. W. by N. 372′ or Miles, or 124 Leagues; I demand her *Difference of Latitude*, and Departure from the *Meridian*?

In the *Triangle* ABC, the *Hypothenuse* AC representeth the Distance sailed, or *Rhumb-line*, BC the Departure from the *Meridian*, and AB the Difference of *Latitude*.

1. To find which, say, Fig. 44.
As Radius or S. 90°,
To Log. Distance sailed 372′;
So is Sc. *V.* A. of the Course 56° 15′,
To Log. cr. AB 309 $\frac{3}{10}$ Minutes: Which being divided by 60′, giveth 5° 9′ 18″ for the *Difference* of *Latitude*.

2. To find the Departure from the *Meridian*, say, Fig 44.
As Radius or S 90°,
To Log. Rhumb-line AC 372′;
So is S. *of V.* of the Course A 33° 45′,
To Log. cr. BC 206 $\frac{6}{10}$ Minutes, the Departure from the *Meridian*: Which divided by 60, giveth 3° 26′ 36″ for the *Difference* of *Longitude*.

§ 1. NOTE, That by this *Propoſition*, you may keep an Account how much you have ſailed either *Eaſt* or *Weſt*, *North* or *South*: The firſt of theſe is call'd *Northing* or *Southing*, the ſecond *Eaſting* or *Weſting*.

PROP. II.

By the Rhumb and Difference of Latitude given; to find the Diſtance, and the Departure from the Meridian.

Admit a Ship ſail N. W. by N. until her Difference of *Latitude* be $309\frac{1}{10}$ Minutes; I demand her *Diſtance* ſailed, and her *Departure* from the *Meridian*?

1. To find the *Diſtance*, ſay,

Fig. 44. *As* Sc. *of V.* of the Courſe A 56° 15′,
To Log. cr. AB the X. of *Lat.* $309\frac{1}{10}$ Minutes;
So is the Radius or S 90°,
To Log. AC 372′ the Diſtance ſailed.

2. For the *Departure*, ſay,

Fig. 44. *As* Sc. of A *V.* of the Courſe 56° 15′,
To Log. cr. AB X. of *Lat.* $309\frac{1}{10}$ Minutes;
So is S. of *V.* of the Courſe A 33° 45′,
To Log. cr. AB $206\frac{6}{10}$ Minutes, the Departure required.

☞ By the help of this *Propoſition*, when your *Latitude*, by *Obſervation*, doth not agree with your Dead-Reckoning, (kept by the former *Propoſition*;) Then, according to this Rule, you may make your Way ſail'd agree with your *obſerved Latitude*, and ſo correct your Account or Dead-Reckoning: For the *obſerv'd Latitude* is moſt to be rely'd on.

PROP.

PROP. III.

By knowing the Diſtance of the Meridians of two Places, and their Difference of Latitude; to find the Rhumb, and Diſtance.

Admit A to repreſent the *Lizard*, AB the *Parallel* thereof, C St. *Mary's Iſland*, (being one of the *Azores*) and CB the *Meridian* thereof.

In the *Triangle* ABC there is given the ſide AB 816 *min.* the Diſtance of the *Lizard* from the *Meridian* of St. *Mary's*, and the ſide CB their Difference of *Latitude* 768 *min.* I demand the *Rhumb*, i. e. the *Angle* at C, and the Diſtance of the *Lizard* from St. *Mary's*? Fig. 45.

1. For the *Rhumb* or Angle at C, ſay,
As Log. cr. CB 768′,
To Radius or S. 90°;
So is Log. cr. AB 816′,
To T. *of the Rhumb*, or Angle at C 46° 44′, and is from the *Lizard* unto St. *Mary's* to the fourth *Rhumb* of the *Meridian*, and 1° 44′ more, *viz.* S. W. and 1° 44′ Weſterly, or from Saint *Mary's* to the *Lizard* N. E. and 1° 44′ Eaſterly: And thus it ſhall be by the *Plain-Chart*.

2. For their *Diſtance* AC, ſay,
As S. *Rhumb* or *V.* at C 46° 44′,
To Log. cr. AB 816′;
So is Radius or S. 90°,
To Log. Hypoth. AC 1120′$\frac{1}{10}$, which is the Diſtance of the *Lizard* unto St. *Mary's Iſland*, and ſuch ſhould be the Diſtance by the *Plain-Chart*. Fig. 45.

§ 1.

PROP. IV.

Admit two Ships to ſet ſail from one Port, one Ship ſails W. S. W. 40′, the other W. by N. ſo far until ſhe finds the firſt Ship to bear from her S. E. by E. I demand the ſecond Ship's Diſtance from the Port, and their Diſtance aſunder?

In the *Triangle* ADE, let A repreſent the *Port*, AD the W. S. W. Courſe, and AE the *Courſe* W. by N.

1. To find the ſecond Ship's Diſtance from the *Port*, ſay,

Fig. 46. *As* S. of *V.* at E 22° 30′,
To Log. cr. AD 40′;
So is S. of *V.* at D 123° 45′,
To Log. cr. AE 86$\frac{23}{100}$ *min.* which is the Diſtance required.

2. To find the two Ships their Diſtance aſunder, ſay,

Fig. 46. *As* S. of *V.* at E 22° 30′,
To Log. cr. AD 40′;
So is S. of *V.* at A 33° 45′,
To Log. cr. DE 58 $\frac{14}{100}$ *min.* which is the Diſtance required.

PROP. V.

Two Ships ſet ſail from two Ports which lie N. and S. of each other, the one ſails from the Northermoſt Port 72$\frac{29}{100}$′, and then meets the other Ship, which came from the Southermoſt Port, on a N. W. Courſe, and had ſailed from thence 56$\frac{80}{100}$′; I demand the Rhumb on which the firſt Ship made her Way, and alſo the Diſtance between the two Ports?

In

In the *Triangle* ADE, let A be the Southermoſt Port, AD the Courſe and Way of the ſecond Ship N. W. $56\frac{8}{100}'$, let E be the Northermoſt Port, ED the Courſe and Way of the other Ship $72\frac{12}{100}'$, and D the Place where they both meet. § 1.

1. To find the *Rhumb* on which the firſt Ship ſailed, ſay,
As Log. cr. DE $72\frac{12}{100}'$,
To S. of *V. at* A 45° 00′;
So is Log. cr. DA $56\frac{8}{100}'$,
To S. of *V.* at E 33° 45′: Which ſheweth the Courſe of the firſt Ship to be S. W. by S. Fig. 47.

2. To find the Diſtance between the two Ports A and E, ſay, Fig. 47.
As S. of *V. at* A. 45° 00′,
To Log. cr. DE $72\frac{12}{100}'$;
So is S. of *V. at* D 101° 15′,
To Log. cr. EA 100′, which is the required Diſtance.

PROP. VI.

Admit a Ship coming off the Main-Ocean, and I had ſight of a Promontory or Cape, by which it is my deſire to ſail; I find it to bear from me S. S. E. and diſtant, by Eſtimation, 33′ or Miles: But keeping ſtill on my Courſe S. until the Evening, having ſailed 36′ or Miles; I wou'd then know how the Cape bears, and its Diſtance from the Ship?

In the *Triangle* ADE, Admit that at A, I do obſerve the *Cape* D to bear from me S. S. E. 33′, and having ſailed from A to E 36′ South; I deſire to know its Diſtance and Bearing? In

§ 1. the *Triangle* there is therefore given AD 33′, AE 36′, and the *Angle* at A 22° 30′.

Fig. 48.

1. To find the *Angle* at E, ſay,

As Z. cr' AE, and AD 69′,
To X. cr' AD, and AE 03′;
So is T. ½*VV* unknown D and E 78° 45′,
To the T. of 12° 20′: Which taken from 78° 45′, leaves the *Angle* at E 66° 25′: So that the *Cape* D then bears from me E.N.E. and 01° 05′ Northerly.

2. To find the Diſtance of the *Cape* ED from the Ship, ſay,

As S *V.* at E 66° 25′,
To Log. cr. AD 33′;
So is S. *V.* at A 22° 30′,
To Log. cr. ED 13$\frac{78}{100}$ Miles diſtant: So that the *Cape* is then diſtant from the Ship 13$\frac{78}{100}$ Miles.

PROP. VII.

Two Ports, both lying in one Latitude, diſtant 64′ or Miles, the Weſtermoſt of thoſe Ports lieth oppoſite to an Iſland more Northerly diſtant therefrom 47′ or Miles, which Iſland is alſo diſtant from the Eaſtermoſt Port 34′ or Miles; I demand the Coaſt from the Weſtermoſt Port to that Iſland?

In the *Triangle* ADE, let A be the *Weſtermoſt Port*, and E the *Eaſtermoſt Port*, diſtant aſunder 64′; and let D be the *Iſland*, diſtant from A 47′, and from E 34′; Then is the *Angle* at A required, which is the *Courſe* or *Rhumb* from the *Weſtermoſt Port* unto the *Iſland*: To find which, ſay,

Fig. 49.

As

As Log. cr. AE 64′, § 1.
To Log. Z. cr' AD, and ED 81′;
So is Log. X cr' AD, and ED 13′,
To Log of a certain Line AO $16\frac{414}{1000}$:
Which added to AE 64′, is $80\frac{414}{1000}$;
The ½ whereof is AB $40\frac{207}{1000}$.

Then again, ſay,
As Log. cr. AD 47′,
To Radius or S 90°;
So is Log. AB $40\frac{207}{1000}$,
To Sc. *V.* at A 58° 51′, that is, N. E. by E. 2° 36′ Eaſterly; which is the Courſe from the *Weſtermoſt Port* A, unto the *Iſland* D.

SECT. II.

Of Sailing by the True Sea-Chart, *commonly call'd* MERCATOR's Chart.

THE True *Sea-Chart*, commonly call'd MERCATOR's *Chart* *, performs the ſame Concluſions as the *Plain-Chart*, and almoſt as ſpeedily, but far more exactly: Becauſe all Places may be laid down hereon, with the ſame Truth as on the *Globe* it ſelf, both to their *Latitudes*, *Longitudes*, *Bearing* and *Diſtance* from each other. § 2.

* *But is indeed the Invention of our Learned Countryman Mr.* Edw. Wright, *although this Stranger hath almoſt got the Name and Praiſe thereof.*

And here it will be neceſſary to have a TABLE of *Meridional Parts*, which I have extracted out of Mr. *Wright's Tables*, to every Tenth Minute of *Latitude*, accounting it in ſingle Miles, or Minutes of the *Equinoctial*; and have hereunto annexed the ſaid Table,

A TA-

§ 2.

A *TABLE* of Meridional Miles.

The Deg. of Lat.	*The Minutes of each Degree.*						The Difference.
	0	10	20	30	40	50	
	The Meridional Miles.						
0	0	10	20	30	40	50	10
1	60	70	80	90	100	110	10
2	120	130	140	150	160	170	10
3	180	190	200	210	220	230	10
4	240	250	260	270	280	290	10
5	300	310	320	330	340	350	10
6	360	370	380	390	400	410	10
7	421	431	441	451	461	471	10
8	481	491	501	511	521	532	10
9	542	552	562	572	582	592	10
10	603	613	623	633	643	653	10
11	664	674	684	694	704	715	10
12	725	735	745	755	766	776	10
13	786	797	807	817	827	838	10
14	848	858	869	879	889	900	10
15	910	920	931	941	951	962	10
16	972	983	993	1004	1014	1024	10
17	1035	1045	1056	1066	1077	1087	10
18	1098	1108	1119	1129	1140	1150	10
19	1161	1172	1182	1193	1203	1214	10
20	1225	1235	1246	1257	1267	1278	11
21	1289	1299	1310	1321	1332	1342	11
22	1353	1364	1375	1386	1396	1407	11
23	1418	1429	1440	1451	1462	1473	11
24	1484	1499	1505	1516	1527	1538	11
25	1549	1561	1572	1583	1594	1605	11
26	1616	1627	1638	1649	1661	1672	11
27	1683	1694	1705	1717	1728	1738	11
28	1751	1761	1773	1785	1796	1808	11
29	1819	1830	1842	1853	1865	1867	12

§ 2.

A *Table* of Meridional Miles.

The Deg. of Lat.	*The Minutes of each Degree.*						The Difference.
	0	10	20	30	40	50	
	The Meridional Miles.						
30	1888	1899	1911	1923	1934	1946	12
31	1958	1969	1981	1993	2004	2016	12
32	2028	2040	2052	2063	2075	2087	12
33	2099	2111	2123	2135	2147	2159	12
34	2171	2183	2195	2207	2219	2231	12
35	2244	2256	2268	2281	2293	2305	12
36	2318	2330	2342	2355	2367	2380	12
37	2392	2405	2417	2430	2442	2455	12
38	2468	2481	2493	2506	2519	2532	13
39	2544	2557	2570	2583	2596	2609	13
40	2622	2635	2648	2662	2675	2668	13
41	2701	2714	2718	2741	2754	2768	13
42	2781	2795	2808	2822	2835	2849	13
43	2863	2876	2890	2904	2918	2931	14
44	2945	2959	2973	2987	3001	3015	14
45	3030	3044	3050	3072	3086	3101	14
46	3115	3130	3144	3159	3173	3188	14
47	3202	3217	3232	3247	3261	3276	15
48	3291	3306	3321	3336	3351	3366	15
49	3382	3397	3412	3428	3443	3459	15
50	3474	3490	3505	3521	3537	3553	16
51	3568	3584	3600	3616	3632	3649	16
52	3665	3681	3697	3714	3730	3747	16
53	3763	3780	3797	3814	3830	3847	17
54	3864	3881	3899	3616	3933	3950	17
55	3968	3985	4003	4020	4038	4056	18
56	4074	4092	4110	4128	4146	4164	19
57	4182	4201	4219	4238	4257	4275	19
58	4294	4313	4331	4351	4370	4390	20
59	4409	4428	4448	4468	4487	4507	20

§ 2.

A *Table* of Meridional Miles.

The Degs. of Lat.	*The Minutes of each Degree.*						The Difference.
	0	10	20	30	40	50	
	The Meridional Miles.						
60	4527	4547	4567	4588	4608	4629	20
61	4643	4670	4691	4711	4733	4754	21
62	4775	4756	4818	4839	4861	4883	22
63	4905	4927	4949	4972	4994	5017	23
64	5039	5062	5085	5018	5132	5155	23
65	5179	5203	5226	5250	5275	5299	24
66	5324	5348	5373	5390	5423	5449	25
67	5474	5500	5520	5552	5678	5704	26
68	5631	5658	5685	5712	5739	5767	27
69	5795	5823	6021	5879	5908	5937	28
70	5966	5996	6125	6055	6085	6115	30
71	6146	6177	6208	6239	6271	6303	31
72	6335	6368	6401	6431	6468	6501	33
73	6535	6570	6605	6640	6675	6718	35
74	6747	6783	6820	6857	6895	6933	37
75	6972	7010	7050	7089	7130	7170	40
76	7211	7253	7295	7338	7381	7424	43
77	7469	7513	7559	7605	7651	7698	46
78	7746	7795	7844	7894	7944	7996	50
79	8048	8100	8154	8209	8264	8320	55
80	8377	8435	8495	8555	8616	8678	60
81	8742	8806	8872	8939	9007	9077	68
82	9148	9221	9295	9371	9449	9528	77
83	9609	9692	9778	9865	9954	10046	88
84	10141	10238	10338	10441	10547	10656	105
85	10770	10887	11007	11133	11263	11398	128
86	11539	11686	11839	11999	12168	12344	165
87	12521	12718	12927	13150	13388	13644	230
88	13920	14221	14550	14914	15321	15783	386
89	16318	16950	17726	18729	20152	22623	

The

§ 2.

The Uſe of the TABLE *is demonſtrated by the ſeveral* Examples *following, after this manner:*

PROP. I.

To find by the Table, *what Meridional Parts are contained in any Difference of Latitude.*

In this *Propoſition*, three Varieties preſent themſelves unto our View.

1. When one Place is under the *Equinoctial*, the other having *North* or *South Latitude*, his *Meridional Parts* correſponding, is to be eſteemed for the *Meridional Difference of Latitude.*

EXAMPLE: One Place upon the *Equator*, the other in 27° of *North Latitude*; What's the *Meridional Difference* of *Latitude?*

Look in the *Table*, under Title *The Deg. of Lat.* and againſt 27 you will find 1683, the *Meridional Difference of Latitude* required.

2. When both Places are towards one of the *Poles*, then the *Meridional Parts* of the *leſſer*, taken from the *Meridional Parts* of the *greater Latitude*, the *Remainder* is the *Meridional Difference* required.

EXAMPLE: Suppoſe one Place in 14° of *North Latitude*, the other in 23° of the ſame *Latitude*; What's the *Meridional Difference of Latitude?*

The *Meridional Parts* anſwering to 14° of *Lat.* is 848, and the *Meridional Parts* anſwering

§ 2. ing to 23° of *Lat.* is 1418; the *Difference* of theſe, which is 570, is the *Meridional Difference of Latitude* required.

3. When one Place hath *North*, and the other *South Latitude*, their correſponding *Meridional Parts* added together gives the *Meridional Difference of Latitude* ſought. Thus having found them out, they may thus be applied.

If the *Degrees of Latitude* had been the ſame as in the laſt *Example*, only one *North*, the other *South*, the *Sum* of the *Meridional Parts*, *viz.* 2266 would be the *Meridional Difference of Latitude*.

PROP. II.

By knowing the Latitudes, and the Difference of Longitude of any two Places; to find the Rhumb, and Diſtance.

Admit there be a *Port* in the *Latitude* of 50° 00′ *North*, and another in the *Latitude* of 13° 12′ *North*, and their *Difference of Longitude* is 52° 57′ *Weſt*; I demand the *Rhumb* and *Diſtance*?

In the *Triangle* Abc, let Ab repreſent the proper *Difference of Latitude*, bc the *Departure*, Ac the *Diſtance* ſailed, A the *Angle* of the *Courſe*, c the *Complement* of the *Courſe*.

Fig. 50.

In the *Triangle* ABC, AB is the *Meridional Difference* of *Latitude*, BC the *Difference* of *Longitude*, A the *Angle* of the *Rhumb*, C the *Compl.* of the *Angle* of the *Rhumb*: Theſe things being underſtood, the Work evidently appears to be the ſame as in *Right-Angled Plain Triangles*.

There

There is then required, firſt, the *Difference* of *Latitude*, and this falls under the ſecond Variety. § 2.

One Port lieth in Lat. 50° 00′ N. 3475
The other lieth in —— 13 12 N. 0799
The Merid. Diff. of Lat. —— 2676 } M. Parts

1. To find the *Rhumb* or *Courſe*, ſay,
As Merid. X. *Lat.* 2676′,
To Radius or S. 90°;
So is X. of *Longitude* 3177′,
To T. of the *Rhumb* 49° 53′: The *Courſe* therefore is S. W. ½ W. *&c.*

2. To find the *Diſtance*, Fig. 50.

Lat. 50° 00′
Lat. 13 12

Proper Diff. of Lat. — 36 48 *which is* 2208′.

As Sc. *Courſe* 40° 7′,
To proper X. of *Lat.* 2208′;
So is Radius or S. 90°,
To the Diſtance 3426 *Minutes*, as required.

PROP. III.

By knowing the Latitudes, and Diſtance of two Places; to find the Rhumb, and Difference of Longitude?

1. To find the *Rhumb* or *Courſe*, ſay,
As the Diſtance ſailed,
To Radius or S. 90°;

So

§ 2. *So is the* X. of *Latitude,*
To Sc. of the *Rhumb required.*

2. To find the *Difference of Longitude,* ſay,
As Radius or S. 90°,
To the X. *of Latitude in Merid. Parts;*
So is T. *of the Rhumb,*
To the X. *of Longitude required.*

PROP. IV.

By knowing the Latitude and Rhumb of two Places; to find their Diſtance, and Difference of Longitude.

1. To find the *Diſtance,* ſay,
As Sc. *of the Rhumb,*
To the X. *of Latitude;*
So is Radius or S. 90°,
To the Diſtance required.

2. To find the *Difference of Longitude,* ſay,
As Radius or S. 90°,
To the X. *of Latitude in Merid. Parts;*
So is T. *of the Rhumb,*
To the X. *of Longitude required.*

PROP. V.

By knowing the Rhumb, Difference of Longitude, and one Latitude; To find the other Latitude, and the Diſtance.

1. To find the other *Latitude,* ſay,
As T. *of the Rhumb,*
To the X. *of Longitude in Parts;*

So

So is Radius or S. 90°,
To the Merid. X. *of Latitude required.*

§ 2.

2. To find the *Diſtance*, ſay,
As Sc. *of the Rhumb,*
To the X. *of Latitude*;
So is Radius or S. 90°,
To the required Diſtance.

PROP. VI.

By knowing the Diſtance, one Latitude, and Rhumb; to find the other Latitude, and Difference of Longitude.

1. To find the *Difference of Latitude*, ſay,
As Radius or S. 90°,
To the Diſtance;
So is Sc. *of the Rhumb,*
To the X. *of Latitude required.*

2. To find the *Difference of Longitude*, ſay,
As Radius or S. 90°,
To the Merid. X. *of Latitude*;
So is T. *of the Rhumb,*
To the X. *of the Longitude required.*

PROP. VII.

By knowing the Latitudes and Departure of two Places; to find their Diſtance, Rhumb, and Difference of Longitude.

Suppoſe one Place in the *Latitude* of 50° 20′ North, the other in 47° 30′ of the ſame *Latitude*, and their *Departure* 130 *Miles*; What's their

§ 2. *true Distance, Course,* and *Difference of Longitude?*

First, find the *common Difference of Latitude*, (by taking the *lesser Latitude* from the *greater*) which is 2° 50′, or 190 *Miles*; and also the *Meridional Difference of Latitude*, which is 258: Then say,

As the common Dif. of Lat. 190 *Miles,*
To the Meridional Dif. of Lat. 258;
So is the Departure 130 *Miles,*
To the Difference of Longitude 177. Which 177 divided by 60, gives 2° 57′, the *true Difference of Longitude.*

Having thus gotten the *Difference of Longitude*; the *Distance* and *Rhumb* is gotten by PROP. II.

This *Case* is very useful, (*though omitted by the former Author*) for by it we sum up each Day's Work when taken from the *Log-board*, and enter it into the *Journal*, in order to find the *true Longitude* and *Latitude* the Ship is in at any time.

SECT. III.

Of Circular-Sailing, *or Sailing by the* Arch *of a* great Circle.

§ 3. THIS is, of all other, the most exact way of Sailing, and above all other most perfect, shewing the Neareſt Way, and Distance between any two Places: And although it is hardly possible

ſible to keep cloſe unto the *Arch* of a *great Circle*, yet it is of great advantage to keep conveniently near it, eſpecially in an *Eaſt* or *Weſt Courſe*. In the former *Propoſitions* of Sailing, we uſed *Meridians*, *Parallels* and *Rhumbs*, as the *Sides* of every *Triangle*, whether by the *Plain* or *Mercator's Chart*: But in *Circular Sailing* the *Rhumbs* are not uſed ſo, becauſe they are *Heliſpherical-lines*, and not *Circles*; nor the *Parallels*, becauſe they are not *great Circles*: Whereas the *Sides* comprehending every *Spherical Triangle* are *Arches* of *great Circles*: Therefore here we uſe *Arches* of the *Meridians*, of the *Equinoctial*, and of other *great Circles* deſcribed, or ſo imagined to be deſcribed, from one Place unto another, on the *Spherical Superficies* of the *Earth* and *Sea*. § 3.

Therefore here ariſeth Two things obſervable: And,

1. If the two Places lie under the *Equinoctial*, then is their Poſition *Eaſt* and *Weſt*; and their Diſtance is their *Difference of Longitude*, converted into *Miles*. Or,

2. If the two Places propoſed be in one and the ſame *Meridian*, then is their Poſition *North* and *South*, and their *Diſtance* is their *Difference* of *Latitude* converted into *Miles*.

And thus far doth *Circular Sailing* agree with the former; their *Difference* will evidently appear, by theſe following *Propoſitions*.

PROP. I.

Two Places, the one under the Equinoctial, the other in any Latitude given; alſo, their Difference of Longitude given; to find,

§ 3. 1. *Their Diſtance in the Arch of a great Circle.*
2. *The direct Poſition of the firſt Place from the ſecond.*
3. *And of the ſecond Place from the firſt.*

Here we call the *Angles* which the *Arch* makes with the *Meridians* of the Places propounded, the *Angles* of the *Direct Poſitions* of thoſe Places one from the other; becauſe the *Arch* of a *great Circle* drawn between two Places is the neareſt Diſtance; and the moſt direct Way of the one, to the other Place. Now I ſhall not here demonſtrate it by *Schemes*, as I have done in the other two *Sections*, but ſhall only lay down the Proportions, whereby the required Parts may be found; and ſo leave the ingenious Seaman to practiſe it with *Schemes* at his leiſure: And,

1. To find the Neareſt Diſtance from Place to Place, in the *Arch* of a *great Circle*, ſay, according to the 10th *Caſe* of *Rectangled Spherical Triangles*,

As the Radius,
To Sc. of X. of *Longitude*;
So is Sc. of X. of *Latitude*,
To Sc. of the *Diſtance* in the *Arch* required.

2. For the *Direct Poſition*, ſay, by the 11th *Caſe*, thus:

As the Radius,
To S. of X. of *Latitude*;
So is Tc. of X. of *Longitude*,
To Tc. of *V.* of *Poſition* required.

3. For

3. For the *Direct Position* of the ſecond Place from the firſt, ſay, by the 11th *Caſe*, thus: § 3.

As the Radius,
To S. of the X. of *Longitude*;
So is Tc. of X. of *Latitude*,
To Tc. of *V.* of *Poſition* required.

PROP. II.

Two Places propoſed, the one lying under the Equinoctial, the other in any Latitude given, with their Diſtance in a great Circle of the ſame Places being alſo known; to find,

1. *Their Difference of Longitude.*
2. *The Direct Poſition from the firſt to the ſecond Place.*
3. *And from the ſecond to the firſt Place.*

1. For their *Difference of Longitude*, ſay, by *Caſe* 12.

As Sc. of the *Latitude*,
To Radius;
So is Sc. of their *Diſtance* in the *Arch*,
To Sc. of their *Difference of Longitude* required.

2. Now to find the *Direct Poſition* from the firſt Place to the ſecond, ſay by the 13th *Caſe*: And, 3dly, For the *Direct Poſition* from the ſecond Place to the firſt, ſay by the 14th *Caſe* of *Rectangulars*.

PROP. III.

Two Places lying in one Latitude given, their Difference of Longitude being alſo known; to find,

§ 3. 1. *The Neareſt Diſtance of thoſe two Places.*

2. *The Direct Poſition of one Place from the other.*

The *Reſolution* of this *Propoſition* depends on the 9th *Caſe* of *Oblique Spherical Triangles*; by ſuppoſing the *Oblique Triangle* to be transfigured or converted into two *Rectangulars*, by a ſuppoſed *Perpendicular*: And then,

1. To find the Neareſt Diſtance in the *Arch*, ſay, by the 8th *Caſe* of *Rectangulars*,

As the Radius,
To Sc. of the *Latitude*;
So is S. of ½ X. of *Longitude*,
To S. of half the required *Diſtance*: Which doubled, giveth the *Diſtance* of the two Places in the *Arches*, as ſought.

2. For the *Direct Poſition*, ſay, by the 9th *Caſe*,

As the Radius,
To S. of the *Latitude*;
So is T of ½ X. of *Longitude*,
To Tc. of *V.* of *Poſition* required.

PROP. IV.

Two Places lying both in one Latitude given, and the Neareſt Diſtance being alſo known; to find,

1. *Their Difference of Longitude.*

2. *The Direct Poſition of one Place from the other.*

The *Reſolution* of this *Propoſition* falls under the 11th *Caſe* of *Oblique Spherical Triangles*; for

for here you have the three Sides of the *Triangle* given, *viz.* the *Arch of Diſtance*, and the other two Sides (are both Equal) being the *Complements* of the Places *Latitude*: And here, ſeeing the two *Sides* are Equal, therefore the two *Angles* of *Poſition* are alſo Equal. Now there is required the Three *Angles*? § 3.

1. To find their *Difference of Longitude*, add the double of the *Complement of Latitude* to the *Arch of Diſtance*; then from half this Sum deduct the *Arch of Diſtance*, and then proceed in all points as you ſee in *Caſe* 11th: So ſhall their *Difference of Longitude* be obtained.

2. To find their *Direct Poſition*:

Firſt, To the double *Complement of Latitude* add the *Arch of Diſtance*, then from half that *aggregate* deduct the *Complement of Latitude*, and then work as before; ſo ſhall the *Direct Poſition* be attained.

PROP. V.

Two Places propoſed lying in one Latitude, and the Diſtance of thoſe Places in their Parallel given; to find,

1. *Their Difference of Longitude.*
2. *Their Diſtance in the Arch of a great Circle.*
3. *The Direct Poſition of the one from the other.*

Now you muſt underſtand, that as the *Semi-diameter* of a *Parallel*, is in Proportion to the *Semi-diameter* of the *Equinoctial*; ſo is any number of *Miles* in that *Parallel*, to the *Minutes* of *Longitude* anſwering to thoſe *Miles*: So that if

§ 3. we ſuppoſe the *Semi-diameter* of the *Equinoctial* to be *Radius*, then the *Semi-diameter* of any *Parallel* is the *Sign* of that *Parallel's Diſtance* from the *Pole*, that is, the *Sc.* of the *Latitude* of that *Parallel*: Therefore,

1. To find the *Difference of Longitude*, ſay,
As Sc. of the *Latitude*,
To the Radius;
So is the Diſtance in that Parallel,
To the Difference of Longitude required.

2. Now the *Difference of Latitude* being obtained the *neareſt Diſtance* may be found, as in the 3d *Prop.* aforegoing: 3. So likewiſe may the *Angles of Poſition* alſo.

PROP. VI.

By knowing the Neareſt Diſtance of two Places, their Difference of Longitude, and one of their Latitudes; to find the Direct Poſition thereof from the other.

This *Propoſition* falls under the 1ſt Caſe of *Oblique Spherical Triangles*, and is thus reſolved: Therefore,
As S. *of the Diſtance* of the two Places,
To S. *of their* ∡. *of Longitude*;
So is Sc. *of the Latitude* of the one Place given,
To S. *of the Direct Poſition* from the other as was ſo required.

PROP. VII.

By knowing the Latitudes of two Places, and likewiſe their Difference of Longitude; to find,

1. The

1. *The Diſtance in the Arch.* § 3.
2. *The direct Poſition from the firſt to the ſecond Place.*
3. *The direct Poſition from the ſecond to the firſt Place.*
4. *The Latitudes and Longitudes by which the Arch paſſeth.*
5. *The Courſe and Diſtance from Place to Place through thoſe Latitudes and Longitudes, according to* Mercator.

I ſhall here make uſe of Mr. *Norwood*'s Example of a Voyage from the *Summer-Iſlands*, unto the *Lizard:* Now becauſe the Work is various, I have therefore illuſtrated it with a *Scheme*, and ſhall be as brief and facile as poſſible. Therefore,

In the *Triangle* ADE, let A be the *Summer-Iſlands*, whoſe *Latitude* is 32° 25′, AD the *Complement* thereof 57° 35′, let E repreſent the *Lizard* whoſe *Latitude* is 50° 00′, and ED the *Complement* thereof 40° 00′, and let their *Difference of Longitude*, namely, the *Angle* ADE, be 70° 00′, now D repreſenteth the *North-Pole*, and AE an *Arch* of a *great Circle* paſſing by theſe two Places: Now ſee the Operation: Fig. 51.

1. By having the *Complements* of the *Latitudes* of the two Places, *viz.* AD 57° 35′, and ED 40° 00′, and their *Difference of Longitude*, namely, the *Angle* EDA, 70° 00′: You may find the Neareſt Diſtance EA to be 53° 24′, by *Caſe* 9. § 5. *Chap.* 5.

2. Then having found the Neareſt Diſtance in the *Arch* EA to be 53° 24′ (or 3204 *Miles*,) the *Angle of Poſition* from the *Summer-Iſlands* to the Fig 51.

§ 3. the *Lizard*, namely, the *Angle* DAE, may be found by *Caſe* 1. § 5. *Chap.* 5. to be 48° 48′, that is N. E. and 03° 48′ Eaſterly.

3. And alſo, by the ſame *Caſe*, may the *Direct Poſition* from the *Lizard* to the *Summer-Iſlands*, namely, the *Angle* AED, be found to be 81° 10′, that is W. by N. and 2° 25′ Weſterly.

4. In order to the finding the *Latitudes* and *Longitudes* by which the *Arch* paſſeth, firſt let fall the *Perpendicular* DB, ſo is the *Oblique Triangle* ADE converted into two *Rectangulars*, *viz.* ABD, and DBE: Secondly, by *Caſe* 8. § 4. *Chap.* 5. you may find the Length of the *Perpendicular* DB to be 39° 26′, whoſe *Complement* is 50° 34′ which is the greateſt *Latitude* by which the *Arch* ABE paſſeth; ſo the greateſt *Obliquity* of the *Equinoctial* from that *Circle* is 50° 34′. — Thirdly, by *Caſe* 9. § 4. *Chap.* 5. you muſt find the *Vertical Angles*, viz. ADB, and BDE, which will appear, the *Angle* ADB to be 58° 31′, and EDB to be 11° 29′: Now theſe things being obtained, the *Latitudes* by which the *Arch* paſſeth at every 10th *deg.* of *Longitude* from A, may be found, by reſolving the ſeveral *Right-Angled Triangles*, viz. BDc, BDf, *&c.* ſubſtracting 10° from ADB 58° 31′, there remains BDc 48° 31′; and ſo for the reſt, as in the *Table*. Now by knowing theſe *Angles* laſt found, and the *Perpendicular* BD before found to be 39° 26′, you may, by *Caſe* 3. § 4. *Chap.* 5. find the *Latitudes* of the ſeveral Points A, c, f, g, h, i, B and E, to be as in the ſubſequent *Table*.

Fig. 51.

Fig. 51.

BDc	48°	31′
BDf	38	31
BDg	28	31
BDh	18	31
BDj	08	31

5. Thus

5. Thus having found the *Latitudes* and *Longitudes* of the *Arch*, and the other required *Parts* afore-mention'd; we now come to ſhew, how the *Courſe*, and the *Diſtance* from Place to Place, according to *Mercator*, may be found. So to find, firſt the *Courſe* and *Diſtance* Ac, now there is given the *Latitude* of A 32° 25′, and of c 38° 51′, and their *Difference of Longitude* is 10° 00′, now the *proper Difference of Latitude* is 6° 26′, or 386′, and *Meridional Difference of Latitude* is 475′. Now knowing theſe things by *Prop.* 2. § 2. *Chap.* 8. you may find the *Courſe* from A to c to be N E. 51° 38′, and the *Diſtance* Ac to be 622′; and ſo thoſe Rules proſecuted, will ſhew the *Courſe* and *Diſtance* from c to f, from f to g, from g to h, *&c.* So of the reſt, which for brevity ſake I ſhall omit, and leave the Ingenious Seaman to calculate at his Pleaſure. § 3.

	Latitude.	*Longitude.*
A	32° 25′	00° 00′
c	38 51	10 00
f	43 34	20 00
g	46 54	30 00
h	49 04	40 00
i	50 15	50 00
B	50 34	60 00
E	50 00	70 00

Fig. 51.

Fig. 51.

I might hereunto annex many more *Propoſitions* of *Circular-Sailing*; but becauſe of the ſmallneſs of this *Treatiſe*, and that thoſe *Propoſitions* already handled, being by the Ingenious Seaman well underſtood, will be ſufficient to enable him to perform any other Concluſion in *Circular-Sailing* whatſoever, I therefore here omit, and haſten forwards unto the other Parts of this *Mathematical Treaſury*.

A TA-

§ 3.

A TABLE of Angles, which every Rhumb maketh with the Meridian.

Theſe on this ſide the WEST, *encline towards the* North-end *of the Meridian.*	*Angles of Inclination with the Meridian.*		*Theſe on this ſide the* EAST, *encline unto the* North-end *of the Meridian.*
Rhumbs.	NORTH		Rhumbs.
N. & by W.	11°	15′	N. & by E.
N. N. W.	22	30	N. N. E.
N.W. & by N.	33	45	N.E. & by N.
North-Weſt	45	00	North-Eaſt
N.W. & by W.	56	15	N.E. & by E.
W. N. W.	67	30	E. N. E.
W. & by. N.	78	45	E. & by N.
WEST	90	00	EAST
W. & by S.	78	45	E. & by S.
W. S. W.	67	30	E. S. E.
S.W. & by W.	56	15	S. E. & by E.
South-Weſt	45	00	South-Eaſt
S. W. & by S.	33	45	S. E. & by S.
S. S. W.	22	30	S. S. E.
S. & by W.	11	15	S. & by E.
Rhumbs.	SOUTH		Rhumbs.
Theſe on this ſide the WEST, *encline unto the* South-end *of the Meridian.*			*Theſe on this ſide the* EAST, *encline towards the* South-end *of the Meridian.*

Note, That if you account in Quarter of Points, add for One Quarter 2° 48′, for One Half 5° 37′, for Three Quarters 8° 26′, (not regarding the *Seconds* in Navigation.)

CHAP.

CHAP. IX.

Of SURVEYING.

IT hath been a Cuſtom among Modern Authors that have treated on this Subject, that before they enter'd on the Work it ſelf, to give the Deſcription of the Inſtruments uſed in, and chiefly appertaining to the Art of SURVEYING, *viz.* the *Circumferentor*, the *Theodolite*, the *Plain-Table*, and the *Semi-circle*; concerning the Deſcriptions of which Inſtruments I ſhall not here treat, but refer you unto thoſe Authors that have largely and amply deſcribed them. I ſhall in this place only demonſtrate the Uſe of the *Semi-circle*, in taking the *Plots* of *Encloſures*, *Champain-Plains*, *Woods* and *Mountains*, divers ways *; and alſo, in taking of *Acceſſible* and *Inacceſſible Heights* and *Diſtances*: And alſo, I ſhall ſhew the Uſe of a little Inſtrument call'd a *Protractor*, in the delineating on Paper the *Plot* of a *Field*, &c. which Inſtrument being ſo commonly known, and ſo generally uſed, makes me omit the Deſcription thereof as ſuperfluous.

* *Which Inſtrument, and the Plain-Table, I eſteem as the two aptest Inſtruments for Surveying of Land*; i. e. *the Plain-Table for ſmall Encloſures, and the Semi-circle for Champain Plains, Woods and Mountains.*

As for your *Chain*, I wou'd have you have it made of good round *Wyre*; to contain in length four *Poles* or *Perch*, to be divided into an hundred equal Parts call'd *Links*.

And here, before we enter on the Work it ſelf, it will be neceſſary to underſtand how, by the *Protractor*, to lay down an *Angle* of any quantity of degrees propounded, or to find the quantity of an *Angle* given.

SECT. I.

Of the Uſe of the Protractor.

PROP. I.

By the Protractor, *to Protract an Angle of any quantity of degrees propounded.*

§ 1. AN *Angle* may be laid down eaſily, according to the Directions of *Prop.* 5. § 1. *Ch.* 4. but becauſe this is more uſeful in *Surveying*, Fig. 52. Know, that if it be required to Protract an *Angle* of 50 deg. having drawn the line AB at pleaſure, place the Centre of your *Protractor* on C, and moving it by your *Protracting-Pin*, until the *Meridional-line* thereof be directly on the line AB, then make a Mark by the diviſion of 50° on the *limb* of the *Protractor*, as at D, and draw the line CD, ſo ſhall the *Angle* DCB be an *Angle* of 50 *degrees*.

PROP.

§ 1.

PROP. II.

By the Protractor given, to Measure an Angle given.

This is performed by the line of *Chords* also, according to *Prop.* 6. § 1. *Chap.* 4. and by the *Protractor* is found thus: Suppose DCB were an *Angle* whose quantity were desired, to find which, first the *Centre* of the *Protractor* applied unto the *Angular* Point C, and its *Meridional-line* lying justly with CB, you shall perceive the Point D to touch the *limb* of the *Circle* at 50 *deg.* Therefore I conclude the Measure of the *Angle* DCB to be 50 *degrees.* Fig. 52.

SECT. II.

Of the manifold Use of the Semi-circle, *in taking the Plots of small Enclosures, Plains, Woods or Mountains, divers Ways.*

PROP. I.

How to take the Plot of a Field, by the Semi-circle, *at one Station taken in any part thereof, from whence all the Angles may be seen, and measuring from the Station unto every Angle thereof.*

§ 2.

SUppose ABCDEF were a Field, and 'tis required to take the Plot thereof: Having placed marks at all the Angles thereof, and made choice of your Station, which let be K; at which place your Instrument, and turning Fig. 53.

it

§ 2. it about until the *Needle* hang over the *Meridian* or *North* and *South-line* of the *Chart*, there ſcrew it faſt: Then directing your ſight to A, you'll find the Degree out, by the *Index*, to be 40° 15′: Then meaſuring K A with your *Chain*, it appears to be 5 *Chains*, and 20 *Links*, which note down in your *Field-book*: And ſo do by all the reſt, untill you have found all the Angles and Diſtances from your Station K, to each reſpective Angle; which finiſhed, your Work will ſtand thus:

Angles.	D	M	C	L
A	40	15	5	20
B	88	00	6	10
C	130	00	5	50
D	200	00	7	00
E	250	00	5	00
F	310	00	5	20

PROP. II.

How to delineate on Paper any Obſervation taken, according to the Doctrine of the laſt Propoſition.

Fig. 53. Upon your Paper draw a Line to repreſent the *Meridian-line*, as M, H; then placing the Centre of your *Protractor* on the Point K, laying the *Meridian-line* of the *Protractor* on the *Meridian-line* M, H; then ſeeing the Angle at A was 40° 15′, make a mark againſt 40° 15′ of the *Protractor*, as at A, and ſo do with all the other Angles, as you find them in your *Table*: Then

Then remove your *Protractor*, and draw the Lines KA, KB, *&c.* This done, lay down on each line his reſpective Meaſure, as it appeareth in the *Table.* Laſtly, Draw the Lines AB, BC, *&c.* So have you on the Paper the exact Figure of the *Field.* § 2.

This Way will ſerve very well for any ſuch Field, Cloſe *or* Common, *from one Point in which you may ſee all the Angles; Provided that the Hedge, Bank, or whatever Fence it be, run ſtraight between every Angle, and not waving in and out: In ſuch Caſe, this Way or Method is as good as any, if there be not too many Angles.*

PROP. III.

How, by the Semi-circle, *to take the Plot of a Field, at one Station, in any Angle thereof, from whence you may view all the other Angles, by meaſuring from the Stationary-Angle, unto all the other Angles.*

Admit A, B, C, D, E, F, G, to be a Field, whoſe Plot is required: Place your *Semi-circle* at G, and turning it about until the *Needle* hang over the *Meridian* or *North* and *South-line* of the *Chart*, and there ſcrew it faſt: Then direct your Sights to the ſeveral Angles, *viz.* B, C, D, *&c.* in order one after the other, and ſo ſhall each reſpective Angle be found, as in the ſubſequent *Table:* Then with your *Chain* meaſure from your *Stationary-Angle* G, to all the other reſpective *Angles*; which done, you have finiſhed, and the Work ſtandeth as in the following *Table.* Fig. 154

§ 2.

Angles.	D	M	C	L
B	40	00	5	00
C	88	00	6	00
D	120	15	6	40
E	165	00	6	30
F	193	00	3	40
A	348	07	4	00

PROP. IV.

How to delineate any Observation taken according to the Doctrine of the last Proposition.

Fig. 54. Upon your Paper draw a straight line as M,N, then take a Point therein as G, to represent the *Stationary-Angle*, to which Point apply the *Centre* of your *Protractor*, (in all respects as is before taught;) then, according to the Notes in the *Table*, prick off all the Angles, *viz.* B, C, *&c.* according to their due quantity; then draw all the lines, *viz.* GB, GC, GD, *&c.* and on them place their respective measure, (as appeareth in your Notes;) lastly, draw the lines AB, BC, CD, *&c.* So is there on the Paper the exact Figure of the *Field*, as was required.

This Method is seldom used in Surveying any Field; because we cannot stand just in the Corner, by reason of the branching out of the Hedges: but where 'tis bounded with Banks and Ditches, this Way is very commodious and short.

PROP.

§ 2.

PROP. V.

How, by the Semi-circle, *to take the Plot of a Field, at two Stations, by meaſuring from each Station to the viſible Angles; the Field being ſo Irregular, that from no one Place thereof all the Angles can be ſeen.*

Admit A, B, C, D, E, F, G, H, I, K, to be the Figure of a Field, whoſe Plot is required: Having made choice of your two Stations, *viz.* Q and P, and placed Marks in all the Angles; then place your *Semi-circle* at Q, and there fix it, with the *Needle* hanging over the *Meridian* of the *Chart*, repreſented by R, Q, X, and direct your Sights unto all the viſible Angles, *viz.* A, B, C, D, E, and F, and note down the quantity of each Angle in your *Field-book*: Then meaſure with your *Chain* from your Station Q, to the Angles A, B, C, D, E and F; and their Length ſo found, note down in your *Field-Book* alſo. Fig. 55.

This done, direct your Sight unto your *ſecond Station* P, and note down in your *Field-book* the *Degree* of *Declination*, of your *ſecond Station* P, from the *Meridian*: Then meaſure the *Stationary-Diſtance* PQ with your *Chain*, and note it down in your *Field-book* alſo. Fig. 55.

Then remove the *Inſtrument* unto P, your *ſecond Station*, and there fix it, with the *Needle* hanging over the *Meridian-line* of the *Chart*, repreſented by TPB; then direct your Sights to the ſeveral *viſible Angles* at this *ſecond Station*, *viz.* F, G, H, I and K, in order one after another, and note down the quantity of each

§ 2. Angle in your *Field-book*; then with your *Chain* meaſure from your *Station* P, to theſe ſeveral Angles G, H, I and K, (in all reſpects as at the *firſt Station* Q;) and their Length ſo found, note down in your *Field-Book* likewiſe: So have you finiſhed your Obſervation, and your Work ſtandeth thus:

The Obſervation taken at the Firſt Station Q.

Fig. 55.

Angles.	D	M	C	L
A	50	0	6	60
B	80	0	7	65
C	140	12	12	0
D	220	7	11	10
E	270	5	12	60
F	330	0	6	0

The *Declination* of the *Station* P, from the *Meridian* R Q X, is 30° 00′, and the *Stationary Diſtance* Q P is 9 *Chains*.

The Obſervation taken at the Second Station P.

Fig. 55.

Angles.	D	M	C	L
F	227	11	7	50
G	297	0	12	0
H	347	16	9	90
I	60	0	6	0
K	90	0	6	26

NOTE,

NOTE, That the manner of taking the Plot of a large *Champain Field*, at many Stations, is almoſt the ſame with this *Propoſition*; for he that can do the one, can alſo perform the other: Therefore, for brevity-ſake, I here omit it, as ſuperfluous. § 2.

PROP. VI.

How to delineate any Obſervation taken according to the Doctrine of the laſt Propoſition.

Upon your Paper draw the *Meridian-line* R Q X, then place the Centre of your *Protractor* on Q (repreſenting your *firſt Station*,) and its *Meridional-line* lay equal to R Q X, then prick off the *Angles* viſible at your *firſt Station* Q, *viz.* A, B C, D, E and F, of their due Quantity; then draw Q A, QB, *&c.* laying on them their correſponding Meaſure, noted in your *Field-book*. Now becauſe your *ſecond Station* P doth decline 30° 00′ from the *Meridian* R Q X, prick off 30° 00′, and draw P Q, making it 9 *Chains*, as in your *Field-book* appeareth, ſo doth P repreſent your *ſecond Station*: Then in all reſpects as before, place your *Protractor* at P your *ſecond Station*, and draw the *Meridian* T P B *parallel* to R Q X; then prick off the ſeveral *Angles*, viz. F, G, H, I and K, of their due Quantity, and then draw P F, P H, P I, *&c.* of their due Length; laſtly, draw the lines A B, B C, C D, *&c.* and ſo ſhall you have on your Paper the exact Figure of the *Field*, as required. Fig. 55.

This Method is very proper to be uſed in Champain-Fields, *or in large* Heaths *or* Commons.

§ 2.

PROP. VII.

How by the Semi-circle *to take the Plot of a Field at two Stations, which lieth remote from you, when either by Oppoſition of Enemies you may not, or by ſome other Impediment you cannot come into the ſame.*

Fig. 56. Admit the Figure A, B, C, D, E, F, to be a Field into which by no means you can poſſibly enter, and yet of neceſſity the Plot thereof muſt be had, for the obtaining of which, chuſe any *two Stations*, it mattereth not whether near at hand, or far off, ſo that all the *Angles* may be ſeen: Let your *two Stations* be H and L, (the full length of the Field, if poſſible) then place your *Inſtrument* at H, and fixing it, as is afore-ſhewed, direct your Sights to the ſeveral *Angles* of the Field, *viz.* A, B, C, *&c.* orderly one after another, obſerving their Degrees as is afore-taught, noting it down in your *Field-book*; then take up your *Inſtrument*, leaving a mark in its room at H, and meaſure with your *Chain* from H unto L your *ſecond Station*, which note down in your *Field-book*; then placing your *Inſtrument* at L your *ſecond Station*, and (as is before taught) fixing it there, make the like Obſervation to the ſeveral *Angles*, viz. A, B, C, D, *&c.* as at the *firſt Station* H, and note it down in your *Field-book* alſo: And having ſo done, you have finiſhed, and your Work ſtandeth as in the following *Table*.

Obſerva-

§ 2.

Observations at the First Station H, *are,*

1. *Angles.*	D	M
A	104	0
B	88	7
C	59	0
D	48	0
E	26	0
F	21	30

Fig. 56.

The *Angle* from H the *first Station*, unto L the *second Station*, is 180° 00', the *Stationary-Distance* H L is 60 *Chains.*

Observations at the Second Station L, *are,*

2. *Angles.*	D	M
A	16	0
B	39	0
C	50	9
D	74	0
E	100	0
F	29	15

PROP. VIII.

How to delineate any Observation taken according to the Doctrine of the last Proposition.

Upon your Paper draw a Line as H L, which make equal to 60 *Chains*; then placing the Centre of your *Protractor* on H your *first Station*, prick off all the Angles A, B, C, *&c.* as you find them in your *Field-book*, and draw H A, H B, H C, *&c.* at pleasure; then remove your *Protractor* unto your *second Station* L, placing it as before, and prick off all the Angles A, B, C, D, *&c.* as you find them in your *Field-notes*, and draw the Lines L A, L B, L C, *&c.* at length, until they intersect the former Lines Fig. 56.

§ 2. HA, HB, *&c.* in the Points A, B, C, *&c.* which *Points of Interſection* are the *Angles* of the *Field*; laſtly, draw A B, BC, C D, *&c.* So ſhall you have on your Paper the Figure of your *Field* required.

PROP. IX.

How by the Semi-circle, *to take the Plot of a great Champain-Plain, Wood, or other over-grown Ground, by meaſuring round about the ſame, and making Obſervation at every Angle thereof.*

Fig. 57. Admit A, B, C, D, be the Figure of a *large overgrown Champain-Field*, whoſe Plot is required: Firſt place your *Inſtrument* at A, laying the *Index* on the Diameter, and turn it about until you eſpy the *Angle* at D, and there fix it faſt, and direct your Sights to B, and note the *Degree* cut by your *Index*, in your *Field-book*, (as afore is taught) then remove your *Inſtrument* to B, and there make the like Obſervation, and ſo to C, and D, noting it down in your *Field-book*, as afore; Then with your *Chain* meaſure the Sides A B, B C, C D and D A, whoſe Length note down in your *Field-book*; and ſo you have finiſhed, and your Work ſtandeth thus:

Angles.	D	M	C	L
DAB	100	0	12	20
ABC	117	15	10	0
BCD	71	30	19	20
CDA	71	15	12	20

PROP.

§ 2.

PROP. X.

How to delineate any Obſervation taken according to the Doctrine of the laſt Propoſition.

Upon your Paper draw the Line A B, at pleaſure, and placing the Centre of your *Protractor* on the Point A, prick off an *Angle* of 100°, and draw A D, ſetting on it, and alſo on A B, their correſponding Meaſure, in your Notes; then on B protract an *Angle* of 117° 15′, draw B C of its due length; then draw the Line C D: So have you the exact Figure of the *Field* on your Paper. Fig. 57.

This is the general, beſt, and ſureſt Way of Surveying, *eſpecially of large Quantities of* Land, *which conſiſt of divers Parcels; alſo, when we take the Plot of a Lordſhip or Manour, we begin to take the Boundary or Out-line after this Method, obſerving (as we go round) the Buttings of the Hedges and Banks, alſo the Croſſings of Roads and Ditches, noting them down in the* Field-Book, *with their Diſtances from their laſt proper Angle: From which Notes and Obſervations, the Principal Line, with the Buttings of the* Fences *or Diviſionary Lines of ſuch Lordſhip or Manour, are firſt delineated, in order to the cutting the ſame into its true and proper Diviſions.*

PROP. XI.

How to take the Plot of any Field, *by the help of the* Chain *only.*

Admit the Figure A, B, C, D, E, to repreſent a Field whoſe Plot is required: To obtain which, Fig. 58.

§ 2. which, firſt meaſure the Sides CD, CB, and BD, and note their due Length down in your *Field-Book*, and then meaſure the Sides CA, and BA, and then note down their Length in your *Field-Book*: Then meaſure the Sides BE, and ED, (for the Sides BC and BD were before known) which note down in your *Field-Book*: So is your *Field* A, B, C, D, E, reduced into three *Triangles*, viz. CBD, CAB, and BED, the Length of whoſe Sides are all known. Thus you have finiſhed, and the Work ſtands as you ſee:

Fig. 58.

	Sides.	*Ch.*	*Lin.*
In the Triangle CBD	CD	5	97
	CB	8	28
	BD	8	25
In the Triangle CAB	CA	4	42
	CB	8	28
	BA	6	51
In the Triangle BED	BE	5	28
	ED	5	25
	BD	8	25

PROP. XII.

How to delineate any Obſervation taken according to the Doctrine of the laſt Propoſition.

Fig. 58. Upon your Paper draw a ſtraight Line, as CD, make it 5 *Chains*, $\frac{27}{100}$; take CB in your *Compaſſes*, and ſtrike an *obſcure Arch*; then take BD,

BD, and with that extent in D croſs the former *Arch* in B, and draw BC, and BD: Then take in your *Compaſſes* BE, and on B ſtrike an *obſcure Arch*; then take DE, and alſo croſs the former *Arch* in E, and draw BE, and ED: Laſtly, take the Line CA, and on C ſtrike an *obſcure Arch*; then take AB, and on B interſect the former *Arch* in A, then draw CA, and AB; ſo have you on your Paper the exact Figure of the *Field* A, B, C, D, E, as was required. § 2. *Fig.* 58.

The beſt way of doing it, (as I find by Practice) is thus: Draw an *Eye-draught*, as near the ſhape as you can, ſuppoſe ABEDC; then meaſure all the Sides, ſetting down each Meaſure upon its correſpondent Side: This done from any one *Angle*, as B, (from whence you may ſee all the reſt) meaſure *Diagonals*, ſetting down upon them their proper Meaſure; from which rough-Draught, as well as from the Field-Book-Notes, the *Field*, *Court*, *Yard*, &c. may be Plotted.

This Method is very exact and true, and very proper for a ſmall Encloſure, or for a Court, Yard, Garden, or the like, where there are not many Angles.

SECT. III.

Of finding the Area *or ſuperficial Content of any Field lying in any Regular or Irregular Form; by reducing the Irregular Fields into Regular Forms.*

HAving already ſhewed how to take the Plot of any Field divers Ways, by the § 3.

Semi-

§ 3. *Semi-circle* and *Chain*, and alſo by the *Protractor* how to delineate the Draught thereof on Paper, *&c.* I come now to ſhew how the *Area* or *ſuperficial Content* of a Field may be attained, *i. e.* how many *Acres*, *Roods* and *Perches* are therein contained. To which end, Know, *That a Statute Pole or Perch contains* 16½ *Feet*; *That* 40 *of thoſe Perches in length, and* 4 *in breadth, makes an Acre*: So that an *Acre* contains 160 *Perches*, and a *Rood* 40 *Perches*, according to the Statute of 33d of *Edw.* I.

Now the Original of the Menſuration of *Land*, and all other *Superficies*, depends on the Menſuration of certain *Geometrical Figure's*; as a *Triangle*, *Square*, &c. which may be meaſured according to the Directions of § 2. *Chap.* 4. of *Geometry*: It would therefore here be ſuperfluous to make a repetition of things already handled: I ſhall therefore omit it, and come to ſhew how any *Field* lying in any *Irregular Form*, may be Meaſured by converting it into *Regular Figures*; for it ſeldom happeneth, but that the Plot of a Field is an Irregular Figure, conſiſting of three, four, five, or more Unequal Sides.

If it be a *Triangle*, it is Meaſur'd by *Chap.* 4. § 2. *Prop.* 3.

If it be a *Four-ſided Irregular Figure*, 'tis divided into two *Triangles*, as in *Plate* 4. *Fig.* 59. The Four-ſided Irregular Figure ABDC is divided by the *Diagonal* AD, drawn from oppoſite Corners; then let fall two *Perpendiculars* from B and C to the *Baſe* AD: This done, find the *Area* of the two *Triangles* ABD, ACD; which may be done at once, (*viz.*) *by Multiplying*

ing

ing the Sum of both the Perpendiculars *by Half the* Baſe, *or the Whole* Baſe *by Half the Sum of the* Perpendiculars. By which Rule, the Content of the Four-ſided Figure or *Trapezium* ABCD is found to be 630 *Perches.* § 3.

If it be an *Irregular Figure*, 'tis Meaſured by *Chap.* 4. § 2. *Prop.* 7.

PROP. I.

How to reduce any Number of Perches *into* Acres, *and, on the contrary,* Acres *into* Perches.

To find how many *Acres* are contained in any Number of *Perches* given, you muſt conſider that 160 *Perches* do make a *Statute Acre*; therefore if you Divide the Number of *Perches* propounded, by 160, the Quotient is the Number of *Acres* contained therein; and if there be a Remainder which exceeds 40, then Divide it by 40, the Quotient ſhall be *Roods*, and the Remainder *Perches*.

But, on the contrary, if it were required to find how many *Perches* are contained in a certain Number of *Acres* propounded; you muſt Multiply the Number of *Acres* by 160; the Product ſhall be the *Perches* contained therein.

PROP. II.

To reduce Dimenſions taken in Chains *and* Links, *into* Acres, Roods *and* Perches.

In regard we generally take our Dimenſions in *Chains* and *Links*, that is, by *Gunter's Chain*, (which contains 4 *Pole*, or 100 *Links*, as before

has

§ 3. has been noted) it will be neceſſary to ſhew you how to Caſt up the *Area* of a Piece of Land, from Dimenſions taken in thoſe Denominations.

Suppoſe the *Four-ſided Figure* A B C D, in *Plate* 4. *Fig.* 60. was Plotted, and the Length of the *Baſe* D B, and two *Perpendiculars* dropt from A and C to be as in the ſaid *Figure*, as followeth, *viz.* the *Baſe* 12 *Chains*, 48 *Links*, or 1248 *Links*; the *Long Perpendicular* 8 *Chains*, 46 *Links*, or 846 *Links*; and the *Leſs Perpendicular* 3 *Chains*, 94 *Links*, or 394 *Links*: Then I take them out, and place them as followeth.

846 *Great Perpendicular.*
394 *Leſs Perpendicular.*

1240 *Sum of them.*

620 *Half Sum.*
1248 *Baſe.*

.4960
2480
1240
620

7.73760 *Acres and Parts.*
4

2.95040 *Roods and Parts.*
40
38.01600 *Perches and Parts.*

And

And here NOTE, That *Gunter's Chain* (as before was hinted) contains 4 *Statute Pole*, or 100 *Links*; so that any Number of *Chain* is so many 100 *Links*; as, 5 *Chain* is 500 *Links*, 7 *Chain* is 700 *Links*, *&c.* And further, you are to Observe, That 160 *Square Statute Pole* is an *Acre*, each *Pole* being 16 *Foot* and ½. Now 'tis evident, In a *Square Chain* there is 16 *Square Pole*: Divide therefore 160 the *Square Pole* in an *Acre*, by 16 the *Square Pole* in a *Square Chain*, the Quote is 10 the *Square Chain* in an *Acre*. But every *Square Chain* contains 10000 *Square Links*, (for 'tis 100 Multiply'd by 100) and consequently an *Acre* 100000 *Square Links*. § 3.

Hence the Reason of the foregoing Operation is evident; for the *Half Sum* of the *Perpendicular* is 620 *Links*, and the *Base* 1248 *Links*, consequently the Product 773760 *Square Links*. Now because 100000 *Square Links* is an *Acre*, therefore I Divide 773760 by 100000 *Links*: But to Divide by 100000, is no more than to cut five figures to the Right; so will the remaining figures on the Left be *Acres*, and those cut off to the Right 73760 *Hundred thousandth Parts* of an *Acre*. If these *Parts* be Multiply'd by 4, and from the Product five figures be again cut off to the Right-hand, those on the Left shall be *Roods*, and those on the Right *Parts* of a *Rood*. And if those so cut off be Multiply'd by 40, and from the Product five figures be cut off to the Right-hand, as before, those on the Left are *Poles*, and those on the Right *Parts* of a *Pole*. In the preceeding Operation, there is 7 *Acres*, 2 *Roods*, 38 *Poles*, and 1600 *Hundred thousandths* of a *Pole*.

PROP.

§ 3.

PROP. IV.

To reduce Statute Meaſure *to* Cuſtomary; *& contrá.*

The *Statute Pole* or *Perch* is 16½ *Foot* long: Now divers Parts of *England* uſes a *Pole* of 18 *Foot* long; others, one of 21 *Foot*; and ſome, of 24 *Foot* long. And therefore, to turn one ſort of Meaſure into another, ſuppoſe *Statute* into *Cuſtomary*, do thus: *Multiply any Number of Acres, Roods or Perches* Statute Meaſure *by the Square* ½ *Yards or Square* ½ *Feet in a Square Pole of the* Statute Meaſure; *the Product Divide by the Square* ½ *Yards, Square* ½ *Feet in a Pole of the* Cuſtomary Meaſure *you would bring it into*; *the Quotient gives you the Acres, Roods or Perches of that* Cuſtomary Meaſure.

EXAMP. I. *In* 786 *Acres* Statute Meaſure, *How many Acres of* 18 *Foot to the Perch?*

NOTE, In a *Statute Pole* there is 5½ *Yards*, that is 11 *Half-Yards*; conſequently, in a *Square* Statute *Pole* there is 121 *Square Half-Yards*; and in a *Square Pole* of 18 *Foot* there is 144 *Square Half-Yards.*

786 *Acres* Statute Meaſure.
121 *Square* ½ *Yards in a* Stat. Pole.

786
1572
786
95106 (660 *Acres* Cuſtomary.
144
870
66

So

So that the *Cuſtomary Acres* (at 18 *Foot* to the *Perch*) in 786 *Statute Acres*, is 660, and $\frac{66}{144}$ of an *Acre*. This *Overplus* Multiply'd by 4, and the *Product* Divided by 144, gives *Roods*. And if the *Remainder*, after this Diviſion for *Roods* be Multiply'd by 40, and the *Product* Divided by 144, will give *Perches*; ſee the following Operation of the precedeing *Remainder*: § 3.

66
4
144) 264 (1 *Rood.*
120
40
144) 4800 (33 *Perches.*
480
48 *Remainder.*

So that 786 Acres *Statute Meaſure*, is 660 Acres, 1 Rood, 33 Perches *Cuſtomary*, with ſome ſmall Remainder over.

NOTE, If there had been any *Roods* or *Perches* with the given *Acres*, then you muſt have turn'd all into *Perches*; and having Multiply'd and Divided as before directed, Divide the Quote by 160; or elſe by 40, and that Quote again by 4.

§ 3. EXAMP. II. *In* 649 *Acres* Cuſtomary, *at* 18 *Foot to the Perch*; *How many Acres* Statute Meaſure?

649 *Acres* Cuſtomary.
144 *Sq.* ℄ *Yards* Cuſtom. *Pole.*

2596
2596
649

121) 93456 (772 *Acres* Stat.
875
286
44 *Remainder.*
4

121) 176 (1 *Rood.*
55
40

121) 2200 (18 *Perches.*
990
22 *Remainder.*

So that in 649 Acres *Cuſtomary*, at 18 Foot to the Perch, there are 772 Acres, 1 Rood, 18 Perches *Statute Meaſure.*

The

The Reaſon of this Operation would be more evident, did we Multiply the *Acres* by 160 to bring them into *Perches*, and thoſe *Perches* by the *Squre* $\frac{1}{2}$ *Yards* in a *Perch*: For by this Way, we ſhould get the *Square* $\frac{1}{2}$ *Yards* in the given Number of *Statute Acres*; and then it would only be Dividing by the *Square* $\frac{1}{2}$ *Yards* in a *Cuſtomary Perch*, and ſo get the Number of *Cuſtomary Perches* in the given *Statute Acres*; which being Divided by 160, will give the *Cuſtomary Acres* required. But Multiplying by 160, and then Dividing again by the ſame, is needleſs; becauſe the Quotient would be the ſame, were thoſe Operations omitted; therefore we only Multiply the *Statute Acres* by the *Square* $\frac{1}{2}$ *Yards* in a *Statute Perch*, and Divide the *Product* by the *Square* $\frac{1}{2}$ *Yards* in a *Cuſtomary Perch*, the *Quotient* gives the *Cuſtomary Acres* in thoſe *Statute Acres*. § 3.

NOTE, That *Cuſtomary*, as well as *Statute Acres*, contain 160 *Square Perch*; ſo that the Difference is only in the Bigneſs of the *Perch*.

SECT. IV.

Of the Uſe of the Semi-circle, *in taking Altitudes, Diſtances,* &c.

PROP. I.

How by the Semi-circle *to take an Acceſſible Altitude.*

ADmit AB be the Height of a *Tower*, which is required to be known. Firſt, placing § 4.

§ 4. your *Semi-circle* at D, (with the *Arch* downwards, and the two *Sights* fixed,) place it *Horizontal* *, and ſcrew it faſt: Then move your *Index*, till through the *Sights* thereof you eſpy the top of the *Tower* at B, and obſerve what *Degree* the lower part of the *Index* cutteth, and that will be equal unto the *Angle* at D 50 *deg.* Then Meaſure the Diſtance D A, which let be 299 *Feet*. Now the Height of the *Tower* A B, is found, according to *Ch.* 5. § 2. *Prop.* 1. thus:

* *Which to do, is no more than thus; with a Thread and Plummet faſtened at the Centre of the* Semi-circle, *ſo that it hath liberty to play, move the* Semi-circle *till the Thread playeth againſt* 90 deg. *then ſcrew it faſt, and it is* Horizontal.

Fig. 61. *As* Sc. *V.* at B 50° 00′,
To Log cr. D A 299 *Feet*;
So is S. *V.* at D 50° 00′,
To Log. A B 356 $\frac{3}{10}$ *Feet*, the Height of the *Tower* required.

PROP. II.

How by the Semi-circle *to take an Inacceſſible Altitude, at two Stations.*

Let A B be a *Tower* whoſe Height is required; Having placed your *Inſtrument* at E, as before, direct your *Sights* unto the Top of the *Tower* at B, and finding the *Degree* cut by the *Index* to be 23° 43′, I ſay, it is the Quantity of the *Angle* at E. Now by reaſon of Water or ſuch-like Impediment, you can approach no nearer the *Baſe* of the *Tower* than D; therefore Meaſure E D, which is found to be 512 *Feet*: Then at D make the like Obſervation, and the *Angle* at D appeareth to be 50° 00′, whoſe

Com-

Complement is the *Angle* D B A 40° 00′, and the *Complement* of the *Angle* E 23° 43′, is the *Angle* E B A 66° 17′: Now if the *leſſer Angle* at B be taken out of the *greater*, the *Remainder* is 26° 17′, the *Angle* E B D. Now, firſt, to find the Side B D of the *Triangle* E B D, ſay, according to *Chap.* 5. § 3. *Prop.* 1. thus: § 4.

As S. of *V.* E B D 26° 17′,
To Log. cr. E D 512 *Feet*;
So is S. of *V.* at E 23° 43′,
To Log. cr. B D 465 $\frac{1}{10}$ *Feet* required. *Fig.* 61.

Now to find the Height of the *Tower* A B, ſay, according to *Chap.* 5. § 2. *Prop.* 2. thus:

As Radius or S 90°,
To Log. cr. D B 465 $\frac{1}{10}$ *Feet* found;
So is S. of *V.* B D A 50° 00′,
To Log. cr. B A 356 $\frac{3}{10}$ *Feet*; which is the Height of the *Tower* required.

NOTE, That in taking any manner of *Altitude*, the Height of your *Inſtrument* muſt be added unto the Height found, and that will give you the *true Altitude* required.

PROP. III.

How by the Semi-circle *to take an Inacceſſible Diſtance, at two Stations.*

Admit A and B be the *two Stations*, from either of which it is required to find the Diſtance unto the *Church* at C: Placing your *Inſtrument* at B, the *Index* lying on the *Diameter*, and direct your *Sights* unto the *Church* at C; faſten your *Inſtrument*, and turn your *Sights* about, until you ſee through your Sights your *ſecond*

§ 4. *Station* at A, ſo will you find your *Index* to cut 30° 00′, which is the Quantity of the *Angle* ABC: Then Meaſure the *Diſtance* AB, which is found to be 250 *Yards*; then with your *Inſtrument* at A, make the like Obſervation as before, and you will find the *Angle* BAC to contain 50° 00′. Now by the Third Maxim of *Plain Triangles*, Chap. 5. § 1. you find alſo the *Angle* ACB to be 100° 00′. Now to find the *Diſtance* AC and BC, you may, by their oppoſite Proportion, according to *Chap.* 5. § 3. *Prop.* 1. find the *Diſtance* of AC, thus:

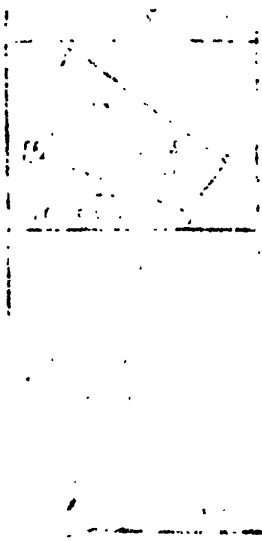
Fig. 62. *As* S. of *V.* at C 100° 00′,
To Log. cr. AB 250 *Yards*;
So is S. of *V.* B 30° 00′,
To Log. cr. AC 127 *Yards*: Which is the *Diſtance* of the *Church* from A.

Now to find the *Diſtance* BC, ſay,
As S. of *V.* at A 100° 00′,
To Log. cr. AB 250 *Yards*;
So is S. of *V.* at A 50° 00′,
To Log. cr. BC 194 $\frac{4}{10}$ *Yards*: Which is the *Diſtance* of the *Station* B from the *Church* at C.

PROP. IV.

How to find the Horizontal-Line of any Hill or Mountain, by the Semi-circle.

Let *Fig.* 63 be a *Mountain*, whoſe *Horizontal-Line* AB is required to be found: To find which, place your *Inſtrument* at A; and having cauſed a Mark to be placed on the Top of the *Mountain* at C, (of the juſt Height of your *Inſtrument*) then move your *Index* until through



through the Sights thereof you eſpy the Mark at C, ſo will you find the Quantity of the *Angle* CAD to be 50° 00′, and by conſequence the *Angle* ACD to be 40 00′; then Meaſure up the *Hill* AC, which is 346 *Yards*. Now having obtained theſe ſeveral things, 'tis required to find the Length of AD part of AB: To find which, ſay, § 4.

As Radius or S. 90°, Fig. 63.
To Log. cr. AC 346 *Feet*;
So is Sc. of *V.* at A 50° 00′,
To Log. cr. AD 222 $\frac{4}{10}$ *Feet*;

Now ſeeing the *Hill* or *Mountain* deſcendeth on the other ſide, you muſt place your *Inſtrument* at C, and direct your *Sights* unto the *Bottom* at B, and the *Angle* DCB will be found 50° 00′, and the *Angle* CBD 40° 00′: Then Meaſuring down the *Mountain* as CB, it appeareth to be 415 *Feet*; then have you the *Angles* DCB, and CBD.

To find DB, part of AB, ſay,

As Radius or S. 90°, Fig 63.
To Log. cr. CB 415 *Feet*;
So is S. of *V.* BCD 50° 00′,
To Log. cr. DB 318 *Feet*. Now AD 222 $\frac{4}{10}$ *Feet* added thereunto, produceth AB 540 $\frac{4}{10}$ *Feet*, which is the *Horizontal-Line* required of the *Mountain* ACBD.

☞ NOTE, That when you come to delineate a Field wherein are *Hills*, you muſt protract the Line AB, inſtead of the *Hypothenuſal*-Lines AC, and CB; and 'twill be neceſſary to diſtinguiſh thoſe kind of Fields, by Shadowing them off with *Hills* and *Dales*.

SECT. V.

How to find whether Water may be conveyed from a Spring-Head, unto any appointed Place.

§ 5. THE Art of conveying of *Water* from a *Spring-Head* unto any appointed Place, hath a ſpecial reſpect unto *Meaſuring*; and therefore I think it not amiſs to inſert it in this Place, and enroll it under the Title of SURVEYING.

In the Performance of which, we make Uſe of a *Water-Level*; the Conſtruction and Making whereof is ſufficiently known to thoſe who make *Mathematical Inſtruments*. Now if it were required to find whether Water may be conveyed in *Pipes*, &c. to any Place aſſigned: To perform which, obſerve theſe Rules.

Firſt, At ſome 10, 20, 30, 40, 60, or 100 Yards diſtance from the *Spring-Head*, in a right-line towards the Place unto which your Water is to be conveyed, place your *Water-Level*, being prepared of two *Station-Staffs* with moveable *Vanes* on each of them, graduated alſo after the uſual manner: Cauſe your *firſt Aſſiſtant* to ſet up one of them at the *Spring-Head*, Perpendicular unto the *Horizon*; and your *ſecond Aſſiſtant* to erect another, as far from your *Water-Level* towards the Place to which the Water is to be conveyed, as your *Water-Level* is diſtant from the *Spring-Head*. Now the *Station-Staffs* in this order erected, and your *Water-Level* placed preciſely *Horizontal*, go unto the

end

end of the *Level*, and looking through the Sights, cauſe your *firſt Aſſiſtant* to move a Leaf of Paper up or down your *Station-Staff*, until through the *Sights* you eſpy the very edge thereof, and then by ſome known *ſign* or *ſound*, intimate to your *Aſſiſtant* that the Paper is then in its true Poſition; then let the *firſt Aſſiſtant* note againſt what Number of *Feet*, *Inches*, and *Parts* of an *Inch* the edge of the Paper reſteth; which he muſt note down in a Paper. Then your *Water-Level* remaining immoveable, go to the other end thereof, and looking through the *Sights* towards your other *Station-Staff*, cauſe your *ſecond Aſſiſtant* to move a Leaf of Paper along the *Staff*, till you ſee the very edge thereof through the *Sights*, and then cauſe him, by ſome known *ſign* or *ſound*, to take notice what number of *Feet*, &c. are cut by the ſaid Paper; which let him keep, as your *firſt Aſſiſtant* did. § 5.

This done, let your *firſt Aſſiſtant* bring his *Station-Staff* from the *Spring-Head*, and cauſe your *ſecond Aſſiſtant* to take that *Staff*, and carry it forwards towards the Place unto which the Water is to be conveyed, ſome 30, 40, 60, or 100 Yards, and there to erect it *Perpendicular*, as before, letting your *ſecond Aſſiſtant*'s *Staff* ſtand immoveable, and your *firſt Aſſiſtant* to ſtand by it: Then in the Midway between your two *Aſſiſtants* place your *Water-Level* exactly *Horizontal*, and looking through the *Sights* thereof, cauſe your *firſt Aſſiſtant*, and after that your *ſecond*, to make their ſeveral Obſervations in all reſpects as before.

In

§ 5. In this manner you muſt go along from the *Spring-Head*, to the Place unto which you wou'd have the *Water* conveyed; and if there be never-ſo-many ſeveral *Stations*, you muſt in all of them obſerve this manner of Work preciſely: So that by comparing the *Notes* of your two *Aſſiſtants* together, you may eaſily know whether the *Water* may be conveyed from the *Spring-Head*, or not, by calling your two *Aſſiſtants* together, and cauſing them to give in their *Notes* of Obſervation at each *Station*; which add together ſeverally: Then if the *Notes* of the *ſecond Aſſiſtant* exceed the *Notes* of the *firſt Aſſiſtant*, take the *leſſer* out of the *greater*, and the *Remainder* will ſhew you how much the appointed *Place*, to which the *Water* is to be conveyed, is Lower than the *Spring-Head*.

The *firſt Aſſiſtant*'s NOTES.

Station.	*Feet.*	*Inch*	*Parts.*
1	15	3	.50
2	2	1	.25
3	1	6	.0
Sum.	18	10	.75

The *ſecond Aſſiſtant*'s NOTES.

Station.	*Feet.*	*Inch.*	*Parts.*
1	3	2	.75
2	14	0	.25
3	3	11	.0
Sum	21	2	.0

By theſe two *Tables* you may perceive, that the *Notes* of the *firſt Aſſiſtant*, collected at his ſeveral *Stations*, being added together, amounts unto 18 *Feet*, 10 *Inches*, and $\frac{75}{100}$ or $\frac{3}{4}$ of an *Inch*; and the *Notes* of your *ſecond Aſſiſtant*, collected

at

at his ſeveral *Stations*, amounts unto 21 *Feet*, 2 *Inches*: So that the Number of the *firſt Aſſiſtant*'s *Obſervations* being taken from the *ſecond*'s, there will remain 2 *Feet*, 3 *Inches*, and $\frac{11}{100}$ or $\frac{1}{4}$ of an *Inch*: And ſo much is the *Place* unto which the *Water* is to be brought, Lower than the *Spring-Head*, (according to the ſleight *Water-Level*) and therefore the *Water* may eaſily be conveyed thither. And here Obſerve theſe *Notes*: § 5.

☞ 1. In your Paſſage between the *Spring-Head* and the appointed *Place*, from *Station* to *Station*, you muſt obſerve this Order; That your *firſt Aſſiſtant*, at every *Station*, muſt ſtand between the *Spring-Head* and your *Water-Level*, otherwiſe great Errors will enſue.

☞ 2. That if the *Notes* of your *firſt Aſſiſtant* exceed the *Notes* of the *ſecond Aſſiſtant*, then 'tis impoſſible to bring the *Water* from that *Spring-Head* unto the *appointed Place*: But if their *Notes* are equal, it may be done, if the Diſtance be but ſhort.

☞ 3. That the moſt approved Authors concerning this Particular, do averr, That at every *Mile*'s end there ought to be allowed $4\frac{1}{2}$ *Inches* more than the *Straight-Level*, for the Current of the *Water*.

☞ 4. That if there be any *Mountains* lying in the Way betwixt the *Spring-Head* and the *Place* to which the *Water* is to be conveyed, you muſt then cut a *Trench* by the ſide of the *Mountain*, in which you muſt lay your *Pipes* equal with the *ſtraight Water-Level*, with the former Allowance: And in caſe there be a *Valley*, you muſt then make a *Trunk* of ſtrong Wood, well under-

propt

§ 5. propt with ſtrong Pieces of Timber, well Pitcht or Leaded, (as is done in divers Places between *Ware* and *London.*)

☞ 5. That when the *Spring* will have too violent a *Current*, you muſt then convey your *Water* to the Place aſſign'd, by a *Crooked* or *Winding-Line*; and you alſo ought to lay the *Pipes*, the one up, and the other down, that thereby the *Violence* of the *Current* may be ſtopt.

CHAP. X.

Of MEASURING,

Of Board, Glaſs, Tiling, Paving, Timber, Stone, and Irregular Solids, ſuch as Geometry *can give no Rule for the Meaſuring thereof.*

SECT. I.

Of the Meaſuring of Board, Glaſs, Paving, Tiling, &c.

§ 1. I Have already, in the 4th *Chapter* of this *Book*, and the 2d *Section* thereof, apply'd *Geometry* to the finding out of the *Superficial Content* of all *Regular Superficies*. I have alſo in the 9th *Chapter*, and the 3d *Section* thereof,

thereof; ſhewed how the *Superficial Content* of any *Irregular Superficies* may be found, by reducing them into *Regular Forms*; which I have explained amply in that *Section*, I ſhall therefore here be as plain and brief as is poſſible. § 1.

PROP. I.

To Meaſure a Piece of Board, Plank, Glaſs, &c.

In Meaſuring of *Board, Glaſs, &c. Carpenters* and other *Mechanicks* Meaſure by the *Foot*, 12 *Inches* unto the *Foot*; ſo that a *Foot* of *Board*, or *Glaſs*, contains 144 *Square Inches*.

Now if a Piece of *Board*, *Plank*, or *Glaſs*, be required to be Meaſured, let it be either a *Parallelogram*, or *Tapering Piece*: Firſt, by the Rules aforegoing, find the *Content* thereof in *Inches*, and that *Product* Divide by 144; the *Quotient* is the *Content* of that *Superficies* in *Feet*.

PROP. II.

To Meaſure Tiling, Flooring, Roofing, and Partitioning-works.

In *Tiling*, *Flooring*, *Roofing*, and *Partitioning-work*, *Carpenters* and other *Workmen* reckon by the *Square*, which is 10 *Feet* every way; ſo that a *Square* containeth 100 *Feet*.

EXAMP. *There is a* Roof 14 Feet *Broad*; *What Length thereof ſhall make a* Square?

Divide 100 by 14, it yields 7 $\frac{1}{7}$ *Feet*.

Now if you have any Number of *Feet* given, and the Number of *Squares* therein contain'd are

§ 1. are required; Divide that Number by 100, the Product is *Squares*.

PROP. III.

To Meaſure Paving, Plaſtering, Wainſcotting, and Painting-work.

In *Paving*, *Plaiſtering*, *Wainſcotting* and *Painting-work*, *Mechanicks* reckon by the *Yard Square*; ſo each *Yard* is equal unto 9 *Square Feet*.

By the Rules aforegoing, find the *Superficial Content* of the *Court*, *Alley*, &c. in *Feet*; which Divide by 9, the *Quotient* is the Number of *Yards* in that Work contained.

PROP. IV.

To find how many Inches in Length will make a Square Foot.

EXAMP. *Suppoſe a Piece of* Board *or* Plank *be* 3 Inches *Broad*; *How much in Length will make a* Foot Square?

Conſider that in a *Square Foot* there are 144 *Square Inches*: Divide therefore 144 by 3, the Quote is 48; which ſhews, that a Piece of *Board* 3 *Inches* Broad, requires 48 *Inches*, or 4 *Foot* in Length, to make a *Foot Square*.

Whence 'tis manifeſt, that if you Divide 144 by any given Breadth, you get the Length that will make a *Foot Square* of that Breadth. The reaſon is clear; for if you Multiply the *Diviſor* and *Quote*, that is, the Length and Breadth, the *Product* will be 144 the *Square Inches* in a *Square Foot*.

And

And after this manner is the TABLE upon the Common *Two-Foot Rules* made, call'd the TABLE *of Board Measure*; which shews, upon any Breadth, how much in Length will make a *Foot Square.* § 1.

A *TABLE* of *Board Measure.*

Inch.	1	2	3	4	5	6	7	
Feet	12	6	4	3	2	2	1	1
Inch.	0	0	0	0	4$\frac{4}{7}$	0	8$\frac{4}{7}$	6

From this TABLE it appears, that if a Piece of *Board* be an *Inch* Broad, 'twill require 12 *Foot* in Length to make a *Foot Square:* If 2 *Inches* Broad, 6 *Foot* in Length will make a *Foot Square:* If 8 *Inches* Broad, 1 *Foot* 6 *Inches* will make a *Foot Square.*

SECT. II.

Of the Measuring of Timber, Stone, and Irregular Solids.

IN *Superficial Measure*, a *Superficial Foot* contains 144 *Square Inches*; but in *Solid Measure*, a *Foot* contains 1728 *Cubick Inches.* Now having already, in the 4th *Chapter* of this *Book*, and the 3d *Section* thereof, largely apply'd *Geometry* unto the Measuring of all *Regular Solids*; I shall therefore in this Place be as brief as possible, only I shall be somewhat larger § 2.

§ 2. larger in the Mensuration of *Irregular Solids*, which is of special Moment in sundry Parts of the *Mathematical Practices*.

PROP. I.

How to Measure any kind of Timber, or Stone, whether Three-square, Four-square, Many-square, Round, or of any other fashion, provided it be straight and equal all along.

To perform which, first, by the Rules aforegoing in *Chap.* 4. § 2. get the *Superficial Content* at the End, and then say,

As 144, the *Inches* of the *Superficial Content* of the End of a *Cubick Foot*,
To a Cubick Foot containing 1000 *Parts*;
So is the Superficial Content of the End of any Piece of *Timber*,
To the Solid Content of One Foot Length of the said Piece of *Timber*.

According to which, Mr. *Philips* calculated the ensuing TABLE, which I have thought fit hereunto to annex.

CASE II. *Or the Solid Content in Feet, &c. may be found otherwise, thus:*

By the Rules aforegoing, find the *Content* of the End of the Piece of *Timber* in *Inches*, which *Content* Multiply by the Length of the said Piece of *Timber* or *Stone* in *Inches*, and that *Product* Divide by 1728, it produceth the *Solid Content* of that Piece of *Timber* or *Stone*, in *Feet*, and *Parts* of a *Foot*.

A TA-

§ 2.

A *TABLE*, shewing the Solid Content of One Foot-length of any Piece of Timber, according to the Superficial Content at the End thereof.

The Inches of the Content at the End.	Feet.	Parts.	The Inches of the Content at the End.	Feet.	Parts.
1	0	7	200	1	398
2	0	14	300	2	83
3	0	21	400	2	778
4	0	28	500	3	472
5	0	35	600	4	167
6	0	42	700	4	861
7	0	49	800	5	556
8	0	56	900	6	250
9	0	62	1000	6	944
10	0	69	2000	13	888
20	0	139	3000	20	833
30	0	208	4000	27	778
40	0	278	5000	34	722
50	0	347	6000	41	666
60	0	417	7000	48	711
70	0	485	8000	55	555
80	0	556	9000	62	500
90	0	625	10000	69	444
100	0	694	20000	138	888

PROP. II.

To Measure Round Timber which is Hollow, or any other Hollow Body.

If *Hollow Timber* be to be Measured, first Measure the *Stick* as though it were not *Hollow*,

§ 2. then find the *Solidity* of the *Concavity* as though it were *Maſſie Timber*; then Subſtract this laſt found *Content* out of the whole *Content* before found, the *Remainder* is the *Content* of that *Hollow Body.*

PROP. III.

To Meaſure Tapering Timber, or Stone.

Thoſe *Tapering Bodies* are either *Segments* of *Cones*, or *Pyramids*: Now the Way to Meaſure ſuch *Bodies*, is demonſtrated in *Chap.* 4. § 3. *Prop.* 4, and 5. But now to find the *Content* of theſe *Segments*, do thus: Meaſure the *Solidity* of the whole *Cone*, or *Pyramid*, and then find the *Content* of the Top-part thereof cut off, (as if it were a *Cone* or *Pyramid* of it ſelf) and the *Content* thereof Deduct from the *Content* of the whole *Cone*, or *Pyramid*; ſo ſhall the *Remainder* be the *Content* of the *Segment* required: which reduced into *Feet*, gives the *Solid Content* of that Piece of *Timber* in *Feet.* Now to find the Length of the Top-part cut off, from the *Cone*, or *Pyramid*, ſay,

As the Difference of the Breadth of the two Ends,
To the Length between them;
So is the Breadth of the greater End,
To the whole Length of the Cone, or Pyramid.

PROP. IV.

How to find the Solid Content of any Solid Body, in any ſtrange form, ſuch as Geometry *can give no Rule for the Meaſuring thereof.*

Theſe ſtrange Forms, are either *Branches* in *Metal*, *Crowns*, *Cups*, *Bowles*, *Pots*, *Screws*, or *Twiſted*

§ 2.

Twisted Ballisters *, or any other *Irregular Solid*, that keep not in thickness one quantity, but are thicker in one place than in another, so that no Man, by *Geometry*, can possibly Measure their *Solidity*.

* *Whose Surface is bounded by a Line call'd* (*by* Proclus) *a* Helicoides; *but it may also be call'd a* Helix, *a Twist or Wreath*, &c.

Now for the finding the *Content* of any such-like *Irregular Body* in *Inches* or *Feet*, do thus: Cause to be made a *Hollow Cube*, or *Parallelopipedon*, so that you may Measure it with an *Inch-Rule* without difficulty; and so to know the true *Content* of the Whole, or any Part thereof, at pleasure, within the *Concavity*: Then take some other convenient Vessel, and put pure Spring-Water therein; then having fill'd the Vessel to a known Measure, make a Mark precisely round the very Edge of the Water, then take the *Solid Body* and put it therein; then take out as much of the Water as (by means of the *Body* put therein) is arisen above the Mark, until the Water do justly touch at the Mark again: Then put the Water taken forth into the *Hollow Cube*, and find the *Solid Content* thereof (being transform'd into a *Cubick Body*) in *Feet*, *Inches*, and *Parts* of an *Inch*: Which *Content* is the just *Solidity* of the *Body* put into the Water. (*Archimedes*, by this Proposition, found the Deceit of the *Crown of Gold*, which *Gelo* the Son of *Hiero* had vowed unto his *Gods*; now the Workmen had mixed *Silver* with the *Gold*; which Theft was discovered by the great Skill of *Archimedes* *.) And herein you must be very curious not to spill any of the Water, or take out of the Vessel, or put into the *Hollow*

* *See* Proclus, *l.* 2. *c.* 3. *&* Vitruvius, *l.* 9. *c.* 3.

§ 2. *Cube*, any more than the juſt quantity ariſen above the Mark; for if you do, it will produce infinite Errors: And thus may the *Solidity* of any *Irregular Body* be found.

PROP. V.

To find how much in Length of any Square Piece will make a Foot Solid.

EXAMP. I. *A Piece of Timber is 8 Inches Square, (conſequently the* Area *is 64 Inches) How much in Length will make a Foot Solid?*

Remember, in a *Solid Foot* are 1728 *Solid Inches*; Divide therefore 1728 by 64, the *Quote* is 27 *Inches*, that is 2 *Foot*, 3 *Inches*, the Length that will make a *Foot Solid* upon 8 *Inches Square*.

If the Piece be not exactly Square, the thing differs not; for the *Area* of the *Oblong* muſt be found.

EXAMP. II. *A Piece of Timber is 9 Inches one way, and 4 Inches the other, (conſequently the* Area *is 36) How much in Length will make a Foot Solid.*

Here, as before, I Divide 1728 by 36, the *Quote* is 48 *Inches*, that is 4 *Foot*, the Length that will make a *Foot Solid*.

The Reaſon is plain; for the *Breadth* Multiply'd into the *Thickneſs*, is the *Diviſor*; and that Multiply'd by the *Quote*, which is the *Length*, will make 1728 the *Inches* in One *Solid Foot*.

And thus is the *Table of Timber Meaſure*, put on the Common *Two-foot Rules*, made; which

which ſhews how much in *Length* any exact *Square* Piece will make a *Foot Solid.* § 2.

A *TABLE* of *Timber Meaſure.*

In. Sq.	1	2	3	4	5	6	7	8	9	10
Feet.	144	36	16	9	5	4	2	2	1	1
Inch.	0	0	0	0	$9\frac{3}{5}$	0	$11\frac{1}{3}$	3	$9\frac{1}{2}$	$5\frac{7}{12}$

The Uſe of this TABLE is thus: If a Piece of *Timber* be 8 *Inches Square*, it ſhews you, that 2 *Foot* 3 *Inches* in *Length* will make a *Foot Solid:* If it be 9 *Inches*, then you will find that 1 *Foot* 9 *Inches* $\frac{1}{3}$ of an *Inch* in *Length* will make a *Solid Foot.*

CHAP. XI.

Of GAUGING.

IN GAUGING, there are Two things chiefly neceſſary to be noted, yet both controverted. 1ſt. That ſeeing all manner of *Casks* made to hold *Liquor* in, are for the moſt part the *Trunk* of a *Spheroid*, cut off with two *Circles*, at *Rightangles* with the *Baſe*, and therefore *Irregular*; Therefore they muſt firſt be reduced into a *Regular Proportion.*

And the 2d thing neceſſary to be noted, is to find the true Quantity of an *Ale*, or *Wine-Gallon*, in *Cubick-Inches* or *Parts* of a *Foot*; that thereby the *Content* of the *Veſſel* or *Cask*, in *Gallons*, may be known.

SECT. I.

Of Gauging any Beer, Ale, or Wine Cask, alſo any manner of Brewers Tuns.

PROP. I.

To find the Solid Content in Inches, of any Cask.

§ 1. I Shall follow Mr. *Oughtred*'s Method, which is; Take the *Diameter* of the *Cask* both at *Head* and *Bung*, by which find the *Area's* of their *Circles*; which done, then take Two-thirds of the *Area* of the *Bung*, and One-third of the *Area* at the *Head*; which added together, ſhall be the *Mean Area* of the *Cask*; which Multiply'd into the *Length* of the Veſſel, it will ſhew how many *Solid Inches* are contained therein.

EXAMP. *Suppoſe the* Diameter *at the Head of a Veſſel be* 18, *and at the* Bung 32, *and* Length *is* 40 Inches.

Now I find the *Aggregate* of the two *Circles* to be 620 and 989 *Cubick Inches*; which Multiply'd by 40 the *Length*, produceth 24839 $\frac{14}{100}$ *Cubick Inches*, for the whole *Content* of that *Cask* in *Cubick Inches*.

PROP.

§ 1.

PROP. II.

To find the Content of a Veſſel, in Wine or Ale-gallons.

The *Wine-Gallon* is eſtabliſh'd by the Conſent of *Artiſts*, in theſe and other Nations, to contain 231 *Cubick Inches* *. Yet Dr. *Wybard* affirms it to be ſomewhat leſs, to wit, 225 at moſt. The *Ale-Gallon* contains 282 *Cubick Inches*, according to the Eſtabliſhment of *Exciſe*. Herein Artiſts differ ſomewhat in their Experiments.

* *See Mr.* Oughtred *in his Book of the* Circles of Proportion, *pag.* 57 *and Mr.* Edm. Gunter *in his Book of the* Croſs-Staff, *Part* 2. *Chap.* 4.

Now having already ſhewed how to find the *Content* in *Inches* of any *Cask*, I now come to ſhew how to find the *Content* in *Gallons*, of any *Beer*, *Ale*, or *Wine-Cask*, which is thus: Divide the Number of *Inches* given by 231 for *Wine Meaſure*, and 282 for *Ale Meaſure*. In the former *Example* I find the ſaid *Cask* to contain 107, 53 *Wine-Gallons*, and 88, 8, *&c. Gallons* in *Ale Meaſure*.

PROP. III.

How to Gauge or Meaſure Brewers Tuns, &c.

Thoſe *Tuns* are moſt commonly *Segments* of *Cones* or *Pyramid*, whoſe *Baſis* is either a *Square Parallelogram*, *Circle*, or *Oval*; To Meaſure which, let their Form be what it will, you muſt do thus: By the former Rules of Meaſuring ſuch *Segments* or *Bodies*, you muſt find their *Solid Content* in *Cubick Inches*, (as in *Chap.* 10. § 2. *Prop.* 3.) which *Content* Divide by 282 *Inches*

§ 1. (the *Inches* in One *Gallon*) it ſheweth the Content in *Gallons*; and Dividing the *Gallons* by 36 (the Number of *Gallons* in a *Barrel*) it ſhews the Content in *Barrels*.

SECT. II.

Of Gauging or Meaſuring, and the Moulding of Ships.

PROP. I.

To Gauge a Ship, thereby to find how many Tuns her Burthen is.

§ 2. IN the *Gauging* or Meaſuring of *Ships*, *Naupegers* (or *Ship-wrights*) obſerve theſe Three particular Rules: 1*ſt*. That if you Meaſure the Ship *within*, you ſhall find the *Content*, or the *Burthen* the Ship will hold or take in. 2*dly*. If the Ship be Meaſured on the *outſide*, to her *light mark* as ſhe ſwims being *unladen*, you ſhall have the Content of the *Empty Ship*. 3*dly*. But if you Meaſure from the *light mark*, to her *full draught* of *Water* being *laden*, you ſhall have the true *Burthen* of the *Ship*.

Now to find the Content of the *Queen*'s *Royal Ships*; Meaſure the *Length* of the *Keel*, the *Breadth* of the *Mid-ſhip Beam*, and the *Depth* of the *Hold*: Which three Multiply into one another, and Divide their *Product* by 100; ſo ſhall you find how many *Tuns* her *Burthen* is.

But for *Merchants Ships*, which give no Allowance for *Ordnance*, *Maſts*, *Sails*, *Cables*, *Anchors*,

Anchors, &c. which are all a *Burthen*, but no *Tonnage*, you muſt Divide the *Product* by 95, ſo ſhall their true *Burthen* be found. § 2.

PROP. II.

By knowing the Meaſure of a Ship of one Burthen, to make another Ship of the ſame Mould, which ſhall be double, or triple, or in any Proportion, either more or leſs than the ſaid Ship.

Firſt, you ſhall Multiply the *Keel Cubically*, and in like manner every *Beam*; the *Mid-ſhip Beams*, Multiply them *Cubically*; and alſo the *Reaking* of the *Ship*, both at *Stem* and *Stem-Poſt*, Multiply them *Cubically*; likewiſe the *principal Timbers*, that doth *mould* the *Ship*, Multiply them *Cubically*; and the *Depth* of the *Hold*, Multiply it *Cubically*; and ſo conſequently every Place, or Places, which doth lead any Work, Multiply them *Cubically*: Then if it be required to have a *Ship* as big again, or thrice as big, *double* or *triple* each reſpective *Cubical Number*: Then by *Chap.* 1. § 1. *Prop.* 9. or by *Chap.* 2. § 2. *Prop.* 4. find the *Cube-Roots* thereunto belonging: Then according unto theſe reſpective *Numbers* make your *Keel*, your *Timbers*, *Beams*, *&c.* Which being done, you ſhall make a *Ship* of the *Mould* and *Proportion* deſir'd.

The *Demonſtration* of this laſt *Propoſition* depends upon the 33d *Prop.* of 11 of *Euclid*, where 'tis prov'd, That all *Similar Solids* are *in Triplicate Ratio* to that of their *Homologous Sides*; that is, they are to each other as the *Cubes* of their *Diameters*.

CHAP.

CHAP. XII.

Of DIALLING.

HOROLOGIOGRAPHIA, *or the* Art of DIALLING, *is an Art Mathematical, which demonſtrateth the preciſe Diſtinction of Times, by the Sun, Moon and Stars, whereby the Time of the Day or Night may be known* *. Now the Demonſtrative Delineation of *Dials*, conſiſteth chiefly in the finding out the *Hour-Lines*, and their true Diſtance one from the other; which *Lines* are *great Circles* of a *Sphere*, which being projected on a *plain Superficies*, become *ſtraight-Lines*; which *Lines* do continually vary, according as the *Planes* on which they are deſcribed, or projected, do lie ſituated in reſpect of the *Horizon* of the Place. Now a *Dial* may be made on any *Plain Superficies*; for all *Plain Superficies* are poſited, either *Perpendicular*, *Parallel*, or *Oblique*, to the *Horizon* of the Place in which the *Plane* is ſeated.

* *Which doth appear to have been in Uſe above* 2400 *Years, for King* Achaz *had a* Dial: *This Art requireth good ſkill in* Geometry *and* Aſtronomy. *Now* Cteſibius *that famous Philoſopher meaſured the Hours and Times by the orderly running of Water. Then by Sand was the Hours meaſured. After that, by* Trochilike *with Weights: And of late, with* Trochilike *with Springs.*

In the Delineation of all which *Dials* in this *Chapter* deſcribed, (which are the moſt Eminent and Uſeful *Dials* now uſed) I have uſed this Method: *1ſt.* I have ſhewed how to Delineate

lineate them by *Geometrical Projection*, by *Scale* and *Compaſs* only: And, 2*dly*. how they may be Deſcribed by *Arithmetical Calculation*: Of both which I have been very plain and large.

SECT. I.

Of the Delineation and Projection of ſundry moſt Uſeful Dials.

PROP. I.

How to draw the Hour-Lines on an Equinoctial Plane.

AN *Equinoctial Plane*, is ſuch which lieth *Parallel* unto the *Equinoctial*, and is an *Horizontal Plane*, under the *Pole*. This is the firſt and plaineſt kind of *Dials*, and is made after this manner: Firſt deſcribe the *Circle* Æ W E R for your *Plane*, then croſs it with the two *Diameters* Æ W and Æ R: Then divide the *Semi-circle* E W R into 12 equal Parts, in the Points ⊙, ⊙, ⊙, *&c.* Then from the *Centre* Q, and through the ſaid Points, draw ſtraight Lines, which ſhall be the true *Hour-Lines* belonging unto this *Equinoctial Plane*. Now becauſe theſe *Planes* are capable of receiving all the *Hour-lines* from *Sun-riſing* unto the *Sun-ſetting*, in *Summer*; therefore the *Hour-lines* of 4 and 5 in the *Morning*, and 7 and 8 in the *Evening*, muſt be delineated as you ſee done in the *Figure*: Theſe *Hours* may be ſub-divided

§ 1. *Fig.* 64.

§ 1. divided into *Half-Hours*, and *Quarters*: The *Stile* of this *Dial* muſt be a ſtraight *Pin*, or
Fig. 64. *Wyre*, ſet *Perpendicular* to the *Plane* on the *Centre* Q, and of any convenient Length. This *Dial* may be made for any *Latitude*, and is of good Uſe for Seamen and others.

If this *Dial* be put up in the Latitude of *London*, it muſt make an *Angle* of 38° 28′; the Under Face next the *Horizon* ſerving for the *Winter* Half-Year, and the Upper Face the *Summer* Half-Year.

PROP. II.

How to draw the Hour-lines on a Polar Plane.

A *Polar Plane* is one that lies *Parallel* unto the *Pole*, and under the *Equinoctial* is an *Horizontal Dial*. The Way to make this *Dial* is thus:
Fig. 65. Firſt draw the Line A B, for the *Horizontal-line* of the *Plane*; and croſs it at the Middle at *Right-angles* with the Line 12 Q 12, which is the *Meridian* or *Hour-line* of 12: Then upon the Line 12 Q 12, either above or below the Point Q, aſſume any Point, as S; then ſetting one foot of your Compaſſes in S, deſcribe the *Semi-circle* C E D, which Divide into 12 Equal Parts, in the Points ⊙, ⊙, ⊙, *&c.* Then lay a Ruler unto S, and unto the ſeveral Points ⊙, ⊙, ⊙, *&c.* and it will croſs the Line A B, in the Points, x, x, x, *&c.* Then through thoſe Points draw (by *Chap.* 4. § 1. *Prop.* 4.) Right-Lines all *Parallel* unto 12 Q 12, and ſo is your *Dial* finiſhed. Then, according unto the Breadth of the *Plane*, you

you may proportion your *Stile*,* whoſe Height must be Equal to the Diſtance between the two *Hour-lines* 12 and 9, or 12 and 3, and then will the *Shadow* of the upper edge thereof ſhew the Hour of the Day. The Height of the *Stile* is alſo found thus: § 1. *Fig.* 56.

* *Which may be either a* Pin *of the length of* QS, *placed on* Q, *and* Perpendicular *unto the* Plane; *or it may be a piece of* Braſs *or elſewhat of the breadth of* 12, *to* 3, *or* 9.

As the Tangent of the Hour-line 4 *or* 5,
To the Diſtance thereof from the Meridian;
So is the Radius,
To the Height of the Stile.

Then for the other *Hour-line*, ſay,
As the Radius,
To the Height of the Stile;
So is the Tangent of any other Hour-line,
To the Diſtance thereof from the Meridian-line.

PROP. III.

How to draw the Hour-lines on a Meridian-Plane, which is an Eaſt or Weſt Dial.

A *Meridian Plane* ſtands upright directly in the *Meridian*, and hath two Faces, one towards the *Eaſt*, and the other towards the *Weſt*.

Now admit it be required to make a Direct *Eaſt Dial*, in the Latitude of 51° 32′: Let ABCD be a *Dial-Plane* on which you would deſcribe a Direct *Eaſt Dial*; on the Point D deſcribe an *obſcure Arch* HG, with the *Radius* of your *Line of Chords*; then take 38° 28′, the *Complement* of your *Latitude*, place it from G to L; *Fig.* 66.

then

§ 1. then draw D L quite through the *Plane:* Then, to proportion your *Stile* unto your *Plane*, *Fig.* 66. ſo that all the *Hours* may be placed thereon, from *Sun-riſing* to 11 *a Clock*, aſſume two Points in the Line L D, as K, for 11, and I, for the 6 *a Clock* Hour-lines; then draw 6, I 6, and 11, K 11, *Perpendicular* to L D. This done, with the *Radius* of your Line of *Chords* on L, ſtrike the *Arch* O P, and from P to O place 15° 00′; and draw O K, to cut 6 I 6 in M, ſo ſhall I M be the Height of the *Stile* proportion'd unto this *Plane*; which may be a Plate of *Braſs*, whoſe Beadth muſt be equal to the Diſtance between the *Hour-lines* of 6 and 9, which muſt be placed *Perpendicular* to the *Plane*, on the Line 6 I 6, whoſe *Shadow* of the upper edge ſhall ſhew the Hour of the Day. Now to draw the *Hour-lines*, with the *Radius* of your *Line of Chords*, on M ſtrike the *Arch* Q N, which divide into five equal Parts, in the Points ●, ●, ●, *&c.* Then lay a Ruler from M unto each of thoſe Points, and it will cut the Line I K in the Points *, *, *, *&c.* through which Points (by *Chap.* 4. § 1. *Prop.* 4.) draw *Parallels* to 6 I 6, as the Lines 77, 88, *&c.* which ſhall be the true *Hour-lines* of an *Eaſt Plane*, from 6 *Fig.* 66. in the *Morning*, till 11 before *Noon*. Then for the *Hour-lines* of 4 and 5, you muſt prick off 5 as far from 6, as 6 is from 7; and 4, as far as 6 is from 8; and draw the *Hour-lines* 55, and 44, as before. Thus is your *Dial* compleated, and in the forming of which, you have made both an *Eaſt* and a *Weſt Dial*; which is the ſame in all reſpects, only whereas the *Arch* H G, through which the *Equinoctial* paſſed in the

the *East Dial* was described on the Right-hand of the *Plane*, in the *West* it must be drawn on the Left-hand; and the *Hour-lines* 4, 5, 6, 7, 8, 9, 10, and 11, in the *Fore-noon* in the *East Dial*, must be 8, 7, 6, 5, 4, 3, 2, and 1, in the *West* in the *Afternoon*; as in the *Figure* plainly appeareth. Or if you have drawn an *East Dial*, 'tis made a *West* by pricking it through, and drawing the Lines on the backside of the Paper: But Note also, That the *Hour-line* of 11 in the *East Dial* will be the *Hour-line* of 1 in the *West Dial*, and the *Hour-line* of 10 in the *East* the *Hour-line* of 2 in the *West*, and so on. Now you may find the Distance of the *Hour-lines* from the *Substile*, by this *Analogy* or *Proportion*: § 1. *Fig.* 67.

As the Radius,
To the Height of the Stile;
So is the Tangent of any Hour's Distance from 6,
To the Distance thereof from the Substile.

PROP. IV.

How to draw the Hour-lines on a Direct South *and* North Plane.

This *Plane* or *Dial* must stand upright, having his Face or *Plane*, if it be a *South Dial*, directly opposite unto the *South*; but if a *North Plane*, directly opposite to the *North*. Now admit it be required to make a Direct *South Dial*, for the *Latitude* of 51° 32′: To make which, first describe the *Circle* ABCD, to represent an *Erect Direct South Plane*; cross it with the *Diameters* CB and AD; then out of your *Line of Chords* take 38° 28′, the *Complement* of the *Latitude*, *Fig.* 68.

§ 1. *titude*, and ſet it from A unto a, and from B unto b: Then lay a *Ruler* from c unto a, and *Fig.* 68. it will cut the *Meridian* A R D in P, the *Poles* of the *World*; a *Ruler* alſo laid from C to b, will cut the *Meridian* A R D in Æ, which is the Point through which the *Equinoctial* muſt paſs; for the drawing of which, you have three Points given, *viz.* C, Æ, and B, and the *Centre* will always be in the *Meridian-line* A R D. To find the *Centre* thereof, do thus: Draw B Æ, and divide it equally in K, erect K M *Perpendicular* to Æ B (by *Chap.* 4. § 1. *Prop.* 1.) and continue it until it cutteth the Line M A R in M, ſo is M the *Centre* of the *Equinoctial-Circle* C Æ B, which deſcribe, then divide it into 12 equal Parts, in the Points ●, ●, ●, *&c.* the *Semi-circle* C D B; then lay a *Ruler* unto R, and to each of thoſe Points, and it will croſs the *Equinoctial-Circle* C Æ B in the Points *, *, *, *&c.* dividing that into 12 unequal Parts; then lay a *Ruler* from P, and to every of thoſe Points, and it will croſs the *Circle* of the *Plane* C D B in the Points I, I, I, *&c.* Laſtly, draw from the *Centre* R, and through thoſe Points the Right Lines, which ſhall be the true *Hour lines* of an *Erect Direct South Dial.* Now for the *Stile*, take 33° 28′, the *Complement* of the *Latitude*, and place it from D unto e, and draw R e for the *Axis* of the *Stile*, which muſt hang directly over the *Hour-line* of 12, or the *Meridian*, and muſt point downward, towards the *South Pole*, becauſe the *Plane* beholdeth the *South-part* of the *Meridian.*

Now in making this *Erect Direct South Dial*, you have alſo made an *Erect Direct North Dial*, which is but the backſide of the *South Dial*, lying

lying in the ſame *prime Vertical*; only as this hath the *South Pole* Elevated above it, and beholdeth the *South-part* of the *Meridian*, ſo the other hath the *North Pole* of the *World* Elevated above it, and beholdeth the *North-part* of the *Meridian*. The *Hour-lines* of 9, 10, 11, or 1, 2, and 3, is not expreſs'd on this *Plane*, becauſe 12, repreſenteth 12 at *Midnight*; neither are the other ſaid *Hours* expreſs'd, becauſe the *Sun* is never above the *Horizon*, at thoſe Hours: Therefore the *North Dial* is capable only to receive theſe *Hours*, namely, 4, 5, 6, 7 and 8 in the *Morning*, and 4, 5, 6, 7 and 8 at *Night*; as doth plainly appear in the *Figure*. Now the Diſtance of the *Hour-lines* from the *Meridian* may be found, by this *Analogy* or *Proportion*: § 1.

As Radius or S 90°,
To Sc. *of the Latitude*;
So is T. *of the Hour from Noon*,
To T. *of the Hour-line from the Meridian.* Fig. 69.

PROP. V.

How to draw the Hour-lines on a Horizontal Plane.

This *Horizontal Plane*, or *Dial*, is one of the beſt and moſt uſeful *Dials* in our *Oblique Hemiſphere*. Admit it be required to make an *Horizontal Dial* for the *Latitude* of 51° 32': To make which, firſt deſcribe the *Circle* ABCD, which repreſenteth your *Horizontal Plane*; then croſs it with the two *Diameters* ARC, and BRD; then take 51° 32' out of your *Line of Chords*, and ſet it from B to a, and from C Fig. 70.

§ 1. to b: Then lay a *Ruler* from A unto a, and it will cut the *Meridian* B D, in P the *Pole* of the *World*; then lay a *Ruler* from A unto b, and it will cut A B D the *Meridian*, in the Point Æ, where the *Equinoctial* cutteth the *Meridian*; then through the three Points A, Æ, and C, draw the *Equinoctial Circle*, whose *Centre* is at H, (and found as in the former *Proposition.*) Then divide the *Semi-circle* A D C into 12 equal Parts, in the Points ●, ●, ●, *&c.* Then lay a *Ruler* to R the *Centre* of the *Plane*, and on those Points, so shall the *Equinoctial Circle* A Æ C be by it divided into 12 Unequal Parts in the
Fig. 70. Points *, *, *, *, *&c.* Then a *Ruler* laid unto P the *Pole* of the *World*, and those Points, shall cut the *Semi-circle* C D A in those Points I, I, I, *&c.* Lastly, from the *Centre* R, and through those Points, let there be drawn *Right Lines*, which shall be the true *Hour-lines* of such an *Horizontal Plane*, from 6 in the *Morning* until 6 at *Night*; but for the Hours of 4 and 5 in the *Morning*, and 7 and 8 in the *Evening*, they are delineated by producing 4 and 5 in the *Evening*, through the *Centre* R, and 7 and 8 in the *Morning*, extending them out unto the other side of the *Plane*, so shall you have those *Hour-lines* also on your *Plane* delineated as you
Fig. 70. see in the *Figure*. The *Stile* of this *Plane* may be a thin Plate of *Brass*, cut exactly unto the quantity of an *Angle* of 51° 32′, and set *Perpendicular* on the *Meridian-line*, for the forming of this *Stile*, take out of your *Line of Chords* 51° 32′, and set it from D unto e, and draw R e, which shall be the *Axis* of the *Stile*. You may also prefix the *Halfs* and *Quarters* of *Hours*,

Hours, in the very ſame manner as the *Hours* themſelves were drawn. § 1.

Now to find out the Diſtance of the *Hour-lines* from the *Meridian*, ſay,

As the Radius or S. 90°, Fig. 70.
To the S. *of the Latitude*;
So is the T. *of the Hour from Noon*,
To the T. *of the Hour-line from the Meridian-line.*

Theſe kinds of *Dials* being ſo frequently uſed with us, in this *Oblique Sphere*; for the help of the ſpeedy Delineating of them, I have annexed hereunto the TABLE of *Longomontanus*, wherein the *Hour-lines*, for many *Latitudes*, are calculated.

The Uſe of the following TABLE is thus: The Firſt Column next the Left-hand is the *Latitude*, beginning at 30 *deg.* and ending at 60 *deg.* The Second Column is the *Diſtances* on an *Horizontal Plane* from xii. to xi. and i. in any of thoſe *Latitudes*. The Third is the *Horizontal Diſtance* betwixt xii. and x. and xii. and ii. and ſo on. The Laſt Column, next the Right-hand, is the *Complement* of the former *Latitudes*; and the Uſe of it is this: The *Diſtances* for an *Horizontal Dial* in the *Latitude* of 30 *deg.* will ſerve for an *Erect Direct South Dial* in the *Latitude* of 60 *deg.* So that the *Diſtances* here calculated ſerve for two *Dials*; as, if it be an *Horizontal* in the *Latitude* of 51 *deg.* 'twill be an *Erect Direct South* in the *Latitude* of 39 *deg.* Underſtand the like of others.

§ 1.

A TABLE, *shewing the Distance of the Hour-lines from the Meridian, in these Degrees of Latitude.*

An Horizontal Dial, *Lat.*	The Hour-lines from the Meridian.												A South Erect Dial, *Lat.*
	xi.		i. x.		ii. ix.		iii. viii.		iv. vii.		v.		vi.	
	D	M	D	M	D	M	D	M	D	M	D	M		
30	7	38	16	6	26	34	40	54	61	49	90	0		60
31	7	51	16	34	27	14	41	42	62	28	90	0		59
32	8	4	17	1	27	53	42	30	63	6	90	0		58
33	8	17	17	27	28	34	43	17	63	45	90	0		57
34	8	30	17	54	29	13	44	5	64	42	90	0		56
35	8	43	18	20	29	49	44	46	64	56	90	0		55
36	8	56	18	45	30	25	45	28	65	27	90	0		54
37	9	9	19	9	31	1	46	9	65	58	90	0		53
38	9	21	19	34	31	37	46	50	66	29	90	0		52
39	9	33	19	57	32	9	47	26	66	55	90	0		51
40	9	46	20	20	32	40	48	1	67	20	90	0		50
41	9	58	20	43	33	14	48	37	67	45	90	0		49
42	10	10	21	7	33	47	49	13	68	11	90	0		48
43	10	22	21	29	34	17	49	44	68	32	90	0		47
44	10	32	21	50	34	46	50	14	68	52	90	0		46
45	10	43	22	12	35	15	50	45	69	14	90	0		45
46	10	54	22	33	35	44	51	16	69	37	90	0		44
47	11	5	22	53	36	10	51	43	69	53	90	0		43
48	11	16	23	12	36	35	52	9	70	10	90	0		42
49	11	26	23	32	37	1	52	35	70	28	90	0		41
50	11	36	23	51	37	27	53	1	70	43	90	0		40
51	11	46	24	9	37	50	53	24	70	58	90	0		39
52	11	56	24	26	38	13	53	46	71	12	90	0		38
53	12	5	24	44	38	36	54	8	71	27	90	0		37
54	12	14	25	2	38	59	54	30	71	41	90	0		36
55	12	23	25	18	39	18	54	50	71	53	90	0		35
56	12	32	25	33	39	38	55	9	72	4	90	0		34
57	12	46	25	49	39	58	55	28	72	16	90	0		33
58	12	48	26	5	40	18	55	46	72	27	90	0		32
59	13	56	26	19	40	36	56	1	72	38	90	0		31
60	13	58	26	30	40	53	56	15	72	47	90	0		30

PROP.

§ 1.

PROP. VI.

How to draw the Hour-lines on an Erect Declining Plane.

Theſe *Planes* are made to ſet on the Sides of Houſes, wherein the *Meridian* is always a *Perpendicular*, drawn on the *Plane*, on whoſe top is the *Centre*, where the *Subſtile* and the *Hour-lines* all meet. Fig. 71.

Now before we can delineate the *Hour-lines* on any ſuch *Planes*, Two things muſt be given; as, *the Latitude of the Place*, and the *Plane's Declination*: By having which, we muſt find theſe Three things; *viz.* the *Pole's Height above the Plane*; *the Diſtance of the Subſtile from the Meridian*; and *the Plane's Difference of Longitude*.

For the finding of which *Requiſites*, by *Geometrical Projection*, we deſcribe on the *Dial Plane* theſe Circles of the *Sphere*, *viz.* the *Horizon*, *Meridian*, and *Equinoctial*; which being deſcribed in their true Poſition on the *Plane*, we proceed thus:

Admit it be required to make a *Direct South Dial*, on an *Erect Direct South Plane*, Declining *Weſtward* 24° 20′, in the *Latitude* of 51° 32′. Fig. 71.

Now in order to find the *Requiſites* beforemention'd, deſcribe the *Circle* ZHNO, and croſs it with the two *Diameters* ZQN, and HQO: Now Z is the *Zenith*, N the *Nadir*, ZQN the *Hour-line* of 12, HQO the *Horizon*. Now ſeeing the *Plane* declines S. W. 34° 20′; make N a, and O b, each equal to 34 20: Then a *Ruler* laid from Z to a, will cut the *Horizon* in S the *South* Point of the *Horizon*, through which draw

§ 1. the *Meridian* Z S N, whoſe *Centre* is at Y, found as in the 4th *Propoſition* aforegoing: Then a *Ruler* laid from Z to b, will cut the *Horizon* in W, the *Weſt* Point thereof. Now the *Horizon*, and the *Meridian* being projected on the *Plane*, take out of your *Line of Chords* 51° 32′, which place from H unto c, and from N unto d; then lay a *Ruler* from W unto c, and it cutteth the *Meridian* in P, the *Pole* of the *World*. Then through P and Q draw the Line P Q D, which repreſenteth the *Axis* of the *World*, and the *Subſtilar-line* of the *Dial*; then lay a *Ruler* from W to d, it cutteth the *Meridian* in Æ, ſo is W Æ two Points through which the *Equinoctial* muſt paſs, whoſe *Centre* is found, as afore to be at M, (being always in the *Axis*) of the *World*,) ſo have you on your *Plane* the *Horizon* H Q O, the *Meridian* Z P S Æ N, and the *Equinoctial* L Æ K W G, deſcribed on the *Plane*, as required.

Fig. 71.

Now, *Firſt*, To find the *Pole's Height above the Plane*, which in this *Scheme* is repreſented by B P, lay a *Ruler* from G unto P, and it ſhall cut the *Plane* in V; then meaſure the Diſtance B V on your *Line of Chords*, and you will find it to contain 34° 33′, which is the *Pole's Height above the Plane*.

Fig. 71. *Secondly*, To find *the Diſtance of the Subſtile from the Meridian*, repreſented in the *Scheme* by the *Arch* Z B, or N D; which meaſured as afore, will appear to be 18° 08′, the Diſtance of the *Subſtile* from the *Meridian*.

Thirdly, To find *the Plane's Difference of Longitude*, which in the *Scheme* is repreſented by the *Angle* Æ P K, lay a *Ruler* from P unto Æ, and

and it cutteth the *Plane* in X, then measure the *Arch* DX, as afore, and so will you find the *Plane's Difference of Longitude*, to be 30° 00′: Thus by *Geometrical Projection* have we found all the three *Requisites*: Now to find them by *Arithmetical Calculation* observe these *Analogies* or *Proportions*. § 1. Fig. 71.

1. For the *Poles* height above the *Plane*, say:
As Radius or S. 90°,
To Sc *of the Latitude* 38° 28′;
So is Sc. *of the Declination* 65° 40′;
To S. *of the Poles height above the Plane* 34° 33′.

2. For the Distance of the *Substile*, from the *Meridian*, say:
As the Radius or S. 90° 00′,
To the S. *of the Plane's Declination* 24° 20′;
So is Tc *of the Latitude* 38° 28′,
To the T. *of the Substilar Distance from the Meridian* 18° 10′.

3. For the *Plane's Difference of Longitude*, say: Fig. 71.
As the Sc. *of the Latitude* 38° 28′,
To the Radius or S. 90° 00′;
So is S. *of the Substilar Distance* 18° 10′,
To the S. *of the Difference of Longitude* 30 *Deg.*

Or it may be found thus:
As the S. *of the Latitude*,
To the Radius;
So is the T. *of the Declination*,
To the T. *of the Difference of Longitude* required.

These things found, we come now to shew how the *Hour-lines* may be projected. To project which observe, First, to lay a Ruler from

§ 1. P the *Pole* of the *World*, to Æ the *Interſection* Fig. 71. of the *Equinoctial* with the *Meridian*, and it will cut the *Plane* in x, where begin to divide the *Semicircle* L x G, into 12 Equal Parts in the Points ●, ●, ●, ●, *&c.* Then lay a Ruler from Q, to every of thoſe Parts, and it ſhall cut the *Equinoctial*; and divide it into 12 unequal Parts, in the Points *, *, *, *, *&c.* Then a Ruler laid from P the *Pole* of the *World* unto each of theſe Points, it will divide the *Plane* into 12 unequal Parts in the Points I, I, I, I, *&c.* Then by a Ruler laid from the *Center* Q, to thoſe Points, draw right Lines, which ſhall be the *true Hour-lines* proper unto ſuch a *Declining Plane*, as you ſee plainly demonſtrated by the *Scheme*.

Now the *Subſtilar Line* falleth in this *Dial*, juſt on the *Hour-line* of 2, in the Afternoon, becauſe the *Plane* declineth *Weſterly*. The *Angle* of the *Stile* is D Q R 34° 33′. which may be either a *Plate* or *Wyre*, brought into ſuch an *Angle*, which muſt be placed *Perpendicular* to the *Plane*, and directly over the *Subſtilar Line* Q D 2.

Now the diſtance of the *Hour-lines*, from the *Subſtilar Line*, may alſo be found by this *Analogy* or *Proportion*.

Fig. 71. *As the Radius,*
To the S. *height of the Pole above the Plane;*
So is the T *of the Hour-line from the Meridian of the Plane,*
To the T. *of the Hour-line from the Subſtile.*

Thus have you compleated your *Dial*; as you ſee in the *Scheme*, and here you may take notice that having finiſhed a *Weſt Decliner*,

you

you have alſo made an *Eaſt Decliner*; if you only convert the *Hour-lines* of the *Weſt Decliner*, in ſuch manner as you ſee in *Fig.* 72. on the *Eaſt Decliner*, and compleat all as you ſee in that *Scheme*. § 1. *Fig.* 72.

Thus I have explained the making and delineating of the beſt and moſt uſeful *Dials*, both by *Geometrical Projection*, and alſo by *Arithmetical Calculations*, in as brief and compendious a manner as poſſible. There are ſundry other kind of *Dials*, as *Incliners*, *Decliners*, and *Recliners*, which being not ſo uſeful, for brevity ſake, they are here omitted: As for *Inſtrumental Dials*, as *Quadrants*, *Rings*, *Cylinders*, *&c.* Which depend on the Sun's height, I refer you to Mr. *Edm. Gunter*'s Book, wherein *they* are largely deſcribed.

As for the Beautifying and Adorning of thoſe *Dials*, &c. by deſcribing on them the *Equinoctial*, *Tropicks*, *Parallels of Declination*, *Parallels of the Sun's Place*, *Length of Days*, *the Sun's Riſing* and *Setting*, *Jewiſh*, *Italian* and *Babyloniſh Hours*, *Almicanthars*, *Azimuths*, *Circles of Poſition*, the *Signs Right Aſcending*, *Deſcending*, *Culminating*, &c. I do adviſe you to conſult Mr. *Gunter*, Mr. *Foſter*, Mr. *Wells*, and Mr. *Holwel*'s *Works*, all which Authors have very learnedly ſhewed the deſcribing of them, by ſeveral large *Schemes* and *Figures*, for the plainer Illuſtration thereof.

Now ſeeing the *Latitude* of a Place muſt be firſt known, before a *Dial* can be made to it, I have therefore hereunto annexed a Table of the *Latitudes* of all the principal *Cities*, *Towns*, and *Iſlands*, in and about *Great Britain* and *Ireland*;

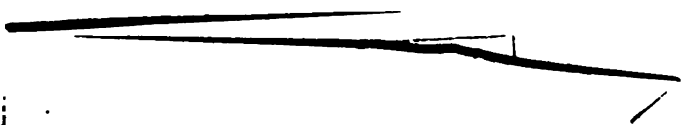
§ 1. *Ireland*; ſo that if you are to make a *Dial*, for any of thoſe Parts, you may have recourſe to this Table, and make your *Dial* to the *Latitude* of that Place, which you find to be the neareſt to the Place, for which you are to make your *Dial*.

A TABLE *of the Names and Latitudes of all the Principal Cities, Towns and Iſlands, in and about* Great Britain *and* Ireland.

ENGLAND.

	D.	M.
Arundel	51	00
Bedford	52	15
Barwick	55	54
Briſtol	51	35
Buckingham	52	10
Cambridge	52	20
Canterbury	51	25
Carliſle	55	20
Chichеſter	50	48
Cheſter	53	18
Colcheſter	52	08
Dover	51	20
Derby	53	00
Dorcheſter	50	50
Durham	54	56
Exeter	50	48
Falmouth	55	22
Gloceſter	51	57
Guilford	51	12
Hartford	51	54
Hereford	52	17
Huntington	52	30
Ipſwich	52	20
London	51	30
Lincoln	53	20
Leiceſter	52	45
Lancaſter	54	15
Northampton	52	24
Norwich	52	45
Nottingham	53	00
Newcaſtle	55	12
Oxford	51	50
Portſmouth	51	08
Plimouth	50	36
Reading	51	40
Salisbury	51	12
Stafford	52	50
Stanford	54	44
Shrewsbury	52	50

Truro



	D.	M.
Truro	50	30
Wincheſter	51	03
Worceſter	52	25
Warwick	52	30
York	54	00

WALES.	D.	M.
Angleſey	53	28
Barmouth	52	50
Brecknock	52	01
Cardigan	52	12
Caermarthen	51	56
Carnarvan	53	16
Denbigh	53	13
Flint	53	17
Landaffe	51	35
Monmouth	51	51
Montgomery	51	56
Pembroke	51	46
Radnor	52	19
St. David	52	00

SCOTLAND.	D	M
Aberdeen	57	30
Dunblain	56	21
Dunkel	56	48
Edenburgh	56	00
Glaſcow	55	58
Kinſaile	57	44
Orkney	60	06
St. Andrews	56	40
Skyraſſin	58	38
Sterling	56	12

§ 1.

IRELAND.	D.	M.
Antrim	54	38
Arglas	54	10
Armagh	54	14
Carterlagh	52	41
Clare	52	34
Corke	51	55
Droghedagh	53	58
Dublin	53	55
Dundalke	53	52
Galloway	53	02
Kenney	52	30
Kildare	53	00
Kings Town	53	08
Knockfergus	54	40
Kynſale	51	41
Lymerick	52	30
Queens Town	52	52
Waterford	52	09
Wexford	52	18
Youghall	51	53

ISLANDS.	D.	M.
Wight	50	48
Portland	50	30
Man	54	24
Lundey	51	22
Jerſey	49	17
Guernſey	49	03

CHAP.

CHAP. XIII.

Of FORTIFICATION.

THE *Utility of this Mathematical Art called* FORTIFICATION, *or* Military Architecture, *is ſo well known, that it needs not my Commendation, and therefore to ſpeak any thing thereto, were but to light a Candle before the Sun.*

In the handling of this Part of the *Mathematicks*, I ſhall be as brief as poſſible, yet as plain as can be deſired: In the proſecution of which, I will uſe this Method. *Firſt*, Give you the moſt principal *Definitions* or *Terms* belonging to this Art. *Secondly*, Preſcribe the moſt conducing *Maxims* or *Rules* herein obſerved. *Thirdly*, Shew how to delineate the Ground-line of any *Fortification*, according to the ſeveral Proportions, uſed by the beſt and moſt experienc'd Ingineers of *Italy*, *France*, *Holland* and *England*; *Fourthly*, Deſcribe the Conſtruction of the chief and principal Outworks now in uſe; and *Laſtly*, lay down ſome general *Maxims* or *Rules*, by moſt *Modern Authors* obſerved in *Irregular Fortifications*.

SECT. I.

Of the Definitions of the Lines and Angles, belonging to the Principal Ground-work of any Regular Fortification.

§ 1. *Defin.* 1. THE *Exterior* or outward Line, which boundeth the *Rampart*, at the Foot next the

the *Ditch*, is the principal and only Line to be regarded in all *Regular*, or *Irregular Fortifications*, being the *Basis* on which all the other Lines and Parts of the *Fortification* do depend. § 1.

2. The *Exterior Polygon* is the outward side of any *Regular Figure*, as in the *Hexagon* (which Figure I shall make use of through this Tract) the side A A, is the *Exterior Polygon*.

3. The *Interior Polygon*, is the inward side of any *Regular Figure*, as in the *Hexagon* is noted by any of the sides between P and P. Fig. 73.

4. The *Bastion* or *Bulwark*, is that great Work of any *Fort*, that advanceth it self towards the *Campaigne*, and here are six all marked with B, the Lines which terminate them, are two *Gorges*, two *Flanks*, and two *Faces*.

5. The *Demi-Gorge* or *Gorge-line* P C, is half the Entrance into the *Bastion*, and terminates the Point C, whereby the *Flank* shall be raised. Fig. 73.

6. The *Flank* is another Out-line of the *Bastion* as C F, which terminateth the *Curtain* and *Face*.

7. The *Face* is the utmost Line of the *Bastion*, as F A, two Lines thereof do form the *Angle* of the *Bastion* A, or the *Flanked Angle*.

8. The Line forming the *Flank* F F, is a prick'd Line, made use of by the *Dutch Ingineers*, and others.

9. The *Capital* is A P, part of the Line coming from the Center ⊙, terminated at the Point of the *Bastion* A. Fig. 73.

10. The *Curtain* is that Part of the *Interior Polygon* C C, which lieth betwixt the two *Bastians* B, and B.

11. The

§ 1. 11. The Line of *Defence* is A C, paſſing from A, the Point of the *Baſtion*, to C the *Angle* of the *Flank* and *Curtain*, and ought never to exceed 800 Engliſh Feet *.

Fig. 73.

* *Becauſe the length of a Muſket-ſhot doth not much exceed that Diſtance.*

12. The Line *Stringent*, is the Line coming from the Point of the *Baſtion* A, and prolonged on the *Face* A F, to the *Curtain* D, which ſheweth that D C, the Part of the *Curtain*, (by ſome called the *ſecond Flank*) will ſcour the *Face*.

13. The *Diameter* of the *Interior Polygon*, is the Line ⊙P, coming from the Center thereof ⊙.

14. The ſhorteſt Line from the Center unto the *Curtain* is ⊙ m. Theſe are the *Definitions* of the principal Lines, appertaining to the Groundwork of any *Regular Fortification*, the *Angles* followeth.

Fig. 73. 15. The *Angle* of the *Center* of the *Polygon* is P ⊙ P.

16. The *Angle* of the *Polygon* PPP, is always the *Complement* of the *Angle* at the *Center*, or remainder unto 180 *Degrees*.

17. The *Angle* of the *Triangle* PP⊙ is always the one half of the *Angle* of the *Polygon* PPP.

Fig. 73. 18. The *Angle* of the *Baſtion*, or the *Flanked Angle* FAF, is expoſed unto the *Batteries* of the *Beſiegers*, and formed by the two *Faces*, F A, and F A, which ought never to be leſs than 60, nor much above 100 *Degrees*.

19. The *Angle* of the *Eſpaule*, or Shoulder, is formed by the *Face* and *Flank*, as A F C.

20. The *Angle* of the *Flank* C C F, is formed by the *Curtain* and the *Flank*, and is moſt commonly a *Right Angle*, but by ſome later *Engineers*,

neers, is made *Obtuſe*, or more than a *Right Angle*, or 90 *Degree*. § 1.

21. The *Angle* made by the two Lines *Fitchant* AT, A, is called the *Angle* of the *Tenaile*.

22. The *Angle* forming the *Flank* is CPF, which *Angle* is made uſe of by moſt of the *Dutch Ingineers*. *Fig.* 73.

SECT. II.

Of General Maxims or Rules obſerved in Fortifications.

1. THAT all the Parts of the Place be of *Cannon-Proof* flanked, *i. e.* defended from another Place, which Place is no farther diſtant than the reach of a *Muſket-ſhot* from the Place to *be Flanked* or defended *: § 2.

* *Becauſe the Defence ought to be eaſie, quick, certain, and of little charge, all which qualities the Muſket hath and the Cannon hath not, therefore the Defence of Fortification ought to be meaſured by the Port of a Muſket, and not by that of a Cannon.*

2. That in all the Place there may be no Part of the *Wall*, or Out-ſide of the *Rampire*, that is not ſeen from the Top to the Bottom of the *Mote* or *Ditch*.

3. That the *Baſtions* are large, and full of Earth, and not empty; the bigger they are, they are the more to be eſteemed, there being the more room to Intrench, in caſe of neceſſity: whoſe *Gorge* let be at leaſt 35 *Fathoms*, and their *Flank* at leaſt 18 *Fathoms*.

4. That

§ 2. 4. That the *Angle* of the *Baſtion*, or *Flanked-Angle*, be not much above 90, nor much leſs than 60 *Degrees*, for in the former it would lie too very *Obtuſe* and open at the Point; and in the latter it would be too ſlender, and ſo eaſily to be Battered down by the Enemies *Cannon*.

5. That the *Angle* of the *Flank* may be ſomewhat *Obtuſe*; neither is there any more virtue in a *Right-Angle* than in any other for the Defence of the *Fort*.

6. That the Length of an extended *Curtain* be not above 135 *Fathoms*, nor the ſingle above 80 *Fathoms*, nor be leſs than 40 *Fathoms*, to be well defended from two *Flanks*.

7. That the *Rampire* be ſo wide, that a *Parapet* of Earth *Cannon-proof* may be erected thereon, and a *Terraplana* left full wide for the *Ordnance* to be *recoiled*.

8. That the *Mote* or *Ditch* be at leaſt 20 *Fathoms* broad, and as deep as poſſible. Now dry *Motes* in great Cities are to be preferred before others that are full of Water, to facilitate the *Sallies*, the Relief and Retreat of the *Beſieged*; and in ſmall *Fortifications* the *Motes* full of Water are the moſt Eſteemable, becauſe in ſuch *Sallies* are not neceſſary, and Surpriſes are very much to be feared.

9. That the Parts that are moſt remote from the *Center*, be commanded by thoſe which are neareſt to it.

10. That the Defence of a *Face* is much ſtronger when the *Angle* made by the *Face* and *Exterior Polygon* is a *Great Angle*; this *Maxim* is ſo very eſſential, that it will try the Goodneſs

Goodneſs of any *Fortification* whatſoever: Thus I have deſcribed the 10 chiefeſt *Maxims* neceſſary for Good *Fortifications*.

SECT. III.

Of the Conſtruction and making of the principal Ground-Line of a Fort, according to the moſt Modern Ways, uſed by the Italian, Dutch, French, *or* Engliſh Ingineers.

I. *Of the* Italian Fortifications.

§ 3. GEnnaro Maria, *Mathematician* to the *Catholick King*, wrote at *Florence*, his *Elements* of *Military Architecture*, Entituled, *Breve Trattato delle Moderne Fortificazioni*. This *Italian* Author was a very Learned and Skilful *Mathematician*, and famous in his Nation. In his ſaid Book, Printed 1665, he makes the *Interior Polygon* 800, and not leſs than 600 Feet, his *Demi-Gorge* he makes $\frac{1}{8}$ of it, and ſo for the *Flank* of the *Quadrangle*. But for the *Pentagon*, and all Figures above, he makes the *Flanks* $\frac{1}{10}$ Part of the *Gorge* more, and he placeth his *Flank* at *Right Angles* with the *Curtain*.

Suppoſing his *Interior Polygon* 1000 Parts, his *Gorges* will be 125, and in the *Quadrangle* the *Flanks* will be 125, but of the *Pentagon*, and all above 138 Parts. For the *Faces*, he makes them to fall on the third Part of the *Curtain*, unleſs in the *Square*, which he allows no *ſecond Flank*.

§ 3.

PROP. I.

To Fortifie a Hexagon *according to this Author's Proportion.*

Fig. 74. Firſt deſcribe the *Hexagon* PPP, *&c.* then divide the *Interior Polygon* PP, into 1000 equal Parts, take 125 for the *Gorges*, and ſet it from P to C. Then on C raiſe a *Perpendicular*, make it equal to 138 Parts, for your *Flanks* CF, then draw the *Face* AF, falling on the third Part of the *Curtain* C C, at D, and ſo do on every *Baſtion*, until the Work is compleated.

II. *Of the* French Fortifications.

Monſieur De la Mont, in his *Fortifications Offenſive* and *Defenſive*, Printed 1671: And *Monſieur Maneſſon Mallet* in his late Work, Entituled, *Travaux de Mars*, Printed 1672, aſſign theſe Proportions for the laying down the Ground-Line of a *Fort*.

Both theſe Authors make the *Interior Polygon* 768 Engliſh *Feet*, which they divide into 5 Parts, and taking one for the *Gorge* 153 $\frac{1}{5}$ *Feet*. Both divide it into 3 Parts, and take one for the *Capital*, that is 256 Feet.

Fig. 74. Now our firſt Author *De la Mont*, makes the *Flank* to ſtand at *Right-Angles* and takes 115 $\frac{1}{7}$ *Feet* for it, which is $\frac{1}{4}$ of the *Curtain*, and ſo draws the *Baſtions*, in all ſave the *Quadrangle*, and *Pentagon*, which he makes to have no *ſecond Flank*.

PROP. II.

To Fortifie a Hexagon *according to the Proportion of* De la Mont.

Firſt deſcribe your *Hexagon* PPP, *&c.* Now ſuppoſing your *Interior Polygon* PP, 1000 Parts, the

the *Capital* 333, the *Gorge* 200, and the *Flank* § 3.
150 Parts, take out of your *Triangular Scale*, Fig. 74.
Fig. 75, (which is made for the more ſpeedy Delineation, according to this Proportion of *De la Mont*) P A for the *Capital*, and prick it off from P, A on all the *Baſtions*. Then take P C, and prick off all the *Gorges* from P to C. Then take F C and prick it off at *Right-Angles*, from C to F. Laſtly draw all the *Faces* A F, A F, *&c.* ſo is your *Hexagon* compleat, as required.

PROP. III.

To Fortifie a Hexagon *according to* Maneſſon Mallet's *Proportion.*

Now our Author *Monſieur Maneſſon Mallet*, in his Works Entituled, *Travaux de Mars*, deviates from our former Author, only in this: that as *De la Mont* did place his *Flanks* at *Right-Angles*, he places them at 98 *Degrees* with the *Curtains*, and leaves no *ſecond Flank* in all his *Fortifications*.

Therefore having deſcribed the *Polygon* P P, Fig. 74.
&c. divide P P into 1000 Parts, prick off the *Capitals* P A 333, and the *Gorges* P C 200, then lay off the *Flanks* C F 150 Parts, at an Angle, of 98 *deg.* with the *Curtain* C C (by *Prop.* 5. §. 1. *Chap.* 4.) and draw all the *Faces* A F, A F, *&c.* Falling on C the Point of the *Flank* and *Curtain*, ſo ſhall your *Hexagon* be fortified, as was required.

III. *Of the* Dutch Fortifications.

The Emperor *Ferdinand* III. hath learnedly Fig. 74.
altered the Method of *Fritach*, *Dogen*, *Goldman*

§ 3. and *Faulhaberus*, all which were *Dutch Ingineers*, and wrote large Volumes on this Subject; In his Works Intituled, *Amuſſis Ferdinandea*, Publiſh'd 1654; by turning their way of working by *Angles* into working by *Sides*.

Thus he ſetteth down a *Catholick* Way of delineating the *Sides* or *Lines* of any *Fort* by his 60 *Prop.* thus: the *Interior Polygon* to be 66, the *Capital* 24, the *Gorge* 15, and the *Flank* 12. Or in making the *Interior Polygon* 22, the *Capital* 8, the *Gorge* 5, and the *Flank* 4. Or yet making the *Interior Polygon* 1000, the *Capital* 363, the *Gorge* 227, and the *Flank* 181, this is an *Epitome* of all the *Dutch Fortifications*, and is general excepting for the *Square*, which
Fig. 74. muſt have no *ſecond Flank*.

PROP. IV.

To Fortifie a Hexagon *according to the Emperor's Proportion.*

Firſt deſcribe the *Polygon* PPP, *&c.* divide PP, *&c.* into 22 Parts, take 8 for the *Capitals* PA, which prick off all round from P to A, take 5 for the *Gorges*, which prick off all round
Fig. 74. from P to C, then take 4 for the *Flank* CF, which prick off all round at *Right-Angles* from C to F, laſtly draw the *Faces* AF, AF, AF, *&c.* So is the *Hexagon* compleated as was required.

IV. *Of the* Engliſh Fortifications.

His Late Majeſty of Great Britain, Carolus *II.* of Ever-Bleſſed Memory, hath much facilitated the Method of Count *Pagan*, who in his *Fortifications*, Printed at *Paris*, 1645, did place the *Flanks*

Flanks at *Right-Angles* with the *Line of Defence*, and he works by the *Exterior Polygon*. Now His *Majesty* places the *Flank* at *Right-Angles* with the *Line of Defence* of the *Interior Polygon*, and works after another manner: Count *Pagan* makes the Proportion of the *Grand Royal Fort*. Supposing the *Exterior Polygon* to be 1000 Parts, will make the *Perpendicular* MT to be 150, and the *Complement* of the *Line of Defence* TC to be 185, which may serve for a general *Proportion* be the Length what it will, only in a *Square* the *Proportions* must thus be altered in the *Grand Royal Fort*, the *Perpendicular* MT must be 162, in the *Mean* R 144, and in the *Petty Royal* 126, the *Complement of the Line of Defence* for the *Grand Royal Fort is* 228, and for the *Mean Royal Fort* 198, also for the *Petty Royal Fort* 198. § 3. Fig. 74.

PROP. V.

How to Fortifie a Hexagon *according to Count* Pagan's *Proportion.*

To delineate this Work draw a *Line*, about the Middle whereof, as at M, set off MA, the half of the *Exterior Polygon* 500 Parts, which makes the *Exterior Polygon* 1000, then on M (by *Prop.* 1. §. 1. *Chap.* 4.) raise the *Perpendicular* Mm, which make MT *Equal* to 150, then draw ATC, and ATC, then take 185, and place it from T to C, and to C, and draw CC for the *Curtain*, then on the Points C raise *Perpendiculars* CF, to the *Line of Defence* CA, for the *Flanks*, so have you also the *Faces* FA. Then on the Points A set off half the *Angle* of the *Figure*, to wit 60° (as you see in the *Table*, Fig. 74.

§ 3. in *Page* 38.) and draw the *Lines* OA and OA, ſo ſhall O be the *Center* of the *Figure*, and PC
Fig. 74. the *Gorge*, and AP the *Capitals*: then finiſh each *Baſtion* at your own Diſcretion, and the Work is finiſhed as required.

PROP. VI.

To Fortifie a Hexagon *according to the way preſcribed by His Majeſty* Carolus *II.*

His Late Majeſty *Charles* II. hath much facilitated this Work, as will appear in this following *Example*, by making the *Line of Defence* ſtand at *Right-Angles* with the *Flank* of the *Interior Polygon*, by this *Table*, which ſuppoſes
Fig. 74. the *Interior Polygon* to be 1000. Then

Polygons	4	5	6	7	8	9	10	*Streight-Lines.*
Capital	398	437	367	333	312	300	291	233
Gorge-Line	155	196	203	242	252	260	263	300

Now to deſcribe the *Hexagon* PP, *&c.* Then divide the *Interior Polygon* PP, into 1000 Parts, take 367 and prick off all the *Capitals* PA; Then take 203 and prick off all the *Gorges*
Fig. 74. from P to C. Now draw the *Lines of Defence* AC and AC, *&c.* Then at C ſet the *Flanks* at *Right-Angles* with the *Line of Defence* AC, ſo ſhall FC be the *Flank*, and FA the *Faces*, then finiſh every *Baſtion*, and your *Hexagon* is Fortified as was required.

☞ Thus

☞ Thus have I ſet down the ſeveral Ways and Rules for laying the Fundamental *Ground Line*, from the moſt conſiderable *Ingineers* of this laſt Age, out of all which it's moſt agreeable to thoſe Authors, and to Practice, to take $\frac{1}{7}$ of the *Interior Polygon* for the *Capital*, $\frac{1}{7}$ for the *Gorge* and *Flank*, which leaves $\frac{5}{7}$ for the *Curtain*, and let this be taken for a general Rule, where the *Flank* and *Curtain* ſtand at *Right-Angles*. § 3. *Obſerve this for a general Rule in Regular Fortification.*

PROP. VII.

By the Semicircle *to lay down on the Ground any of the former* Fortifications.

Having drawn the *Plot* of your *Fort* on *Imperial Paper*, or *Vellom*, and if it be a *Regular Fort* you need not deſcribe it but two *Half Baſtions* from the *Center*, for that will be ſufficient. Having ſuch a *Plate*, whoſe Length is ſet down on each reſpective *Line*, and all proper *Angles* expreſſed, will not only be uſeful for laying down the Work, but for finding the Solidity of the *Ramparts*, *Parapets*, and the other *Earth-Works*. *See* Fig. 76. *Fig.* 76.

If it be in ſuch a Place, that from the *Center* of the *Fort*, all the *Angles* may be ſeen, place your *Semicircle* at Z, and lay off all the *Angles* of the *Center*, which here is 60°; then mark out the *Diametrical Lines*, and making them their due Length, as by your *Plate* they appear to be, ſet *Piquets* on all the P P' upright with the *Plane*: Then take up your Inſtrument and place a *Piquet* at Z: Then *lock-ſpit* out all the *Polygons* P P: Then mark out the *Gorges* C P, then ſet out the *Flanks* C F, *Caſe* 1. *Fig.* 76.

§ 3. either at *Right-Angles*, or as otherwiſe required: Then *lock-ſpit* out the *Flanks* CF, and the *Faces* AF, having firſt ſet off the *Capital* PA, ſo is the *Fort* lined out for the *Ground-Line*.

Caſe 2. But if there be *Houſes* and *Obſtacles* in the way, that from the *Center* all may not be ſeen, then muſt you mark out any one Side and meaſure it, and at each End ſet off the *Angles* of the *Polygon*, (which here is 120°) and draw Side after Side until all be finiſhed: Then finiſh the *Baſtions* as before, and here great Care muſt be had, or elſe you will run into infinite Errors.

☞ But you have Liberty Experimentally to alter any of the former *Proportions*, as you have occaſion, and as will beſt ſerve the Place; as you ſee by the Fortifying a ſtraight-lined *Fig.* 77. Figure: *Fig.* 77. where in Count *Pagan*'s or in *Maneſſon*'s way it may not be allowed without ſome Alterations.

SECT. IV.

The Dimenſions and Meaſures of the Rampires, Parapets, Mote, Covert way, and it's Glacis, with the Breaſt-work.

§ 4. THE *Rampire*'s Thickneſs and Height muſt receive its Determination from the *Judgment* of the *Ingineers* and Purſe of the *Prince*.

Fig. 78. The *Height* TS, muſt not exceed 18 Feet, nor be leſs than 10; the Thickneſs may be from 50 to 80 in all *Royal Works*, and according as *Earth* is to be had; the *Slope* of the Inward

ward Side *Rampire* TR, is commonly a Foot for a Foot, therefore RS the *Talu*, will be equal to the *Height* TS; ſo if TS be 18, RS will be 18, if 15 then 15 Feet. § 4.

The *Outward Slope* is generally proportioned ½ a Foot for a Foot, ſo if the *Height* OQ be 18, the *Talu* OA will be 9 Feet, *&c.*

The *Height* of the *Parapet* ZD muſt always be 6 Feet, the *Exterior Height* PM muſt be 4 Feet, the *Thickneſs* of the *Parapet* DM in *light Earth* muſt be 20 Feet, in *ſtiff Earth* 16, and in ſolid *Rough Clay* 14 Feet. Suppoſe it be 18 Feet, PM will be 4 Feet, MQ 2 Feet, LD 1 Foot, ſo will the lower *Thickneſs* LQ be 21 Feet.

The *Height* of the *Foot-Bank* VX is 1 ½ Foot, and *Thickneſs* VL 3 Feet.

The *Lizier* muſt be made ſo wide as to ſupport the *Rampire* from ſlipping into the *Ditch*, and is taken from 3 to 10 Feet; the *Mote* or *Ditch* may be from 70 to 130 Feet, that is, from E to G, and the *Depth* IF may be 8, 10 or 12 Feet, the little *Ditch* at the Bottom of the *Mote* repreſented by c q g, muſt be as large and deep, as the Earth and Work will give leave.

The *Covert-way* is left about 18 Feet wide from G to C, on which is placed a *Parapet* and *Bank*, like that on the *Rampire*, which *Parapet* muſt ſlope ſo into the *Campaigne*, that a ſtraight *Line* drawn from Z the Top of the *Rampire*, may terminate F d the Slope of the *Glacis*.

PROP.

§ 4. **PROP. VIII.**

How to lay down the Profile of the Work, according to this Table.

	Feet.
The Base of the Rampart RA	70
Height TS *or* OQ	16
Inward Talu RS	16
Outward Talu OA	8
Base of the Parapet LQ	21
Inward Height ZD	6
Outward Height MP	4
Outward Talu MQ	2
Inward Talu LD	1
Breadth of the Foot-bank VL	3
Height of it VX	1½
The Terra Plana TV	22
The Berm or Lizier AE	3
The Mote's Breadth EG	112
The Depth of it IF	12
Breadth at Bottom FH	88
The Talu's EI *or* KG	12
Breadth of the little Ditch cg	18
Depth of it	5
The Covert-way GC	18
The Base of the Glacis or Counterscarp Cd	60
The Height and Breadth of the Foot-bank, and Height of the Parapet, as before.	

Now to lay down this *Profile* do thus; draw a *Base* or *Ground-Line* of convenient Length, as RSOACGd, then by a *Scale* of 20 or 30 at most in an Inch, representing Feet, take out of it 70 for RA, 16 for RS, 8 for OA, 3 for AE, 112 for EG, 18 for GC, and 60 for

for CD, and mark them off on your *Paper*, as in *Fig.* 78. ſet alſo from E to I and K to G, 12 Feet for the *Talu's* of the *Mote*, at S, O and C raiſe, and from I and K let fall *Perpendiculars*. Then take 16 Feet for ST and OQ, 12 for IF and KH, and 6 for Cf, and draw RT, TQ, QA, EF, HG, Cf, fd. Then from Q ſet off QL 21, LV 3, QM 2, and LD 1, and raiſe the *Perpendiculars* MP 4, DZ 6, and VX 1 ½. Then draw VX, XY, YZ, ZP and PQ, and make the little *Ditch* by its Meaſure, ſo is the *Profile* perfected. § 4.

The Solid *Content* of thoſe *Earth-Works* may eaſily be attained by the former Rules, which *Content* being got in Feet, Divide the *Product* by 324, the *Quotient* ſhall be the *Solid Floors* contained therein, a *Floor* being 18 Feet Square and 1 Foot Deep.

SECT. V.

Of the Dimenſions and Conſtruction of Platforms, Caveleers and Casemate in the Flanks,

1. *Platforms* are *Plantations* where *Guns* are to be placed, and are commonly made of *Plank* and *Sleepers*; there needeth for one *Gun* to be but one *Platform*, which muſt be 8 Feet broad next the *Parapet*, and 14 Feet wide at the other End, and their Length ſhould be 18 Feet. § 5.

2. An *Embraſure* is the *Port-Hole* made in the *Parapet*, which towards the *Gun* muſt be 4 Feet wide, and towards the *Campaigne* 8 Feet wide, whoſe *Height* muſt be proportioned unto the *Wheel* of the *Carriage*, and are 16, 18 or 20 Feet aſunder.

3. *Ca-*

§ 5. 3. *Cavaleers* or *Mounts*, are Maſſy-pieces of *Earth* raiſed on or near the *Rampart*, above the *Parapet*, on which *Ordnance* and *Small-ſhot* may be planted. As to their Conſtruction I ſhall follow the Method of *Maneſſon*, who places them in the *Gorge* of the *Baſtion*, and gives this Rule for it, [ſaith he] "Lengthen out the *Line of* "*Defence* to E, until it cut the *Capital*, the "*Center* of your *Cavaleer* ſhall be the middle "Point betwixt P and E, to wit at F, then "with the Diſtance of 84 Feet on the *Center* F "ſtrike a *Circle*, which ſhall be the *Baſe* of "your *Cavaleer*": Now its *Height* ought to be at leaſt 20 Feet; and if the Work be to be faced with *Stone* or *Brick*, it needs not have a *Talu* above 3 ½ Feet, ſo that the *Diameter* at the Top will be about 153 Feet, whereon ſet a *Parapet* of 20 Feet Seat, and 4 Feet high, and other Dimenſions as aforeſaid, in the *Rampire*, and there will be a *Terra-plana* at the Top of above 100 Feet, whereon ſix Pieces of *Ordnance* may be planted, making *Embraſures* and *Platforms* as was laſt directed. *Fig.* 79.

4. *Cazemats* are made in the Corners of the *Flanks*, and are ſeveral *Platforms* for *Guns* to to be planted on, thereby to be hid from the *Battery* of the *Enemy*: As to the Conſtruction I ſhall follow *Maneſſon*'s Directions; Firſt as to the Form, and alſo to the Meaſure: [ſaith he] "The *Cazemate* ſhall take up one half of the "*Flank*, and no more; The *Grand Cazemate* D "B is about 7, 8 or 9 Feet from the *Level* of "the *Plane* of the *Fort*, and hath a Paſſage into "it from within the *Fort*. A, C is its *Parapet* of 20 "or 22 Feet Seat, and in it let there be 3 or 4

"*Em-*

" *Embraſures*; D is the Part thereof moſt hid § 5.
" from the Enemies *Cannon*; F is the *Magazine*
" for this *Battery*; H is the ſecond *Cazemate*, G Fig. 79.
" the *Ladder*, and L the *Magazine*, and M the
" *Parapet*; this is to hold but one *Gun*; M is the
" third *Cazemate* on the Level of the *Baſtion*,
" which let be all firm, in which let there be no
" void place. The *Dimenſions* and *Conſtruction*, according to the Method of this our Author are thus. [Saith he] " Lengthen the *Line of Defence*
" from C to G ſome 40 Feet, then draw CD,
" *parallel* to C f, (by *Prop.* 4. § 1. *Chap.* 4.) let
" CF be half of c f, ſo that c F may be equal to
" F f, then from the middle of the *Face* oppoſed,
" draw K F, and let it cut GD, in I, then make I
" L, and FM equal to 6 Feet, then make MN 66 Fig. 79.
" Feet, and draw NO *parallel* to the *Flank*,
" which let be 24 Feet: Laſtly [ſaith he] for
" the *Orillon* or Blind, prolong the *Face* F T 36
" Feet, and alſo F V 36 Feet, then join T V, and
" make that Part all ſolid: So is your *Cazemate*
" finiſhed: Let the *Height* of the lower *Caze-*
" *mate* be 6 Feet as before, and let all the reſt
" be compleated as you ſee in the *Figure*.

SECT. VI.

Of the Dimenſions and Conſtructions of thoſe Out-Works called Ravelins, Horn-works, Crown Works, &c.

THE *Ravelin* is a certain Work lying be- § 6.
yond the *Mote* or *Ditch*, for the covering the *Curtain*, *Bridge* and *Gate*; the *Angle* of the *Ravelin*

§ 6. *Ravelin* muſt nor be leſs than 60, not much above 100 *deg.* The manner of delineating it is thus: Lengthen out the *Middle Line* of the
Fig. 80. *Curtain* OM unto a convenient Length, then take with your *Compaſſes* the Length of the *Curtain* CC, and ſetting one Foot in F, the Point of the *Face* and *Flank*, croſs the *Middle Line* in q; then laying the *Ruler* at q, and to the *Points* F, draw the *Lines* of the *Ravelin* qR and q S, which ſhall be the *Ground-Lines* of the *Ravelin*: The *Mote* ſurrounding it muſt be half the Breadth of the *Great Mote*; the *Rampart* may be 30 Feet thick, and ſome 6, 7 or 8 Feet high, on which may ſtand a *Parapet* equal to
Fig. 81. that of the *Rampire*.

Now if from the *Points* F you raiſe ſtreight *Lines* into the *Campaign*, at *Right-Angles* to the *Curtain*, and from the *Points* F ſet off F E, and F E 720 or 750 Feet, then may you join E E either with, A;

Single Tenaile: which is done by joining *E E*, and dividing it into four equal Parts; take one and place from D to N, and ſo draw E N and E N, ſo have you a *Single-Tenaile* I E N E L, which muſt have a *Mote Rampire*, and *Parapet* like the *Ravelin*. *Secondly* it may, if occaſion require, be Fortified with, A;

Horn-Work: which is done by joining the *Points* E E, and Fortifying the *Exterior Polygon*
Fig. 83. E E as is afore taught: Or divide E E into three Parts; make M E, and E N equal to M O; then draw N M, which divide ſo likewiſe at O and P; then draw E O and E P; then at P and O raiſe *Perpendiculars* O Q and P R, ſo ſhall M, E, Q, O, P, R, E, N, be the *Horn-Work* which

was

was desired: which must likewise have small *Rampires* and *Parapets*, as afore. § 6.

For the *Crown-Work*: From the *Center* of the *Fort* O draw O M B of a convenient Length, then from the middle of the *Ravelin* set off 1000 or 800 Feet to B, then on q, strike the *Arch* D B E, set off the *Curtain* and *Demi-Gorge* P C C, from B to F, and G both ways, then draw C F and C E, to terminate the Points I and H on the *Counterscarp*; then take $\frac{1}{4}$ Part of B F or B E, and set it from B to M, and from F to L, and from E to M; then draw L M, and M N; then for your *Demi-Bastions* make N P and L O equal to N E, *&c.* Then for the *Demi-Gorges* of the whole *Bastion* in the Middle, let them be equal to $\frac{1}{5}$ of the *Interior Polygon* L M or M N, *viz.* M Y or M X; then finish the *Bastions* by drawing the *Lines of Defence*, and raising *Perpendiculars*, or making *Angles* of 98° at O, X, Y, and P, then the *Crown-work* is finished as desired. You may make *Ravelins* and other Works (beforementioned) before these *Curtains*, if occasion require. There are some other Works which are used; as *Half-Moons*, *Bonnets*, *Double Tenails*, *Counter-guards*, *Horse-shooes*, *Priests-Caps*, &c. which would be superfluous to speak of in this Place. *Fig.* 84.

5. *Cittadels* are *Castles* or *Forts* of the least sort, and are the *Out-works* lastly used, which are * commonly of 4 or 5 *Bastions*, and are placed in such Order, that there may be two *Faces* and a *Curtain* towards the *Town*: the *Construction* whereof is after this manner: Lengthen out the *Line* O M, and

* *Built to bridle the Town or the Place, left the Burghers should be rebellious, and to be the last refuge or place of retreat.*

therein

§ 6. therein find the *Center* of the *Cittadel*, the *Interior Polygon* of the *Pentagon* may be ¾ of the *Curtain* adjoining, or a little more; the *Center* of the *Square* may be on P the *Point* of the *Interior Polygon*, the *Center* of the *Hexagon* may be near the outward *Point* of the *Baſtion* of the *Town*, taken away to make the *Cittadel* in, which may be delineated as afore: The *Motes* and other Works in proportion accordingly, and the *Rampires* as high as thoſe of the *City* or *Town*.

SECT. VII.

Of ſome Maxims or Rules neceſſary to be known in Irregular Fortification.

§ 7. IRregular *Fortification* is when any *Town* or Place is to be Fortified, which lieth in an Irregular Form; *i. e.* whoſe *Sides* and *Angles* are unequal in the Fortifying of *Irregular Figures* *. I ſhall here ſay very little, only I ſhall lay down ſome Precepts that are of immediate concern in Fortifying of *Irregular Figures*, and ſhall refer you to peruſe *Marlois*, *Dogen*, *Fritach*, *Taurnier*, *Dilichius*, &c. which will greatly ſatisfie and help you: To this end know,

* *The* Ingineer *muſt firſt form a Map of the Town or Place, with all the Ways, Paſſages, Old Walls, Rivers, Pools, Incloſures, and all other matters fit to be known in the Draught, and then he is to deſign what Works he findeth moſt agreeing to the Place to be Fortified.*

1. That the ſame *Laws* and *Maxims* for *Regular Fortifications* ſtand and be in Force for *Irregular*; *i. e.* that the *Line of Defence* muſt not

not exceed the *Port* of a *Musket*, nor the *Angles* of the *Baſtion* be leſs than 60°, nor much above 90°, *&c.* §7.

2. That no *Inward Angle* of the Place be leſs than 90°, if it be ſo it muſt be altered, and that *Point* may be made the *Outward Point* of a *Baſtion.*

3. That between *Regular* and *Irregular Fortifications*, there is no other Difference, but by rectifying the *Sides* that are too ſhort, or too long, and altering the *Angles* that are too little; as for the *Sides*, if they be above 500, and under 1000 Feet, they may be Fortified by *Baſtions* placed according to the uſual manner, at the *Extreme Points* thereof; But if the *Sides* be between 1000 and 1700 Feet, then in the midſt you may place a *Plat-Baſtion*, and at the *Extreme Points* place two *Baſtions*, as before: But if the *Line* be leſs than 500 Feet, you may lengthen it, by producing it into the *Plane*: As for the *Angles*, they are made greater or leſſer, according as occaſion requireth. For the Raiſing the *Rampires*, *Parapets*, and other *Out-works*, they are to be as in the *Regular*, and the *Out-work* may be placed before the *Curtains*, as was before mentioned.

4. That the *Capital*, in any *Regular* or *Irregular Baſtion*, is found by dividing the *Angle* of the *Polygon* into two equal Parts (by *Prop.* 7. §. 1. *Chap.* 4.) and by producing the *Line of Angular Diviſion* or *Separation*, on which the due Length of the *Capital* muſt be placed, which obſerve for a general Rule.

SECT. VIII.

Of the Dimenſions and Conſtruction of ſmall Forts or Sconces, which are built for the Defence of ſome Paſs, River, or other Place.

§ 8. WHEN they are made *Regular*, of 4, 5, or 6 *Baſtions*, then they may be Fortified by the precedent Rules, but there are others of ſmaller Dimenſions fit for the ſame Purpoſe: *viz. Triangle with Demi-Baſtions*, *Square* with *Demi-Baſtions*, *Parallelograms* with *Demi-Baſtions* and *Tong*, *Star-Redoubts* of four, five or ſix Points, and *Plain Redoubts*.

PROP. IX.

To Fortifie a Triangle *with* Demi-Baſtions.

This *Triangle* may conſiſt and be comprehended of three equal or unequal Sides in this
Fig. 84. *Example*: let it be an *Equilateral Triangle* PPP. Now divide PP into three Parts, then take 1, and prick off the *Capitals* PA, *&c.* and the *Gorges* make equal thereunto, as PC, PC, *&c.* then make the *Flanks* FC to ſtand at *Right-Angles*, and to be $\frac{1}{4}$ of PC or PA, then draw the *Faces* AF, AF, *&c.* and the Work is finiſhed as required.

PROP. X.

To Fortifie a Square *with* Demi-Baſtions.

The Sides of the *Square* may be from 100 to
Fig. 85. 200 Feet, let PP be 180 Feet, which divide into 3 Parts, take one for the *Gorges* PC, and for the *Capitals* PA, and prick them off all round as

as you ſee, then take $\frac{1}{7}$ of PP, and at *Right-Angles* prick off the *Flanks* CF, then draw the *Faces* AF, AF, *&c.* and the *Figure* is compleated. § 8.

PROP. XI.

To Fortifie a Parallelogram *with* Demi-Baſtions *and* Tong.

Firſt deſcribe the *Parallelogram* or *Long-Square*, PPPP, then divide PP into 6 Parts (the Side on which the *Tong*, or *Tenaile*, is placed) and make MC equal unto $\frac{1}{7}$ thereof, and alſo MG and MH. Draw CG, GC, and CH, HC, then finiſh the *Demi-Baſtions* as before, ſo ſhall the Work be compleated as was required. A *Long Square* may alſo be Fortified, as *Fig.* 77. *Fig.* 86.

PROP. XII.

To Fortifie a Star-Redoubt *of* 4, 5 *or* 6 Points.

1. A *Star-Redoubt of Four Points* may have his Side from 40 to 60 Feet: Firſt deſcribe the *Square* PPPP, then divide PP into two Parts at M, take $\frac{1}{7}$ of PM, (and by *Prop.* 1. § 1. *Ch.* 4.) raiſe *Perpendiculars* round at M, make MA equal to $\frac{1}{7}$ of PM, and draw all as in the *Figure*. *Fig.* 87.

2. *A Star-Redoubt of Five Points* is thus Fortified. Deſcribe the *Pentagon* PP, *&c.* then divide PP into halves at M, raiſe the *Perpendiculars* MA, make MA equal to $\frac{1}{7}$ of PM, and draw the *Fort* in all reſpects as the *Figure* repreſenteth. *Fig.* 88.

3. *A Star-Redoubt of Six Points* is thus Fortified. Deſcribe the *Hexagon* PPP, *&c.* divide PP into two equal Parts at M, then raiſe *Perpendiculars* at the M's, then make MA equal to $\frac{1}{7}$ of PM, or $\frac{1}{7}$ of PP, and draw every reſpective Line, as you ſee in the *Figure*. *Fig.* 89.

§ 8. PROP. XIII.

To Delineate a Plain Redoubt.

Plain Redoubts are called *Grand Redoubts*, which are uſed as *Batteries* in *Approaches*, whoſe Fig. 90. Side may be from 60 to 80 Feet, or *Petit Redoubts*, which are uſed for a *Court of Guards* in the *Trenches*, and may be from 20 to 50 Feet, and are framed and delineated in all reſpects, as you ſee in *Fig.* 90.

The *Profiles* to be ſet on theſe ſeveral Works, and the *Motes*, are alterable and uncertain, for they being ſometimes uſed in *Approaches*; then they do require the *Breaſt-work* at the Bottom to be 7 or 8 Feet wide, and the *Interior Height* 6, and the *Exterior* 5 Feet, and the *Mote* to be either 8, 10 or 12 Feet, ſometimes 14 or 20 Feet wide at the Bottom, and the Height of 7, 8 or 9 Feet, to have two or three Aſcents to riſe to the *Parapet*. There are many other things belonging to this Art, which the Limitation I am bound to, will not permit here to be treated of.

CHAP. XIV.

Of Military Orders, *or the Embattelling and Encamping of Soldiers.*

SECT. I.

Of the Embatteling and Ordering of Soldiers.

§ 1. BATTAILS *are conſidered either in reſpect of the Number of Men, or in reſpect of the form of Ground. In the reſpect of the*

the Number of Men, it is either a Square Battail, *a* Double Battail, *a* Battail *of the* Grand Front, *or a* Battail *of any Proportion, of the Number in* Rank *to the Number in* File. *In reſpect of the form of the Ground, the* Battail *is either a* Geometrical Square *of Ground, or a* Long Square *of Ground. For the* Diſtance *or* Order *of Soldiers, martialled in Array, is diſtinguiſhed either into* Open Order, *which is when the* Centers *of their Places are* 7 *Feet diſtant aſunder, both in* Rank *and* File, *or* Order; *which is when the* Centers *of the Places are* 3 ½ *Feet diſtant both in* Rank *and* File; *or elſe* 3 ½ *Feet in* Rank, *and* 7 *Feet in* File. §. 1.

PROP. I.

To place any Number of Soldiers (ſuppoſe 36000*) in ſuch Order of* Battail, *that there ſhall be any Number in* Rank *or* File. *Suppoſe* 500 *in* Rank, *how many muſt there be in* File?

Divide the Whole Number of the Army by the Aſſigned Men in *Rank* (or *File*) the *Quote* (72) is the Number that will be in *File* (or *Rank.*)

PROP. II.

To Order any Number of Soldiers into a Square Battail *of Men.*

Admit it were required to Martial into a *Square Battail* 16129 Men: To do which, extract the *Square Root* of 16129 (by *Prop.* 8. §. 1. *Chap.* 1.) which is 127, therefore you are to place 127 Men in *Rank*, and alſo in *File.*

§ 1.

PROP. III.

To Order any Number of Soldiers into a Double Battail.

Admit 16928 Men were to be Martialled into a *Double Battail*, extract the Root of half the Number of Men; *i.e.* of 8464, whose Root is 92, therefore I say that 92 Men must be placed in *File*, and 184 in *Rank*, to order that Number of Men propounded into a *Double Battail.*

PROP. IV.

To Order any Number of Soldiers into a Battail *of the* Grand Front.

Admit 16900 *Soldiers* were to be Martialled into a *Battail* of the *Grand Front*, that is, *Quadruple.* Extract the *Square Root* of 4225 (that is $\frac{1}{4}$ of the Men) the *Root* is 65; therefore I say 65 must be placed in *File*, and 260 in *Rank*, to form a *Battail* of the *Grand Front.*

PROP. V.

Any Number of Men, together with their Distance in Rank *and* File, *being propounded, to Order them into a* Square Battail *of Ground.*

Admit 2500 *Soldiers* were to be Martialled into a *Square Battail* of Ground, in such sort that their Distance in *File* should be 7 Feet, and in *Rank* 3 Feet, and 'tis required to know how many Men must be placed in *Rank* and in *File* to draw up 2500 Men into *Square Battail* of the Ground. According to *Prop.* 1. §. 1. *Ch.* 1. say, *As* —— 7 to 3; *So is* 2500 to 1071, *&c.* whose *Square Root* is 32, *&c.* therefore I say 32 Men are to be placed in *File*, Now to find how

how many Men are to be placed in *Rank*, Divide 2500 by 32, the *Quotient* is 78, which are the Number of Men to be placed in *Rank*, and 4 Men to be diſpoſed elſewhere. § 1.

PROP. VI.

Any Number of Soldiers propounded, to Order them in Rank *and* File, *according to the reaſon of any two Numbers given.*

Admit 6400 *Soldiers* are to be Martialled into *Array*, in ſuch *Order* that the Number of Men placed in *File*, ſhall bear ſuch Proportion to the Number in *Rank* as 7 to 13; (according to *Prop.* 1. §. 1. *Chap.* 1.) ſay as 7 to 13; ſo is 6400 to 11885, *&c.* whoſe *Square Root* is 109, *&c.* the Number of Men to be placed in *Rank*, by which divide 6400, it produces, 58, *&c.* the Number of Men to be placed in *File*, and 78 Men to be employed elſewhere.

SECT. II.

Of Caſtrametation, or Quartering and Encamping of Soldiers.

IN *Quartering* and *Encamping* of *Soldiers*, it is requiſite the *Quarter-Maſter General*, and all other under *Quarter-Maſters*, be skilled at *Foot-Meaſure*, that ſo they may lay out their *Quarters* as directed. § 2.

The common Allowance for the Depth of Ground, that a *Regiment of Horſe* or *Foot* will take up, the Wideneſs muſt be anſwerable to the Number of Men 200 Feet for the *Hats* in

§ 7. Length, and 100 for the *Commanders* and *Sutlers* before them; every two *Soldiers* to a *Hut*, 8 Feet broad, and 8 Feet deep, 2 Feet *Hut* from *Hut*, ſo that there may ſtand 20 *Huts* in the 200 Feet, the Alley betwixt *Hut* and *Hut* may be 8 Feet, that is 16 Feet in width, and 200 in Length for 40 Men, which is 3200 Feet, and for the 100 Feet more, 1600 Feet, in all 4800 Feet, and there muſt be 25 Rows of *Huts* for 1000 Men; ſo that for a *Regiment of Foot* containing 1000 Men, with *Officers* and *Sutlers*, will take up 120000 Feet, which is 2 *Acres* and 3 *Roods*, which becauſe of Ways may be allowed 3 *Acres* of Ground for every *Regiment*, which may be 350 Feet deep and 370 Feet wide, or near 360 Feet Square: Now if 1000 Men, *Officers*, *Sutlers*, *High-ways*, and all take up a Square of 360 Feet, how many Feet ſhall the Side of a Square be wherein 10000 *Foot-men*, &c. may be encamped? ſay (by *Prop.* 1. *Chap.* 1.) as 1000 to 10000; ſo is the Square of 360, *viz.* 129600, to 1296000, the Square of 1138 Feet, which is very near 30 Acres of Ground.

For the *Quartering* of *Horſe*, you muſt keep the ſame Depth of 300 Feet for all, and take 200 Feet for the *Huts*, the *Horſe-Huts* muſt be 10 Feet deep and 4 wide; ſo that 12 *Horſes* may ſtand in one *Hut* together, which is 48 Feet long and 10 wide, and 6 Feet a Street; The *Huts* for the *Troops*, will be 6 for 12 *Troops*; now conceive a *Regiment* to conſiſt of 8 *Troops*, ſo to a *Troop*, it will take up, leaving 20 Feet Streets, and Croſs-ways, very near as much Ground as a *Regiment of Foot*, Ways and all muſt be allow'd 3 Acres, near 360 Feet Square, ſo that 10 *Regiments*

ments of Horſe will take up 30 Acres: Moreover, it will be needful and you may very well allow as much Ground as both *Horſe* and *Foot* will take, for the *Train of Artillery*, *Victuallers*, *Parade Places*, &c. From theſe Conſiderations the young Beginner, nay even the better practiſed *Soldier* may receive Help, and thereby be enabled to Encamp an *Army* if required. § 2.

CHAP. XV.
Of GUNNERY.

SECT. I.

Of the Names of the Principal Members of a Piece of Ordnance.

1. *Defin.* A CANNON is a long round Body, either of *Braſs* or *Iron*, formed and made hollow by Art, and Proportion, to offend afar of, with a *Ball* of *Iron*, *Stone*, or any Artificial Subſtance, Charged with *Gun-Powder*, in it's charged *Cylinder*, which being fired, in an inſtant performs its deſired Effect. This *Machine* was invented by an *Engliſh-man*, and firſt put in Practice by the *Venetians* againſt the *Genoueſes* at *Chiezza*, Anno 1376. § 1.

This Military Engine Bombarda, *Gun*, *Cannon*, &c. *ſo called from* Bombo, *a reſounding Noiſe*, Cannone, *or Cannon, from the likeneſs it holds with his* Canna, *Bore or Concavity*; Artegleria, *from* Artiglio, *the Talons or Claws of Ravenous Fowls, becauſe its Shot flying afar off, tears and defaces all that it doth meet; from whence ſome Natures of this Machine are called* Smeriglii, *long-winged Hawks*, Falconi, *Falconets*; Paſſa volanti, *ſwift flying Arrows*, &c.

2. The

§ 1. 2. The *Superficies* of the *Metal*, is the Outside round about the *Piece*.

3. The Body is the Subſtance of the whole Maſs of *Metal*.

4. The *Chaſe* is the Concavity of the Piece, in which they put the Charge.

5. The *Muzzel* is the Extremity of the *Chaſe* by which you load and unload the *Piece*.

Fig. 91. 6. The *Calibre* is A B the *Diameter* of the *Muzzel* or *Mouth*.

7. The *Touch-hole* is that little *Vent* which paſſeth from the *Convex Superficies* to the very *Chamber* of the *Piece*, made to give fire to the *Powder* within, as C; that which encloſeth the Extremity of the *Chaſe* about the *Touch-hole* is called the *Breech* or *Coyl*.

8. The *Caſcabel* is the *Pummel* at the *Breech* or *Coyl*, as D.

The *Trunnions* are pieces of *Metal* fixed unto the *Exterior Superficies* of the *Gun*, on which he moves in the *Carriage*, as E E.

The *Body* of the *Piece* is that which is comprehended betwixt the *Center* of the *Trunnions* Fig. 91. and the *Caſcabel* E G.

The *Vacant Cylinder* is comprehended betwixt the *Center* of the *Trunnions* and the *Muzzel*, as E B.

The *Frees* or *Muzzel Ring*, is that thick *Corniſh* which encompaſſeth the *Convex Superficies* of the *Piece* at I, the *Baſe Ring* is K L G, the *Reinforced Ring* is M, the *Trunnion Ring* is N, and the *Corniſh Ring* is O.

The *Line of the Cylinder* is a direct Line imagined to be deſcribed along the *Chaſe Parallel* unto the Middle of the *Chaſe*, as X Z.

The

The Line of Metal is a Line touching both *Corniſhes*, as M N I. § 1. Fig. 91.

The *Diſpart Line* of the *Piece*, is the Difference betwixt the *Semidiameter* of the *Muzzel* and *Baſe Ring*, as the Line I H.

The *Vent* of the *Piece* is the Difference betwixt the *Diameter* of the *Shot* and the *Mouth* of the *Piece*, as e d.

The *Chamber* or Charged *Cylinder*, is that Part of the *Chaſe* towards the *Touch-hole* equally large, nor narrower in one Place than in another, and doth contain the *Powder* and *Ball*.

SECT. II.

Of the Dimenſion of our Uſual Engliſh Cannon, and other Ordnance, &c.

IN the following *Table* I have ſet down the Length and Weight of our moſt uſual *Engliſh Ordnance*, the *Diameters* and Weight of their *Bullets*, the Length and Breadth of their *Ladles*, the Weight of *Powder* to Charge them, *&c.* § 2.

PROP. I.

How to know the different Fortification of a Piece of Ordnance.

In Fortifying any Piece of *Ordnance* there are three Degrees obſerved, as firſt *Legitimate Pieces*, which are thoſe that are ordinarily Fortified; ſecondly *Baſtard Pieces*, which are ſuch whoſe *Fortification* is leſſened; thirdly *Double Fortified Pieces*, or extraordinary *Pieces*.

The

The Names of the ſeveral Pieces of Ordnance.	Length of Guns		Guns weight	Bore of Guns		diamet. of Bullets		weight of Bullets		length of Ladles		breadth of Ladles		weight of Powder		Shoots Level Random	
	Feet	Inches	Pounds	Inches	8 parts	Inches	8 parts	Pounds	Ounces	Inches	8 parts	Inches	8 parts	Pounds	Ounces	Paces	Paces
A Baſe.	4	6	200	1	2	1	1	0	5	4	0	2	0	0	8	60	600
A Rabinet.	5	6	300	1	4	1	3	0	8	4	1	2	4	0	2	70	700
A Falconet.	6	0	400	2	2	2	1	1	5	7	4	4	0	1	4	90	900
A Falcon.	7	0	750	2	6	2	5	2	8	8	2	4	4	2	4	130	1300
Minion ordinary.	7	0	800	3	0	2	7	3	4	8	4	5	0	2	8	120	1200
Minion largeſt.	8	0	1000	3	2	3	0	3	12	9	0	5	0	3	4	125	1250
Saker leaſt.	8	0	1400	3	4	3	2	4	12	9	6	5	6	3	6	150	1500
Saker ordinary.	9	0	1500	3	6	3	4	6	0	10	4	6	6	4	0	160	1600
Saker old ſort.	10	0	1800	4	0	3	6	7	5	11	0	7	2	5	0	163	1630
Demiculver leaſt.	10	0	2000	4	2	4	0	9	0	12	0	8	0	6	4	174	1740
Demiculver ordinary.	11	0	2700	4	4	4	2	10	11	12	6	8	0	7	4	175	1750
Demiculver old ſort.	11	0	3000	4	6	4	4	12	11	13	4	8	4	8	8	178	1780
Culverin leaſt.	11	0	4000	5	0	4	6	15	0	14	2	9	0	10	0	180	1800
Culverin ordinary.	12	0	4500	5	2	5	0	17	5	16	0	9	4	11	6	181	1810
Culverin largeſt.	12	0	4800	5	4	5	2	20	0	16	0	10	0	11	8	183	1830
Demicannon leaſt.	11	0	5400	6	2	6	0	30	0	20	0	11	4	14	0	156	1560
Demicannon ordinary.	12	0	5600	6	4	6	1	32	0	22	0	12	0	17	8	162	1620
Demicannon large.	12	0	6000	6	6	6	3	36	0	22	6	12	6	18	0	180	1800
Cannon Royal.	12	0	8000	8	0	7	4	58	0	24	0	14	0	32	8	185	1850

The *Fortification* of any Piece of *Ordnance*, is accounted, by the Thickneſs of the *Metal*, at the *Touch-hole*, *Trunnions*, and at the *Muzzel*, in Proportion to the *Diameter* of the *Bore*. § 2.

The *Legitimate Pieces*, or the ordinary *Fortified Cannons*, have $\frac{7}{8}$ at the *Touch-hole*, $\frac{5}{8}$ at the *Trunnions*, and $\frac{3}{8}$ at the *Muzzel* of the thickneſs of the *Bore*, in thickneſs of *Metal*. *Baſtard Cannons*, or *leſned Cannons*, have $\frac{3}{4}$ at their *Touch-hole*, or $\frac{12}{16}$, and $\frac{9}{16}$ at their *Trunnions*, and $\frac{7}{16}$ at their *Muzzel*: the *Double Fortified Cannons* have full one *Diameter* of the *Bore* in thickneſs of *Metal* at the *Touch-hole*, and $\frac{11}{16}$ at the *Trunnions*, and $\frac{7}{16}$ at their *Muzzel*. Now all *Double Fortified Calverins*, &c. are $\frac{1}{8}$ at the *Touch-hole*, $\frac{11}{16}$ at the *Trunnions*, and $\frac{9}{16}$ at the *Muzzel*, and the *Ordinary Fortified Culverins*, are Fortified every way as *Double Fortified Cannons*, and *leſned Culverins* as *Ordinary Cannons* in all reſpects.

PROP. II.

How to know how much Powder is fit for Proof, and what for Service for any Piece of Ordnance.

For *Cannons* take $\frac{4}{7}$ of the Weight of their *Iron Bullet* of good *Corn Powder* for Proof, and for Service $\frac{1}{3}$ the Weight of the *Iron Bullet* is ſufficient, eſpecially for *Iron Ordnance*, which will not endure ſo much *Powder* as *Braſs* ones will receive by $\frac{1}{4}$ in Weight, for *Culverins* allow the whole Weight of the Shoot for Proof, and $\frac{2}{3}$ for Service. For *Sakers* and *Falcons*, take $\frac{4}{7}$ of the Weight of the Shot, and for leſſer Pieces the whole Weight may be uſed in Service, until they grow hot; but then there muſt be ſome Abatement

§ 2. ment made at Diſcretion, and take 1 $\frac{1}{8}$ of the Weight of their *Iron Bullet* for *Proof*.

PROP. III.

To know what Bullet is fit to be uſed in any Piece of Ordnance.

The *Bullet* muſt be ſomewhat leſs than the *Bore* of the *Gun*, that ſo it may have Vent in the Diſcharge; ſome Authors affirm $\frac{1}{8}$ of an Inch leſs than the *Bore* will ſerve all *Ordnance*, but this Vent is too much for a *Falcon*, &c. and too little for a *Cannon*: therefore I approve them not, but commend Mr. *Philip*'s Proportion * to your Uſe, which is to divide the *Bore* of the *Gun* into 20 equal Parts; and let the *Diameter* of the *Bullet* be $\frac{19}{20}$ thereof, according to which Proportion the precedent *Table* is calculated.

* *In his Mathematical Manual.* Page 165.

PROP. IV.

By knowing the Proportion of Metals one to another, and by knowing the Weight of one Ball, to know what any other ſhall weigh.

The common received Proportions for *Metals* are theſe:

Lead is to *Iron* as 2 to 3.
Lead is to *Braſs* as 24 to 19.
Lead is to *Stone* as 4 to 1.
Iron is to *Lead* as 3 to 2.
Iron is to *Braſs* as 16 to 18.
Iron is to *Stone* as 3 to 8.

The more exact Proportion betwixt *Metals* are thus known: Admit a *Cube*, or *Ball* of *Gold*, weigh 100 *l.* a *Cube* of any of thoſe *Metals* enſuing of the ſame bigneſs, ſhall bear ſuch Proportion as followeth, to the ſaid *Cube* of *Gold*.

Gold

	li.	*pts.*		*li.*	*pts.*
Gold	100	00	*Iron*	42	10
Quickſilver	71	43	*Tin*	38	95
Lead	60	53	*Stone*	15	80
Silver	54	39	*Water*	05	68
Braſs	47	37			

§ 2.

It is the Opinion of Dr. *Wybard* in his *Taſſometria*, that a *Bullet* of *Caſt Iron*, whoſe *Diameter* is 4 Inches doth weigh 9 *l. Averdupoize Weight.*

Now to find what any other *Bullet* or *Cube* ſhall weigh; ſay (as in *Prop.* 4. *Chap.* 1.) As the *Cube* of the *Bullet* propounded, is to his Weight, ſo is the *Cube* of another *Bullet* given, to his Weight, and ſo obſerve ſtill this Proportion.

SECT. III.

Of the Qualification of an able Gunner, and neceſſary Operations before Shooting, and in ſhooting.

§ 3.

A *Gunner* ought to be a Man of Courage, Experience, and Vigilant; he ought to have good Skill in *Arithmetick*, to know the Extraction of the *Roots*, &c. He ought to have Skill in *Geometry*, to take *Heights*, *Diſtances*, &c. to know the Diviſions and Uſe of his *Circle*, *Quadrant* and *Quadret*; to know how to level and to lay *Platforms*, and to raiſe *Batteries*. He muſt know the Names of all ſorts of *Ordnance*, their *Weight*, the *Height* of their *Bore*, the *Height* and *Weight* of their *Shot*, the *Length* and *Breadth* of their *Ladles*, how much *Powder* to uſe for

Proof

§ 3. *Proof* and *Action*; The *Shoots Level* and the *Shoots Random*; He muſt know the Names of all the Members of a *Piece of Ordnance*; he muſt alſo know the *Length*, *Thickneſs* and *Breadth* of all manner of *Carriages*, and muſt know all the Parts thereof: *viz.* the *Cheeks* or *Sides*, the *Ax-tree*, *Spokes*, *Nave*, *Hoops*, *Tranſomes*, *Bolts*, *Plates*, *Drawing-Hooks*, the *Clout*, the *Hole* for the *Linſpin*, the *Shafts*, the *Thill* and *Thill-bolt*, the *Fore-lock* and *Fore-lock-keys*, *Cap-ſquares*, the *Fore-lock-pins* and *Chain*, the *Pintle* and *Bolt-hole*, *Fellows*, *Nails*, *Fellow-bars*, *Stirrops*, the *Ruts* of the *Wheel*, *Dowledges*, *Beds*, *Coins*, *Levers*, *Hand-ſcrews*, &c. He muſt alſo know how to make his *Ladles*, *Spunges*, *Cartridges*, whether of *Paper* or *Canvas*, and to have by him *Formeres* of all ſorts, *Sheep-ſkins* undreſt to make *Spunges*, *Powder*, *Shot*, *Needles*, *Thread*, *Paſte* and *Starch*, *Marlin*, *Twine*, *Nails*, *Hand-ſpikes*, *Crows* of *Iron*, *Granado-ſhells*, and Materials for *Compoſition*, *Faſces*, *Budg-Barrels*, *Cannon-Baskets*, &c. Theſe being general things he is to know, and at all times to have ready by him, and he is more particularly to know theſe following Parts of his Art: As,

PROP. I.

How to Tertiate, Quadrate, and to Diſpart a Piece of Ordnance.

1. *To Tertiate a Piece*, is to find whether it hath its due Thickneſs at the *Trunnions*, *Touch-hole* and *Neck*; if the *Trunnions* and the *Neck* are in its due Order, and the *Chaſe* ſtreight.

2. *To Quadrate a Piece* mounted, is to ſee whether it be directly placed, and equally poized in

in the *Carriage*; which is known by finding in the *Convex Superficies* of the *Base* and *Muzzel Ring*; the Point which is *Perpendicular*, over the *Soul* of the *Piece*: which may be found by the *Gunner's* Instrument, called a *Level*; an Instrument whose Use is so vulgarly known, that it needeth not my Explanation. § 3.

3. To *Dispart a Piece*, is to fix or elevate on the *Convex Point* of the *Muzzel Ring*, a Mark as far distant from the *Cylinder*, or *Soul* of the *Piece*, as is the Point of the *Base-Ring*; to the end that the *Visual-ray* which passeth by these Marks, may be *Parallel* to the *Chase*, *Soul*, or *Cylinder* of the Piece. Now the *Dispart*, i. e. the Difference of the *Semidiameters* of the *Cornishes*, may be by a pair of *Calliper Compasses* attained: Which found, place on the Top of the *Cornish Ring* near the *Muzzel*, over the middle of the *Inferior Cylinder*.

PROP. II.

To know how far any Piece of Ordnance will shoot, &c.

As to the several Shootings in *Artillery*, Authors differ much in their Judgments and Opinions, but they all unanimously agree that the *Ball* being shot forth flies through the Air, with a *Violent*, *Mixt* and *Natural Motion*; describing a *Parabolical Line*, in whose Beginning and Ending are Lines sensibly streight, and in Middle Curved: In the Beginning the Impreſt Force driving forward by the Fire, the *Natural Gravity* of the *Ball* doth describe a *Right-Line*, called the *Direct Line*, or *Ranges* of the *Ball's Circuit*. Fig. 92.

§ 3. In the Middle that Force diminiſheth, and the *Natural Gravity* prevaileth, ſo that it deſcribeth a Curved Line, called the *Ball's Middle Helical* or *Conical Arch**; In the End the *Natural Gravity* overcoming the Impreſt Violence, (which becomes altogether weak and faint) deſcribes a new Right Line, called the *Ball's Declining Line*, in which the *Ball* tends towards the *Center* of the Earth, as towards a Place natural unto all heavy Bodies: See *Figure* 92. Theſe Motions are ſomewhat longer, according as the *Piece* is mounted from the *Level* unto the *Angle* of 45 *deg.* which is called the *Utmoſt Random*: The *Elevation* of which is regulated by the *Gunners Quadrant*, the Uſe of which Inſtrument is ſo generally known, and by ſo many Authors fully explained, that I here crave Leave to omit it: But take theſe for General Rules.

* *See Mr.* Digges *in his* Pantometria, *Page* 179.

General Rules to be obſerved in the Battering down of a Place, or making of Breaches.

1. *That a Shot at* Right-Angles *ſtrikes more violently and furiouſly than at* Oblique Angles; *therefore* Gunners *uſe, when they are to Batter down a* Tower, Wall *or* Earth-work, *to ſhoot Point Blank at the Object, Tire by Tire; by diſcharging all the Pieces in the Battery againſt the ſelf-ſame Object in the ſame Inſtant,* holding it for a Maxim, *that Ten Cannons diſcharged together, do far more Execution than diſcharged one after another.* Now at *Oblique Angles* they ſhoot either Croſs-ways or by rebounding.

2. *That the ſpeedieſt way to make a Breach in a* Wall, *&c. is by ſhooting at the Object from Two* Batteries, *which ruins far more ſpeedily than by ſtriking the Object with one* Battery *at* Right-Angles, *although that one* Battery *hath as many Cannon as the other Two hath.*

3. *That*

3. *That if you were to batter a* Flank *covered with an* Orillion, *(which becauſe you cannot poſſibly Batter it right forward) you muſt therefore of neceſſity Batter it Obliquely, by way of Rebounding*, thus: Chuſe a fit Place in the *Curtain* to be your *Object*, on which you may play with your *Battery* Obliquely, ſo that by a Rebound the Shot may leap into the *Flanks*, holding for a *Maxim*, in this Operation, * *That the Angles of Incidence and Reflexion are Equal.* § 3.

* *According to Learned* D' Chales, *on the 4th Prop. of the firſt Book of* Euclid.

Now we come to ſhew the Length of the *Right Range*, of all our Common Engliſh *Ordnance*, which is ſet down in the precedent *Table*, in which the *Cannon* exceed not 185 *Paces*, &c. Eſteeming the *Pace* 5 Engliſh *Feet*, nor his *utmoſt Random* above 1850 *Paces*, which *Table* ſo ſheweth for all other Natures.

As for the *Ranges* and *Randoms*, to the ſeveral *Degrees* and *Points* of *Mounture* of the *Quadrant*, I have hereunto annexed ſome *Tables* calculated by the Experiments of ſundry moſt Eminent *Artiſts*, whoſe Works will perpetuate their Worth and Name to ſucceeding Generations.

The Uſe of the following Table of Randoms.

This *Table* is moſt agreeing to *Cannons* and *Culverins*, and the greateſt ſort of *Ordnance*, the Uſe whereof is thus:

Admit a *Saker* to be mounted to 3 *deg.* ſhoots the *Bullet* 323 *Paces*, how far will it ſhoot being mounted unto 7 *deg.*? Say (*by Prop.* 1. *Chap.* 1.)

As 510 the *Tabular Diſtance* for 3 *deg.* of *Mounture*, to 323, the *Diſtance* found;

So is 934 the *Tabular Diſtance* for 7 *deg.* of *Mounture*, to 592 $\frac{17}{30}$, the *Diſtance* required,

§ 3. which the *Saker*, according to this Experiment, ſhall ſhoot at 7 *deg.* of *Mounture*.

A Table of Ranges and Randoms, to the ſeveral Degrees of Mounture of the Quadrant.

A TABLE OF

Right Ranges, or Point Blanks:		*Randoms, or the Firſt Graze.*	
The Degrees of the Pieces Mounture.	*The Right Range in Paces of 5 Feet.*	*The Degrees of Mounture.*	*The Paces of the Random 5 Feet a Pace.*
0	192	0	192
1	209	1	298
2	227	2	404
3	249	3	510
4	261	4	610
5	278	5	722
6	285	6	828
7	302	7	934
8	320	8	1044
9	337	9	1129
10	354	10	1214
20	454	20	1017
30	693	30	2185
40	855	40	2289
50	1000	50	2283
60	1140	60	1792
70	1220	70	1214
80	1300	80	1000
90	1350	90	

Mr. *NYE* in his Book of *Gunnery*, Printed *Anno* 1647. ſaith he made an Experiment by a *Saker* of 8 Feet long, which he loaded with Three Pounds of *Powder*, of an exact Weight, both *Powder* and *Wad* at every Charge, every time ramming it down with three equal ſtroaks, as near as poſſible; but on the *Bullet* he put no *Wad*, becauſe the *Saker* was mounted; And thus

this he made four Shoots, each of them half an Hour after the other, that ſo the *Piece* might be of equal temper, and mounted his *Piece* to theſe four *Degrees of Mounture*, viz. 1 *deg.* 5 *deg.* 7 *deg.* 10 *deg.* and found theſe *Randoms*. § 3.

At 1 *Deg. the Random* was 225
At 5 *Deg. the Random* was 416
At 7 *Deg. the Random* was 505
At 10 *Deg. the Random* was 630
} *Paces.*

According to which Experiment, he framed this Table of *Randoms*.

Deg.	*Paces*	*Deg.*	*Paces*
0	206	6	461
1	225	7	505
2	274	8	548
3	323	9	589
4	370	10	630
5	416		

Captain HEXAM in his Book of *Gunnery*, ſhews how by finding out the *Random* of a *Cannon*, for the firſt *Degree of Mounture*, thereby to find the *Random* for every *Degree* to 45 *deg.* or utmoſt *Random*, and this is his Rule to perform it. Firſt find how many *Paces* the *Cannon* will ſhoot being laid level by the *Metal*, (which by him is accounted 1 *deg.*) Then divide the *Diſtance* found by 50, then multiply the *Quotient* by 11, ſo ſhall the *Product* be the greateſt *Digreſſion*, or *Difference* betwixt *Ranges* and *Ranges*; which being divided by 44, the *Quotient* giveth the Number of *Paces*, which the *Bullet* will *loſe* in the other *Ranges*, from

§ 3. *Degree* unto *Degree*; according to this Rule, this *Table* is calculated.

A Table of Randoms to 45 Degrees, accounting a Pace 2 ½ Foot.

D. Mount.	Paces.	Diff.	D. Mount.	Paces.	Diff.
0	0775	225	23	4685	110
1	1000	220	24	4795	105
2	1220	215	25	4900	100
3	1435	210	26	5000	95
4	1645	205	27	5095	90
5	1850	200	28	5185	85
6	2050	195	29	5270	80
7	2245	190	30	5350	75
8	2435	185	31	5425	70
9	2620	180	32	5595	65
10	2800	175	33	5560	60
11	2975	170	34	5620	55
12	3145	165	35	5675	50
13	3310	160	36	5725	45
14	3470	155	37	5770	40
15	3625	150	38	5810	35
16	3775	145	39	5845	30
17	3920	140	40	5875	25
18	4060	135	41	5900	20
19	4595	130	42	5920	15
20	4325	125	43	5935	10
21	4450	120	44	5945	5
22	4570	115	45	5950	

I have

I have hereunto also annexed the *Table* calculated by *Alexander Bianco*, for all sorts of *Ordnance*, (which *Table* I account one of the best that was ever yet found Extant) In his Work Printed 1648. § 3.

A Table of Randoms for the first Six Points of the Gunner's Quadrant.

Points.	1	2	3	4	5	6
Falconet.	375	637	795	885	892	900
Falcon.	550	935	1166	1254	1309	1320
Minion.	450	765	954	1026	1071	1080
Saker.	625	1062	1325	1125	1487	1500
Demi-culver.	725	1232	1537	1653	1725	1740
Culverin.	750	1275	1590	1710	1785	1800
Demi-cannon.	625	1062	1325	1425	1487	1500
Cannon of 7.	675	1147	1431	1489	1606	1620
Double Cannon.	750	1275	1590	1710	1785	1800

SECT. IV.

Of Shooting in Morters.

A *Mortar-Piece* is a short *Piece* with which they shoot *Bombs*, *Granado-Shells*, *Stone-Bolls*, &c. and their *Flight* is not in a Streight Line, but in a Curve Line from on high, so that it may fall where it should be desired; This *Mortar* is placed in the *Carriage* in all respects, as you see in *Fig.* 93. in which A signifies the *Carriage*, B the *Mortar*, C the *Course* § 4.

the *Shot* flies, and D the *Place* on which it falls.

Bombs are great *Hollow Balls* or *Carcaſſes* of *Iron*, in which are put fine Sifted *Gun-Powder*, which by a *Fuſe* they proportion to them a due Fire, that ſo they may break in ſuch a determinate Time, or at ſuch a determinate Diſtance; theſe *Fuſes* are ſmall Trunks of *Wood*, *Tin* or *Iron*, filled with a prepared Compoſition for that Purpoſe. *Granadoes* are of the ſame Form, and for the like Uſe with *Bombs*, only ſmaller, and many times are thrown by the Hand, and are made not only of *Iron*, but of *Braſs*, *Glaſs* or *Earth*.

Now in Order to the right and true Shooting in thoſe kind of *Machines*, 'tis requiſite to let you know that Authors have differed very much in their Opinions, but now they all unanimouſly agree that all *Projectiles* are in the *Curve* of a *Parabola*, as has been prov'd by *Galilæus*, *Torricellius*, and of late by the Ingenious Captain *Halley*, in a *Diſcourſe of Gravity*, Publiſhed in *Philoſ. Tranſact.* Numb. 179. In which *Diſcourſe* he Mathematically Demonſtrates that the Motion of all *Projects* is in the *Curve* of a *Parabola*; from the Properties of which *Curve* theſe uſeful Propoſitions have been deduced.

Firſt, That the *Diſtance* of *Horizontal Ranges* are one to another, *as the Sines of the Double Angles of Elevation.*

Secondly, That the *Heights* of *Shots* in the *Air* are one to another, *as the Verſed-Sines of the Double Angles of Elevation.*

Thirdly,

Thirdly, That the *Times* of Continuance of a Shot in the *Air* at different *Elevations*, are one to another *as the Sines of the Angles of Elevation.* §. 4.

From theſe Three *Propoſitions*, all that is neceſſary in the *Practice of Gunnery* may be known.

As Firſt: Suppoſe I make a Shot upon ſome given *Elevation*, and then meaſure the Length of the *Horizontal Range* of that Shot (which is only the Diſtance between the *Mouth* of the *Piece* and where the Shot falls) you may from thence, by one of the preceding *Propoſitions* tell the Length of the *Ranges* of all Shots upon any other *Elevation*, uſing the ſame *Force* or *Charge* of *Powder*.

But before we proceed to the *Propoſitions* for *Shooting*, it will be neceſſary to give the following *Rules*.

1. *That before you make a Shoot at any Place, you find the Diſtances thereof from your* Mortar, *which may be obtain'd by* Prop. 3. §. 4. Chap. 9.

2. *That the* Bombs, *or other Bodies that are to be ſhot be of equal Weight, otherwiſe the Shots will vary.*

3. *That the* Carriage *in Breadth be always on a Level, and without any Deſcent, that ſo it may not leap in Diſcharging.*

4. *That the* Powder *with which the* Mortar *is Loaded, be always of the ſame Force and Weight.*

5. *That the Charge of the* Mortar, *as well in* Powder *as in* Wadding, *be always Rammed in with Blows equally Heavy, and of equal Number.*

6. *That the* Wads *be always either of* Wood *or* Tampeons, *or elſe of* Oakam, *for the ſtrongeſt drives it fartheſt.*

7. *That*

§ 4. 7. *That the* Fuſes *be newly made, that is, in thoſe Days they are to be uſed, and that they be made of a Compoſition proportionable to the Range that the Shot will make in the* Air, *ſo that the* Bomb *may break in the very Moment of its Fall; which Compoſition muſt be ſuch, that tho' it fall in the Water yet not to Extinguiſh, but the* Bomb *there to break.* And therefore, before we proceed any further, 'twill be neceſſary to ſhew how to Compoſe the Ingredients for your *Fuſe*.

PROP. I.

To make Fuſes for Bombs, &c.

The Compoſition for *Bombs* muſt be of a ſlow Motion, that ſo time enough may be given to throw either *Bombs*, *Granadoes*, *Fire-balls*, *Thundring-Barrels*, &c. They are Compounded of theſe Ingredients, thus: Take a Pound of *Gun-Powder*, $1\frac{1}{2}$ of *Sulphur*, $1\frac{1}{2}$ of *Salt-Peter*, well beaten, dry, and ſifted ſeparately, then mix it and make up your *Fuſe* thereof: Or take Powder of *Benjamin* and *Small-Coal*, all well beaten and mixed together with ſome *Oyl of Peter*, and ſo fill your *Fuſe* therewith.

PROP. II.

Suppoſe a Shot with a Mortar *upon* 15 *Degrees of Elevation throws a* Bomb 30 *Yards, How far will it throw it upon* 35 *Deg. of Elevation?*

Say, *As Sine* 30 *deg. Double Angle Elev.*	9.698970
To Sine of 70 *deg. Double Angle Elev.*	9.973986
So is 30 *Yards the firſt Range*	1.477121
	11.451107
To 57 *Yards the ſecond Range*	1.752137

So

So that upon 35 *Deg. of Elevation* the ſame *Mortar* with the ſame Charge will throw a *Bomb* 57 Yards. And after this way are the *Diſtances* or *Ranges* upon all *Elevations* found. § 4.

Again: *Admit a Shot with a* Mortar *upon* 20 *deg. Elevation ſends a* Bomb 420 *Yards, What is the greateſt Diſtance ſuch a* Mortar *will throw a* Bomb.

☞ *Note*, The greateſt Diſtance a *Mortar* can throw a *Bomb*, is, when the *Mortar* is Elevated to 45 *deg.* for the *Sine* of double 45 *deg.* which is 90, is the greateſt of all *Sines*.

Say, *As Sine of* 40 *deg.*	9.808067
To Sine of 90 *deg.*	10.000000
So is 420 *Yards the firſt Range*	2.623249
To 652 *Yards the ſecond Range*	2.814182

So the greateſt Space ſuch a *Mortar* can throw a *Ball*, is 652 Yards.

PROP. III.

Suppoſe I had a mind to let fall a Bomb 400 *Yards diſtant from me, what Elevation is requiſite thereto, from a* Mortar *whoſe greateſt Range is* 700 *Yards.*

This *Propoſition* is the Converſe of the former, and therefore you muſt ſay,

As 700 *Yards*	2.845098
To 400 *Yards*	2.602060
So is Sine of 90 *deg.*	10.000000
To the Sine of 34 *deg.* 51 *min.*	9.756962

The Half of this 34° 51′ is 17° 25′ ½ the true *Angle of Elevation.*

The

§ 4 The ſame may be found from any other *Range* as well as the greateſt, if the *Angle of Elevation* were given.

From theſe *Proportions* is the *Table of Ranges* made.

PROP. IV.

Suppoſe the greateſt Range of a Shot be 600 *Yards, 'tis requir'd to find how high ſuch a Piece will throw a Ball upon* 40 *deg. of Elevation.*

☞ *Note:* The greateſt Height that a *Ball* will aſcend to when ſhot Perpendicular upright, is $\frac{1}{2}$ the greateſt *Range.*

Say therefore: *As the Verſed Sine of* 180 *deg. (viz.* twice the *Rad.) is to the Verſed Sine of* 80 *deg. (viz.* twice 40 *deg.) So is* $\frac{1}{2}$ *the greateſt Range, to the Altitude required.*

Multiply therefore the *Verſed Sine* of 80 *deg.* which is 8263 by 300 *Yards*, which is half the greateſt *Range*, and dividing the *Product* by 20000 the *Verſed Sine* of 180, the *Quote* is 124 *Yards*, the Perpendicular Height. See the following Operation.

8263
300

2 | 0000) 247 | 8900 (124 *Prope.*

And ſo on the Contrary: *If the greateſt Horizontal Range was given, and the Perpendicular Height of any Shot known, to find the Angle of Elevation*

Say: As 300 *Yards* half the greateſt *Range*, to the *Verſed Sine* of 180 *deg.* viz. 20000; ſo is the given *Perpendicular Height* of a Shot; Suppoſe

poſe 124 *Yards* to 8263 the *Verſed Sine* of 80 *deg.* half of which is 40 *deg.* the *Angle of Elevation* required. § 4.

PROP. V.

Suppoſe a Shot made upon 45 *deg. of Elevation, and by a Watch with a Hand to ſhow Seconds, I find it continues* 12 *Seconds in the Air, 'tis requir'd to know how long it will continue in the Air when ſhot at* 75 *deg. of Elevation.*

Say, As Sine of 45 deg.	9.849485
Is to 12 Seconds	1.079181
So is Sine of 75 deg.	9.984944
	11.054125
To 16 Seconds	1.214640

By this Rule *Fuſees* are adjuſted as to their time of Burning.

The Contrary of this *Propoſition* is alſo manifeſt, *viz. That if upon* 54 *deg. a Shot continues in the Air* 14 *Seconds, how many Degrees was the Piece Elevated to when the Shot continued* 16 *Seconds in the Air?*

Say, As 14 Seconds	1.146943
To 16 Seconds	1.204120
So is Sine 54 deg.	9.907958
	11.112073
To the Sine of 67° 35′	9.965130

which 67° 35′ is the *Angle* the Piece muſt be ſet to for the *Ball* to continue 16 Seconds in the *Air*.

Further I might have gone in ſhewing from ſome *Data's* to find the *Powder requiſite*, *Length* of the *Chaſe*, *Diameter* of the *Ball*, &c. But theſe being not ſo abſolutely neceſſary to be acquired

§ 4. acquired in this Caſe I omit them, and will conclude with a uſeful *Table* by which may be found the *Horizontal Random* of any Shot from ſome given *Elevation*; as alſo from a given *Elevation* to caſt a Shot to any Place at Demand, provided it be within the reach of the *Piece*, and that without the Help of *Logarithms*.

The Uſe of the following Table *is thus:* The left Column is the *Degrees of Elevation*, and the Top are the *Minutes*, being to every 6 *Minutes*, beginning with 6, 12, 18, *&c.*

Suppoſe a Piece Elevated 13° 18′, *lays a Ball* 1837 *Paces, what is his greateſt Random?*

In the *Table* you will find againſt 13 *deg.* and under 18 *min.* 896. Say therefore, *As* 896 *to* 2000; *So is* 1837 *Paces to* 4100, the greateſt *Random* of ſuch *Piece*.

If the *Elevation* had been 76° 42′, which is as much above 45 *deg.* as 13° 18′ is under, the ſame would have come out.

The contrary of this is alſo evident; for, *If the greateſt Horizontal Range be* 4100 *Paces, what muſt be the Elevation of the Piece to throw a Ball* 1837 *Paces?*

Say, *As* 4100 *to* 2000; *So is* 1837 *to* 896, againſt which in the following *Table* you have in the left hand Column 13 *deg.* and in the Head of the *Table* over the Number you have 18 *Minutes*, which is the *Elevation* of the *Piece* to lay a *Ball* at 1837 *Paces* diſtance. Or if you look in the Right-Hand Column, at the Bottom of the *Table* you will find 76° 42′ the *Elevation* over 45 *deg.* to throw a *Ball* the aforeſaid *Diſtance*.

A TA-

A TABLE of the Horizontal Distances of Projections.

Deg. Elev.	0	6	12	18	24	30	Min.
0	0	7	14	21	28	35	89
1	70	77	84	91	98	105	88
2	140	146	153	160	167	174	87
3	209	216	223	230	237	244	86
4	278	285	292	299	306	313	85
5	347	354	361	368	375	382	84
6	416	423	429	436	443	450	83
7	484	491	497	504	511	518	82
8	551	558	565	571	578	585	81
9	618	625	631	638	645	651	80
10	684	691	697	704	710	717	79
11	749	756	762	769	775	781	78
12	813	820	826	833	839	845	77
13	877	883	889	896	902	908	76
14	939	945	951	957	964	970	75
15	1000	1006	1012	1018	1024	1030	74
16	1060	1066	1072	1078	1083	1089	73
17	1118	1124	1130	1136	1141	1147	72
18	1176	1181	1187	1192	1198	1284	71
19	1231	1237	1242	1248	1253	1259	70
20	1286	1291	1256	1302	1307	1312	69
21	1338	1343	1349	1354	1359	1364	68
22	1389	1394	1399	1404	1409	1414	67
23	1439	1444	1448	1453	1458	1463	66
24	1486	1491	1456	1500	1505	1509	65
25	1532	1537	1541	1545	1550	1554	64
26	1576	1580	1585	1589	1593	1597	63
27	1681	1622	1626	1630	1634	1638	62
28	1658	1662	1666	1670	1674	1677	61
29	1656	1700	1703	1707	1711	1714	60
30	1732	1736	1739	1742	1746	1749	59
31	1766	1769	1772	1776	1779	1782	58
32	1798	1801	1804	1807	1810	1831	57
33	1827	1830	1833	1836	1838	1841	56
34	1854	1857	1860	1862	1865	1867	55
35	1879	1882	1884	1886	1889	1891	54
36	1902	1904	1906	1906	1911	1913	53
37	1923	1924	1926	1928	1930	1932	52
38	1941	1942	1944	1946	1947	1949	51
39	1956	1958	1959	1961	1562	1963	50
40	1970	1971	1972	1973	1974	1975	49
41	1981	1981	1982	1983	1984	1985	48
42	1989	1990	1990	1991	1992	1992	47
43	1995	1956	1996	1996	1997	1997	46
44	1999	1999	1999	1999	2000	2000	45
Min.	60	54	48	42	36	30	Deg. Elev.

The TABLE continued.

Deg. Elev.	36	42	48	54	60	Min.
0	42	49	56	63	70	89
1	112	119	126	133	140	88
2	181	188	195	202	209	87
3	251	258	265	271	278	86
4	320	327	334	340	347	85
5	388	395	402	409	416	84
6	457	463	470	477	484	83
7	524	531	538	545	551	82
8	591	598	605	611	618	81
9	658	664	671	677	684	80
10	723	730	736	743	749	79
11	788	794	801	807	813	78
12	852	858	864	870	877	77
13	914	920	927	933	939	76
14	976	982	988	994	1000	75
15	1036	1042	1048	1054	1060	74
16	1095	1101	1107	1113	1118	73
17	1153	1159	1164	1170	1176	72
18	1209	1215	1220	1226	1231	71
19	1264	1269	1275	1280	1286	70
20	1317	1323	1328	1333	1338	69
21	1369	1374	1379	1384	1389	68
22	1419	1424	1429	1434	1439	67
23	1467	1472	1477	1482	1486	66
24	1514	1519	1523	1528	1532	65
25	1559	1563	1567	1572	1576	64
26	1601	1606	1610	1614	1618	63
27	1642	1646	1650	1654	1658	62
28	1681	1685	1689	1692	1696	61
29	1718	1721	1725	1729	1732	60
30	1753	1756	1759	1763	1766	59
31	1785	1788	1791	1795	1798	58
32	1816	1818	1821	1824	1827	57
33	1844	1846	1849	1852	1854	56
34	1870	1872	1875	1877	1879	55
35	1893	1896	1898	1900	1902	54
36	1915	1917	1919	1921	1923	53
37	1934	1935	1937	1939	1941	52
38	1950	1952	1953	1955	1956	51
39	1965	1966	1967	1968	1970	50
40	1976	1978	1979	1980	1981	49
41	1986	1987	1988	1988	1989	48
42	1993	1994	1994	1995	1995	47
43	1998	1998	1998	1999	1999	46
44	2000	2000	2000	2000	2000	45
Min.	24	18	12	6	0	Deg. Elev.

More *Examples* might be given, but theſe are ſufficient for *Practice:* what I have elſe to ſay, is, That the Reſiſtance of *Air* is ſome Impediment in *Nice-ſhooting* (for by the Reſiſtance of that *Medium* all Projects are diverted from flying in a true *Parabola*) and conſequently that the greateſt *Random* is not made upon 45 *Degrees of Elevation*, but at ſome few *Minutes* under, and ſo will cauſe ſome ſmall Difference in the *Diſtances*. § 4.

But conſidering that in ordinary and common Practice (eſpecially in Service) that with the common Inſtruments we can ſcarce come to the Certainty of a *Degree* or two, the foregoing *Table* will be near enough, and ſerve very well for ordinary Practice, although the Allowance for the *Air*'s Reſiſtance be not therein Conſidered.

Vive, vale: ſi quid noviſti rectius iſtis,
Candidus imperti: ſi non his utere mecum.
Horat. *lib.* 1 *Epiſt.*

A SHORT

TREATISE

OF THE

DESCRIPTION and USE

Of the following

Mathematical Inſtruments;

VIZ.

Globes,	*Nocturnal*,
Sector,	*Univerſal Dial*,
Quadrant,	*Gauging Rod*,
Foreſtaff,	*Coggerſhal's Rule.*

By *W. ALINGHAM.*

The Deſcription and Uſe of the GLOBES.

NOtwithſtanding *Aſtronomical Definitions* have already been given before, I muſt upon the *Deſcription* and *Uſe* of the GLOBES, re-iterate ſome of them here again, as followeth.

A *Sphere* or *Globe*, is a perfect round *Solid* contained under one *Surface*, in the midſt of which is a Point called the *Center*, from whence all Lines drawn to the outſide are equal.

In

In this Form the *Heavens* and *Earth* are prov'd nearly to be, and therefore *Aſtronomers* have contrived Two *Artificial Globes*, one to repreſent the *Heavens*, on which is depicted the Principal fixt *Stars* in ſuch Proportion and Diſtance as they are found to have in the *Heavens*; on the other is deſcribed the *Earth* and *Water* in all their Parts, and according to their true Situation, Form and Diſtance, as found by the late Diſcoveries of our Modern *Navigators*, ſo that in reference to our preſent Buſineſs a *Sphere* or *Globe* is only an Artificial Reſemblance of *Heaven* and *Earth*.

The Line the *Globe* turns about on is termed the *Axis* of the *World*, the Extremities of it are the *Poles* of the *World*, one of which is the *North Pole*, the other the *South Pole*. It is repreſented in the *Artificial Globe* by the two *Wires* the *Globe* turns upon in the *Meridian*.

Of the Circles *of the* Sphere.

Circles of the *Sphere* are Lines imagined to be drawn in the *Cavity* of the *Heavens*: Their Uſe is to determine ſeveral Propoſitions in *Aſtronomy*, *Geography* and *Navigation*.

Theſe *Circles* are *Greater* or *Leſſer*; *Greater* are ſuch as cut the *Globe* into two equal Parts, *Leſſer* cut the *Globe* unequally.

Every *Circle*, whether Great or Small, is conceived to be divided into 360 equal Parts called *Degrees*, and every *Degree* they again divide into 60 *Minutes*, and every *Minute* into 60 *Seconds*, &c. ſo that one *Degree* is the $\frac{1}{360}$ of a *Circle*, and one *Minute* is the $\frac{1}{60}$ of a *Degree*.

Of the Great Circles.

The Principal *Great Circles* are the *Horizon*, the *Meridian*, the *Equinoctial* or *Equator*, the *Zodiack* or *Ecliptick*, and the two *Colures*.

The *Horizon* divides the Viſible Part of *Heavens* from the Inviſible, and when at *Sea*, or on Plain Ground, is the *Circle* that ſeemingly parts *Heaven* and *Earth*; this *Circle* limits the Length of the Artificial Day, for when the *Sun* is above this *Circle* 'tis then *Day*, when under 'tis *Night*. It bounds the Riſing and Setting of the *Sun*, *Moon* and *Stars*, and ſhews the Point of the *Compaſs* they Riſe or Set at. 'Tis artificially repreſented by the broad Wooden *Circle* in which the Braſs *Meridian* and the *Ball* turns; Upon this *Circle* is a *Calendar*, ſhewing the Place of the *Sun* by having the Day of the Month, according to either Account; as alſo next the outer Edge is ſet the *Winds* or *Points* of the *Compaſs*.

The *Meridian* paſſeth through the *Zenith* and *Nadir*, and both *Poles* of the *World*, and cuts the *Horizon* at *Right Angles*, and ſhows the North and South Points thereof. It divides the Viſible Part of the *Heavens* into two equal Parts, one of which is the Eaſtern, the other the Weſtern *Hemiſphere*; it alſo determines the *Noon* and *Midnight* of all Places, for when the *Sun* is upon this *Circle* in the Day-time it is *Noon*, when upon it in the Night, 'tis *Mid-night*.

It is repreſented on the *Artificial Globe* by the Great Braſs Hoop the *Ball* hangs in, and is divided into 360 *Degrees*.

The

The *Equinoctial* or *Equator*, is equally distant from both *Poles* of the *World*; It divides the *Globe* into two *Hemiſpheres*, that next the *North Pole* is the *Northern Hemiſphere*: the other next the *South* is the *Southern Hemiſphere*, All Places lying under this *Circle* have their *Day* and *Night* always equal; for in all Poſitions of the *Globe*, this *Circle* is half above and half under the *Horizon*. When the *Sun* comes to this *Circle*, which is twice a Year, he makes the *Day* and *Night* equal to all the Inhabitants of the Earth; 'tis upon this *Circle* we number the *Right* and *Oblique Aſcenſion* of the *Sun* and *Stars*, this *Circle* is delineated or depicted on the *Ball* of an *Artificial Globe*, and is actually divided into 360 equal Parts.

The *Zodiack* is one of the greateſt *Circles* of the *Sphere*, becauſe by *Aſtronomers* it has allowed 18 *Degrees* of Breadth, which Breadth is limited by the wandring of the *Planets*; upon it is placed the 12 *Signs*; In the middle of the *Zodiack* runs the *Ecliptick Line*, ſo named from the *Greek* Word *Eclipo*, which ſignifies to fail or want; 'tis under this Line the *Sun* and *Moon* ſuffer their *Eclipſes*, this *Circle* cuts the *Equinoctial* obliquely, and is called *Via Solis*, becauſe the *Sun* deſcribes it by his Yearly Motion.

This *Circle* is deſcribed on the *Artificial Globe* with the *Pourtraiture* of the 12 *Signs*, and is divided into 360 equal Parts, as was the *Equator*.

The *Colures* are Two great *Circles* of the *Sphere*, being drawn through both *Poles* of the *World*, in which Points they cut one another, and alſo the *Equator*, at *Right Angles*. One of

theſe is named the *Solſticial Colure* from its paſſing through the firſt Points of *Cancer* and *Capricorn*, in which Points the *Sun* ſeems to ſtand ſtill, the other is called the *Vernal Colure*, from its paſſing through the firſt Points of *Aries* and *Libra*. They determine the Four Seaſons of the Year, for when the *Sun* paſſes over the *Solſticial Colure* he ſhews the Beginning of *Summer* and *Winter* Quarters, the firſt when he is in his greateſt *North Declination*, the latter when he is in his greateſt *South Declination*. When he paſſes the *Vernal Colure* he ſhews the Beginning of *Spring* and *Autumn* Quarters, the former beginning when he paſſes over it in his *Aſcenſion*, the latter when he paſſes over it in his *Deſcenſion*.

Theſe Two *Circles* are deſcribed on an *Artificial Globe*, and actually divided into 360 equal Parts or Degrees.

Of the Leſſer Circles.

The *Leſſer Circles* are the Two *Tropicks* and Two *Polars*.

The *Tropicks* are diſtant on each Side the *Equinoctial* or *Equator* 23° ½; That on the North Side is called the *Tropick of Cancer*, the other on the South Side is named the *Tropick of Capricorn*; theſe *Circles* limit the *Torrid Zone*: they alſo bound the *Sun*'s Courſe, he never ſwerving ſo far North or South, as to go beyond either of them*.

**These are delineated on an Artificial Globe by a red or gilded Line.*

The *Polars* are as far diſtant from the *Poles* of the *World* as the *Tropicks* are from the *Equator*, that at the North is termed the *Arctick Circle*, that next the South the *Antarctick*.

They

They bound the *Frigid Zones*, and ſhew us what Places have their *Artificial Day* longer than 24 Hours. Theſe *Circles* are alſo actually deſcribed on an *Artificial Globe* by ſuch Lines as were the *Troticks*.

Beſides the *Circles* before mentioned, there are ſome other both of the greater and leſſer ſort, which are very neceſſary for Solving *Aſtronomical* and *Geographical Problems*, and are as followeth.

Hour Circles or *Meridians*, are great *Circles* meeting in the *Poles* of the *World*. They are on the *Artificial Terreſtrial Globe* drawn thro' every 15 *deg.* of the *Equator*, and at *Right Angles* thereto.

Azimuths or *Vertical Circles*, are great *Circles* all meeting in the *Zenith* and *Nadir*, and cut the *Horizon* at *Right Angles*. Theſe are never deſcribed on an *Artificial Globe*, becauſe the *Quadrant* of *Altitude* ſupplies their Place.

Circles of *Longitude*, or *Poſition*, are great *Circles* meeting in the *Poles* of the *Ecliptick*, and therefore cut it at *Right Angles*. They are drawn on the *Artificial Cæleſtial Globe* through every 30 Degrees of the *Ecliptick*.

Circles (or as commonly called *Parallels*) of *Declination*, are ſmall *Circles* parallel to the *Equator* on both Sides thereof, and on the *Terreſtrial Globe* are drawn through every 10° of the *Meridian*, gradually diminiſhing till they end in the *Pole*.

Almicanthars, or *Circles* of *Altitude*, are ſmall *Imaginary Circles* parallel to the *Horizon*, but are never deſcribed on an *Artificial Globe*, the *Quadrant* of *Altitude* ſhewing the Points thro' which they muſt paſs.

Parallels of *Latitude* are ſmall *Circles* imagined to be drawn parallel to the *Ecliptick*, and are ſometimes delineated on an *Artificial Cæleſtial Globe*.

Rhomb Lines are neither *Circles* nor *Straight Lines*, but *Spiral*, imagin'd to proceed from the Place where we ſtand, and wind about the *Globe* till they loſe themſelves in the *Pole*. They are drawn on an *Artificial Terreſtrial Globe*, from the Four Points where the *Colures* cut the *Equator*, and their Uſe is to ſhew the bearing of Places one from another.

Of the Appendants to an Artificial Globe.

To a *Globe*, (beſides the forementioned *Circles*) belongs a ſmall *Braſs Circle* called the *Hour Circle*, whoſe *Center* is the *Pole* of the *Globe*; 'tis divided into 24 equal Parts, repreſenting the *Hours* in a *Natural Day*. This *Circle* is fixed in a Grove on the *Braſs Meridian*; on the *Pole* is put a ſmall Bit called the *Index*, which, as the *Globe* turns about, points to the Diviſions on the *Circle*: Their *Uſe* is to ſhew the Time of the *Sun*, *Moon*, or a *Star*'s Riſing and Setting, and what a Clock 'tis in any Part of the *World*.

There is alſo another Appendant called the *Quadrant* of *Altitude*; 'tis a thin Braſs-Plate divided into 90 equal Parts or Degrees, to one End is fitted a Nut and a Skrew for it to move, and then to be fixt to any Degree upon the *Braſs Meridian*. Its Uſe is to limit *Azimuths* and *Almicanthars*, and to meaſure *Diſtances* and *Altitudes*.

Moreover to ſome *Globes* belong a ſmall *Compaſs Box* and *Needle* for the ſetting the

Globe

Globe due North or South, alſo half a *Braſs Circle* called the *Semicircle* of *Poſition*; but in regard none of the following *Problems* require them, they being but for ſome particular Uſes, we ſhall omit a further Deſcription of them.

Aſtronomical Terms Explained.

Declination is the *Diſtance* of the *Sun*, *Moon* or *Stars* from the *Equinoctial*, if they be on the North Side they have *North Declination*, if on the South Side, *South Declination*.

Right Aſcenſion is an *Arch* of the *Equinoctial* contained betwixt the Beginning of *Aries*, and that Point of the *Equinoctial* which comes to the *Meridian* with the *Sun*, *Moon* or *Star*.

Oblique Aſcenſion is an *Arch* of the *Equinoctial* comprehended betwixt the Beginning of *Aries* and that Point of the *Equinoctial* which ariſes with the *Sun*, *Moon* or *Star*.

Oblique Deſcenſion is an *Arch* of the *Equinoctial* lying betwixt the Beginning of *Aries* and that Point of the *Equinoctial* which ſets with the *Sun*, *Moon* or *Star*.

Aſcenſional Difference is the Difference betwixt the *Right* and *Oblique Aſcenſion* which, if converted into Time, gives the *Hour* and *Minutes* of the *Sun*'s Riſing and Setting before or after Six.

Amplitude is an *Arch* of the *Horizon* betwixt the true Eaſt or Weſt Point, and that Point againſt which the *Sun*, *Moon* or *Star* riſes, and is named according as it is, *viz.* either *Amplitude* of Riſing or Setting.

Azimuth is the ſame with *Amplitude*, but only 'tis reckon'd when the *Planets* or *Stars* are above the *Horizon*. *Alti-*

Altitude is the *Height* of the *Sun*, *Moon* or *Stars* above the *Horizon*.

Longitude of a *Star* or *Planet*, is an *Arch* of the *Ecliptick* contain'd between the Beginning of *Aries* and the Point where any *Star* or *Planet* is; or elſe 'tis the Place where a *Circle* (that paſſeth thro' the *Star* and *Poles* of the *Ecliptick*) cuts it.

Latitude of a *Star* or *Planet*, is the Diſtance of it from the *Ecliptick*.

Longitude of a Place, is the Diſtance in *Degrees* and *Minutes* of the *Equator* from the *Aſſign'd* or *Fixt Meridian*.

Latitude of a Place is its Diſtance from the *Equator*, or 'tis an *Arch* of the *Meridian* contain'd betwixt the *Elevated Pole* and *Horizon* of the Place.

The *Zenith* is the Point directly over our Heads where-ever we be.

The *Nadir* is oppoſite to the *Zenith*, and therefore the Point directly under our Feet.

Of the Superficies of the Cœleſtial Globe.

On the Surface of this *Globe* is depicted in Proportion and Diſtance moſt of the Principal Fixt *Stars*, in that Order and Poſition as they appear in the *Heavens*; and tho' the Number of them ſeems Infinite, yet the greater are not only numbred but have Names to diſtinguiſh them. The Number at preſent taken notice of are about 1400, the Biggeſt and Brighteſt of them are called *Stars* of the *Firſt Magnitude*, the next to them *Stars* of the *Second Magnitude*, and ſo gradually decreaſing to the *Sixth Magnitude*, which are the leaſt that are pictur'd on the *Globe*.

Now

Now *Aſtronomers* are deſirous to bring theſe Bodies into ſuch Order whereby they might the more readier be known and found, having divided them into certain Forms and Shapes of *Men*, *Beaſts*, *Birds*, and other Creatures called *Conſtellations*, by which Method of Diviſion they know where to ſeek them in the *Natural Sphere*, and being found, how to expreſs them on the *Globe* or *Hemiſphere*. The Number of *Conſtellations* depicted on the *Globe* are 62, of which 23 are on the North Side of the *Ecliptick*, 12 upon it, and 27 on the South Side; in all which *Conſtellations* the *Stars* are expreſs'd according to their ſeveral *Magnitudes*.

Aſtronomical Problems ſolv'd by the Globe.

PROB. 1. *Day of the Month (ſuppoſe* April 1.*) given, to find the Place of the* Sun.

Find the Day of the Month on the *Horizon*, according to the *Old* or *New* Account, then right againſt the ſame you have (*Aries* 22°) the Sine and Degree the *Sun* is in.

PROB. 2. *The* Sun's *Place (ſuppoſe* 15° ♒*) given, to find the Day of the Month.*

Seek for his Place on the *Horizon*, againſt which you have (*Jan.* 24.) the Day of the Month, according to the *Old* or *New* Account.

PROB. 3. *To rectifie the* Globe *fit for Uſe in any particular Latitude*; (*ſuppoſe* London, *which is* 51° 30′.

Count from the *North Pole* downwards on the *Braſs Meridian* 51° ½, and bring that Point to the *Horizon*, the *Quadrant* of *Altitude* muſt be ſcrew'd to the *Zenith*, which is found by telling

telling the *Complement*, which is 38° ½, from the *Elevated Pole* to the Top of the *Globe*, the Place of the *Sun* muſt be brought under the graduated Edge of the *Braſs Meridian*, and then the *Index* muſt be ſet to point at 12 on the *Hour Wheel.*

PROB. 4. *Latitude of the Place* (51° ½) *and Place of the* Sun (♎ 23°) *given*;

To find his { *Declination*, *Right Aſcenſion*, *Height at Noon.*

Firſt, Rectifie the *Globe* by the foregoing *Problem*, then find out the Place of the *Sun* on the *Ball*, which Point bring under the graduated Edge of the *Braſs Meridian* and there fix it. This done count on the *Braſs Meridian* the *Degrees* betwixt his Place and the *Equinoctial*, it gives 8 ½ *Deg.* his *South Declination.*

Secondly, The *Degrees* on the *Equinoctial* betwixt the firſt Point of *Aries*, and the Point where the *Braſs Meridian* cuts the *Equinoctial* is (201 ½) his *Right Aſcenſion.*

Thirdly, The *Degrees* on the *Braſs Meridian* betwixt the *Sun*'s Place and the *Horizon* is (30° 00′) his Height at *Noon.*

PROB. 5. *Elevation of the* Pole (51° ½) *and* Sun's *Place* (27° *Cancer*) *given*,

To find { *His Amplitude of Riſing*, *Time of Riſing*, *Length of the Night*, *Oblique Aſcenſion*, *Point of the Compaſs he riſeth on*, *Difference of Aſcenſion.*

Having

Having rectified the *Globe* turn the *Ball* Eaſtward till the Point of the *Sun's* Place comes right againſt the *Horizon*, then count the *Degrees* on the *Horizon* from the Eaſt Point thereof to the Point right againſt the *Sun*'s Place, it gives (35° 00′) his *Amplitude* of *Riſing*. *Secondly*, The *Hour* the *Index* points to is (4 a Clock) the Time of Riſing. *Thirdly*, Double the *Hour* and *Minute* of his Riſing, it gives (8 *Hours*) the Length of the Night. *Fourthly*, The *Degrees* on the *Equinoctial* betwixt the firſt Point of *Aries*, and Point where the *Horizon* cuts it is (90°) his *Oblique Aſcenſion*. *Fifthly*, The Point of the *Compaſs* he riſes on is (N. E. *by* E.) found upon the *Horizon* right againſt his Place on the *Ball*. *Sixthly*, Subſtract the *Right Aſcenſion* from the *Oblique Aſcenſion* or *Contra*, the Remainder (29°) gives the *Difference* of *Aſcenſion*.

PROB. 6. *Elevation of the* Pole (51° ½) *and Place of the* Sun (21° ♑) *given*,

To find { *His Amplitude of Setting*, *Time of Setting*, *Length of the Day*, *Oblique Deſcenſion*, *Point of the Compaſs he Sets on.*

The *Globe* rectify'd as before, turn the *Ball* Weſtward till the *Sun*'s Place be juſt againſt the *Horizon*, then tell the *Degrees* on the *Horizon* betwixt the Weſt Point and the *Sun*'s Place, for that is (38° 00′) his *Amplitude* of *Setting*. *Secondly*, See what time the *Index* points to, for that is (4 a Clock) the *Hour* of his *Setting*. *Thirdly*, Double the Time of his *Setting*,

Setting, it gives (8 *Hours*) the Length of the *Day*. *Fourthly*, The *Degrees* on the *Equinoctial* betwixt the Point where the *Horizon* cuts it and the Beginning of *Aries*, is (318°) his *Oblique Deſcenſion*. *Laſtly*, The Point of the *Compaſs* he *Sets* on is (S. W. *by* W.) found on the *Horizon* right againſt his Place on the *Ball*.

PROB. 7. Sun's *Place* (25° 00′ ♌) *Latitude of the Place* (51° ½) *given, and any Morning Hour (ſuppoſe 9 a Clock) propoſed*;

To find { *His Azimuth from the Eaſt*, *His Height*, *Point of the Compaſs he is upon.*

The *Globe* and all things rectified, move round the *Ball* Eaſtward till the *Index* points to the *Hour* propos'd, and there fix it; Then if you lay the *Quadrant* of *Altitude*'s graduated Edge over the *Sun*'s Place, and tell the *Degrees* from the Eaſt Point of the *Horizon* to the Point where the *Quadrant* cuts the *Horizon*, it will give you (32° 00′) his *Azimuth* from the Eaſt. *Secondly*, You have his *Altitude* (38° 00′) by counting the *Degrees* on the *Quadrant* betwixt the *Horizon* and his Place. *Laſtly*, Juſt againſt the *Quadrant*'s Edge on the *Outer Circle* of the *Horizon* you have (S.E. *by* E.) the Point of the *Compaſs* he is then upon.

If an *Afternoon Hour* had been propos'd, the *Globe* muſt have been turn'd Weſtward, and the *Azimuth* found from the Weſt.

Note, 'Tis by this *Problem* we find the *Sun*'s *Azimuth* at 6 in the *Morning* or *Afternoon*, which he has only in the *Summer* Half Year to all *Northern Inhabitants*.

PROB.

PROB. 8. *Again, Place of the* Sun (17° 00′ ♌) *Latitude of the Place* (51° ½) *and any Azimuth (ſuppoſe* 28 *Degrees) from the Eaſt Southward given;*

To find { *His Altitude,* / *Hour of the Day,* / *Point of the Compaſs he is upon.*

All things rectified, bring the *Quadrant* of *Altitude* about till it's graduated Edge ſtands juſt againſt the given *Azimuth*, then turn about the *Ball* till the *Sun*'s Place comes under the *Quadrant*'s Edge and there fix it; then the *Degrees* on the *Quadrant* betwixt the *Sun*'s Place and *Horizon* is (39° 00′) his *Height*. *Secondly*, The *Hour* the *Index* points to is the Time of the Day. *Thirdly*, The Point of the *Compaſs* is (S. E. *by* E.) had on the *Outer Circle* of the *Horizon* right againſt the *Sun*'s Place on the *Ball*.

Note: If the *Azimuth* given be Weſtward, the *Hour* found will be *Afternoon*, otherwiſe 'tis *Morning*.

By this *Prob.* the *Sun*'s *Height* or *Hour* of the Day is found when he is due Eaſt or Weſt, which can only be in the *Summer* Half Year, in Places that have *North Latitude*.

PROB. 9. Sun's *Place* (6° 00′ ♉) *and Elevation of the Pole* (51° ½) *with any Afternoon Altitude (ſuppoſe* 32° 00′) *given;*

To find { *The Hour of the Day,* / *Azimuth from the Weſt,* / *Point of the Compaſs he is upon.*

All

All things rectified, bring the *Quadrant* of *Altitude* on the Weſt Side of the *Horizon*, then turn the *Ball* and it together till the *Altitude* counted on the *Quadrant* from the *Horizon* lies exactly over the *Sun*'s Place, where fix it, then the *Hour* the *Index* points to is ($\frac{1}{4}$ after 3) the Time of the *Day*; alſo the *Degrees* on the *Horizon* from the Weſt to the Divided Edge of the *Quadrant* is (20° 00′) his *Azimuth* from the Weſt. *Thirdly*, Juſt againſt the *Quadrant*'s Edge on the *Outer Circle* of the *Horizon* is (W. S. W.) the Point of the *Compaſs* he is then upon.

If it had been *Morning Altitude* the *Globe* muſt have been turn'd Eaſtward, and then the *Index* would have pointed to a *Morning Hour* and the *Azimuth* will be *Eaſterly*.

PROB. 10. *Latitude of the Place* ($51\frac{1}{2}$) *and Place of the* Sun (20° 00′ ♒) *given, to find the Length of the longeſt preſent (which ſuppoſe* Jan. 29.*) and ſhorteſt Day.*

From ſeveral of the preceding *Problems* and Nature of the *Sun*'s Motion, 'tis manifeſt the *Sun*'s *Declination* is the Cauſe of the Increaſe and Decreaſe of the *Artificial Day*, for his Increaſing in *North Declination* lengthens the *Day* to all *Northern Inhabitants*, but the contrary happens to all *Southern*; and conſequently, when he is in his greateſt *North Declination*, the Longeſt Day is made to all that have *North Latitude*, but when in his greateſt *South Declination*, then the Longeſt Day is made to all *Southern Inhabitants*.

Rectify therefore the *Globe* to the *Latitude* of the Place, and find the Place of the *Sun* for the

the preſent Day (which is 20° 00′♒) on the *Ball*, this done, bring his ſaid Place to the Eaſt Side of the *Horizon*, and ſet the *Index* to the lower 12 on the *Hour Wheel*. Then (having made two Marks on the *Tropicks* where the Eaſt Part of the *Horizon* cuts them,) turn about the *Ball* Weſtward, till the Mark on the *Tropick of Capricorn* comes right againſt the Weſt Side of the *Horizon*, ſo the *Hours* the *Index* has paſt over from the Point 'twas ſet at is (7 *Hours* ½) the Length of the *Shorteſt Day*. This done, turn it more Weſtward till (20° 00′♒) the *Sun*'s Place for the *preſent Day* comes againſt the *Horizon*, then, as before, the *Hours* the *Index* has paſt over from the Point 'twas firſt ſet at, which (is 9 *Hours* ½) will be the Length of that Day in our *Latitude*, *Thirdly*, Turn the *Globe* more Weſtward till the Mark on the *Tropick of Cancer* come juſt againſt the Weſt Point of the *Horizon*, ſo will the *Index* point out (16 *Hours* ½) the Length of the *Longeſt Day* in the *Latitude* of 51° 00′ North.

The Length of the *Night* is found by taking the Length of the *Day* from 24 *Hours*.

PROB. 11. *Latitude of the Place* (51° 30′) *and Place of the* Sun (20° 00′♉) *given*;

To find { Sun's *Depreſſion at Midnight*, *Time of Twilight's Beginning*, *Time of Twilight's Ending*, *Time of its Continuance*, and *Length of the quite dark Night.*

All things rectified, bring the oppoſite Point to the *Sun*'s Place under the *Braſs Meridian*,

(which Point is had in the *Ecliptick* under the *Braſs Meridian* below the *Horizon.*) Then the *Degrees* on the *Meridian* betwixt that Point and the *Horizon* is (21° 00′) his Depreſſion at Midnight. *Secondly*, Move the *Ball* and *Quadrant* together Weſtward, till the aforeſaid oppoſite Point of the *Sun*'s Place lies juſt againſt 18 *Degrees* of the *Quadrant*, then the *Hours* the *Index* has paſs'd is (1 *Hour* ¾) the Time after *Midnight*, *Twilight* begins at; Or, if you move the *Ball* and *Quadrant* Eaſtward till the ſaid Point be under 18 *Degrees* of the *Quadrant*, 'twill give you (19 *Hours* ¼) the Time *Afternoon Twilight* ends at. *Fourthly*, If the *Hour Twilight* begins at be taken from the *Hour* of *Sun-riſing*, 'twill leave (5 *Hours* ¾) the Time of *Twilight*'s Continuance. *Fifthly*, The Time that *Twilight* begins at, doubled, gives (3 *Hours* ½) the Length of the *quite dark Night.*

PROB. 12. *The Day of the Month, (ſuppoſe* Aug. 17. *) according to either Account, given, to find what other Day in the Year is of the ſame Length.*

From *Prob.* 10. 'tis evident that in the *Sun*'s decreaſing in *Declination*, there is one *Day* in which he had the ſame *North Declination* as he had in his Encreaſing, which two *Days* are of equal Length. To find which, look the Place of the *Sun* for the given *Day* on the *Horizon*, which got, find it on the *Ball*, and note the *Declination*; then turn about the *Ball* till ſome other Point of the *Ecliptick* comes under the afore-found *Declination*, which Point *note*, and find out on the *Horizon*, and right againſt it

is

is (*Apr.* 4.) the Day of the Month that is of the ſame Length with the given *Day*.

PROB. 13. *To find in what Latitude the Longeſt Day is an Hour longer than with us at* London.

Set the *Globe* to *London*'s *Latitude*, and bring the *Solſticial Colure* under the *Meridian*, and ſet the *Index* to 12, then at the Interſection of the *Horizon* with the *Tropick of Cancer* make a *Prick* on the ſaid *Tropick*. After this move the *Ball* Weſtward till 7° 30′ of the *Equinoctial* have paſs'd the *Meridian*, and againſt the *Horizon* on the *Tropick* make a ſecond Mark, then bring back the *Colure* to the *Meridian* and there fix it; after which Elevate the *Globe* 'till the ſecond *Prick* appears, ſo will the *Globe* reſt at (56°) the *Latitude* of the Place Inquir'd.

PROB. 14. *To find how much the* Sun's *Declination muſt increaſe or decreaſe, to make the Artificial Day an Hour longer (or ſhorter) than it is at* (March 8.) *the Time propos'd.*

Having Elevated the *Globe* to the *Latitude* of the Place, find the *Sun*'s *Declination* (for *March* 8.) this *Declination* mark on the *Vernal Colure*, which Mark bring to the Eaſt Part of the *Horizon*, then if the *Days* lengthen, turn the *Globe* Eaſtward, but if they ſhorten, turn it Weſtward, till 7° 30′ of the *Equator* has paſs'd the *Meridian*, and where the *Horizon* now cuts it makes another Mark upon it, ſo the Diſtance on the *Colure* betwixt theſe two Marks is (6.) the *Degrees* that the *Declination* muſt

increaſe or decreaſe to make the *Day* an *Hour* longer or ſhorter than it was at the time propos'd. *If the Days have not an Hour to lengthen or ſhorten, then this* Prob. *is improper.*

PROB. 15. *Having the Latitude of the Place, to find the Number of Days contain'd betwixt the Days of lengthening or ſhortening one Hour at any time propos'd. Suppoſe from* March 8.

Make two *Pricks* on the *Colure*, as in the laſt *Example*, then find the Days on which the *Sun*'s *Declination* anſwers to thoſe *Pricks*, ſo the Time included betwixt thoſe two Days are the Days that he is lengthening or ſhortening one *Hour* at that time of the *Year*. In our *Example* 'twill be about 31 Days.

PROB. 16. *To find the Height of the* Sun *by the* Globe, *when the* Sun *ſhines.*

Set the *Globe* on ſome Plain (where the *Sun* ſhines) as level as you can, then turn the *North Pole* of the *Globe* to the *Sun*, moving it up and down till the *Pole* has no Shade, then will the *Globe* reſt at the *Altitude* counted on the *Braſs Meridian* betwixt the *Pole* and *Horizon.*

PROB. 17. *To know whether 'tis before or after Noon.*

Having made an Obſervation of the *Sun*'s Height, make a ſecond a little while after, then if the *Altitude* increaſe 'tis before *Noon*, if it decreaſe 'tis after *Noon.*

PROB.

PROP. 18. *To find the Hour of the Day by the* Globe *when the* Sun *ſhines.*

Rectify the *Globe* and ſet it upon ſome plain truly Level, with the *North Pole* towards the North Part of the *Heaven*, then when the *Sun* ſhines, obſerve the Shade of the *Axis* upon the *Hour Circle*, 'twill give you the Hour of the Day.

Note: This is only for the *Summer* Half-Year, and therefore for to find the Hour for the other Half-Year, depreſs the ſaid *Pole* as much below the South Part of the *Horizon*, by which means 'tis directly oppoſite to its firſt Poſition, ſo will the Shade of the *Axis* give the time of the Day in the *Hour Circle* as before.

Of the STARS.

PROB. 19. *To find the Longitude and Latitude of any* Star.

Lay the *Quadrant* of *Altitude* over that *Pole* of the *Ecliptick* neareſt the *Star* whoſe *Longitude* you ſeek. Then place the *Quadrant's* Edge over the *Center* of the *Star*, and look what *Degrees* of the *Ecliptick* (reckoned from the Beginning of *Aries*) it cuts; for that is the *Star's Longitude:* Alſo the *Degrees* on the *Quadrant* betwixt it and the *Star* is its *Latitude.* Proceed thus and you'll find the *Longitude* of *Arcturus* in *Bootes* to be about 20 *Degrees*, and his *Latitude* near 34 *Degrees*.

PROB. 20. *To find the Diſtance of* 2 Stars *wherever the* Stars *be.*

Lay the *Quadrant* of *Altitude* over them, and if the *Quadrant* be too ſhort, elevate or depreſs the *Globe* till they both come againſt the *Horizon*, ſo the *Degrees* betwixt the *Stars*, either on the *Quadrant* or *Horizon*, is their Diſtance. Proceed thus and you will find the Diſtance betwixt *Aldebaran* and *Cor Scorpio* 171 *Degrees.*

PROB. 21. *To know what* Stars *will be upon the Meridian at any Hour of the Night and any time of the Year, the Latitude being given.*

The *Globe* and all things rectified, bring the *Sun*'s Place for the given Day under the *Braſs Meridian*, and ſet the *Index* to 12, then turn the *Ball* Weſtward till the *Index* points to the Hour propos'd, where fix it; then all the *Stars* that are under the *Braſs Meridian*, are then upon the *Meridian*. Proceed thus and you ſhall find that *Scheat* and *Marſhal* both in *Pegaſus*, with ſome other ſmall *Stars*, will be under the *Meridian* on *Oct.* 8. at ¼ paſt 9 at Night.

PROB. 22. *To know what Day in the Year any* Star *will be upon the Meridian at* 9 *at Night.*

Bring the *Star* to the *Meridian*, then turn the *Ball* Eaſtward till the *Index* points at as many Hours before Noon as the *Star* is to be on the *Meridian* after Noon, that is, in our *Example*, till the *Index* points at 3; the *Globe* reſting in this Poſition, ſee what Point of the *Ecliptick* is under the *Braſs Meridian*, which Point is the Place of the *Sun*, and if ſought out

on

on the *Horizon*, right againſt it is the Day of the Month the *Star* propos'd will be on the *Meridian*. Proceed thus and you ſhall find that *Aldebaran* will be on the *Meridian*, *December* 30. near 9 at Night.

PROB. 23. *Latitude, Day of the Month, and Altitude of a* Star *given, to find the Hour of the Night.*

All things rectified, and oppoſite Point of the *Sun*'s Place brought under the *Braſs Meridian*, move the *Ball* till the *Altitude* propos'd lies juſt under the *Quadrant*'s Edge, then will the *Index* point at the Hour of the Night.

Note, That if the *Sun*'s Place and *Quadrant* be brought on the Eaſt Side, the Hour found is after *Midnight*, but if on the Weſt Side, 'tis before *Midnight*. Proceed thus, and you'll find that *Cor Leonis* on *November* 1. is 25 *deg.* high at ½ an Hour paſt 2 in the Morning.

PROB. 24. *To find how many Hours any* Star *continues above or under the* Horizon *in any Latitude.*

The *Globe* ſet to the *Latitude*, bring the *Star* to the *Meridian* and ſet the *Index* to 12, then turn the *Globe* Weſtward till the *Star* propos'd comes juſt againſt the *Horizon*, then ſhall the Hour the *Index* points to be half the Time, which doubled gives the whole Time; as ſuppoſe it points at 7 Hours ½, this doubled gives 15 Hours ½, the time it continues above the *Horizon*. Proceed thus and you ſhall find that *Fomahaut* in *Aquarius* will continue 5 Hours above the *Horizon* in our *Latitude*.

PROB. 25. *At what Hour any time of the Year, any fixt* Star *riſes or ſets in any Latitude.*

Let it be propos'd to find what Hour any *Star* riſes or ſets *Sept.* 14. The *Globe* being rectified to the *Latitude* and *Day* propos'd, that is, the *Sun*'s Place brought under the *Braſs Meridian*, turn about the *Globe* till the *Star* enquir'd after comes juſt againſt the Eaſt Part of the *Horizon*, then will the *Index* point at the Hour of its riſing. If you turn it Weſtward till it comes againſt the *Horizon*, the *Index* will ſhew the time of its ſetting. Proceed thus, and you ſhall find that in our *Latitude*, *Syrius* the *Dog-Star*, on the 14. of *September*, will riſe at 7 a Clock.

PROB. 26. *Latitude of the Place given, to find what* Stars *never riſe or ſet.*

Obſerve what *Stars* have 38° ½ *North Declination*, for all ſuch *Stars* never ſet in our *Latitude*, neither doth any *Star* riſe that hath above 38° ½ of *South Declination.*

PROB. 27. *To find the Coſmical and Acronical Riſing and Setting of the* Stars.

The *Coſmical* Riſing of a *Star* is when it riſeth with the *Sun*, and the *Coſmical* Setting when it ſets in the Morning or goes down when the *Sun* is riſing. The *Acronical* Riſing is when a *Star* riſeth when the *Sun* ſets, and the *Acronical* Setting is when a *Star* ſets with the *Sun.*

The *Globe* rectified, bring the *Sun*'s Place for the Day propoſed to the Eaſt Part of the *Horizon*,

Horizon, then all the *Stars* that appear on the *Ball* againſt the Eaſtern Edge of the *Horizon*, do that Day riſe *Coſmically*; and thoſe *Stars* which are againſt the Weſtern Rim of the *Horizon*, do that Day ſet *Coſmically*. But if you bring the *Sun*'s Place to the Weſtern Side of the *Horizon*, then all the *Stars* againſt the Eaſt Part of the *Horizon* be riſing *Acronically*, and all the *Stars* againſt the Weſt Part of the *Horizon* be ſetting *Acronically*.

Thus proceeding, you ſhall find that on *May* the 27*th*, *Aldebaran* or the *Bull*'s *Eye*, with other ſmall *Stars* riſe *Coſmically*, and the *Right Leg* of *Serpentarius*, with ſeveral ſmaller *Stars* ſetting *Coſmically*.

Alſo upon the 18*th* of *October* you ſhall find a *Star* in the *Whale*'s *Tail*, and other ſmaller *Stars Acronically* riſing, and the *Tail* of the *Lion*, *South Ballance*, and other little *Stars* ſetting *Acronically*.

The Uſe of the Globes *in* Geography *and* Navigation.

Before we enter on the *Geographical* and *Nautical Problems*, it will be neceſſary to inform the *Reader*, that among the *Meridians* before deſcribed to be on the Surface of the *Globe*, one is more generally noted than the reſt, which *Meridian* is (as it were) the *Landmark* of the whole *Sphere*, and is the Bounds from whence the *Longitude* of all Places is accounted quite round the *Globe*. In our common *Engliſh Globes* it is made to paſs thro' St *Michael*'s, one of the *Azores* Iſlands.

In

In *Navigation* we count that the *Grand Meridian* from whence we begin our Reckoning.

PROB. I. *Any Place being found on the* Globe, *to find its Longitude and Latitude.*

Bring the Place under the *Braſs Meridian*, then the *Degrees* the *Meridian* cuts on the *Equator* is its *Longitude*; alſo the Diſtance on the *Braſs Meridian* betwixt the Place and *Equator* is the *Latitude*.

Thus proceeding, you will find

Jeruſalem { *Longitude* of 66° 40′.
Latitude of 32° 10′. N.

The Converſe of this *Problem* is eaſily perform'd, that is, by having the *Longitude* and *Latitude*, to find out the Place on the *Globe*.

PROB. 2. *To find the Bearing and Diſtance of one Place from another*; *ſuppoſe* London.

Elevate the *Globe* to the *Latitude* of *London*, and bring *London* to the *Meridian*, ſo will it fall juſt in the *Zenith*, where fix the *Quadrant* of *Altitude*; then lay its graduated Edge over any other Place, whoſe *Bearing* and *Diſtance* you require from *London*. Then right againſt the *Quadrant* on the *Horizon*, is the Point of the *Compaſs* the ſaid Place bears from *London*; alſo the *Degrees* on the *Quadrant*, betwixt the *Zenith* and *Place*, is their Diſtance in *Degrees*, which Multiplied by 70, gives their Diſtance in *Engliſh* Miles.

Proceed thus, and you ſhall find the *Bearing* and *Diſtance*

From

From *London* of	*Tangier*	S.S.W.	1225
	Aleppo in *Syria*	E. *by* S.	2170
	Babylon in *Caldea*	E. *by* S.	2660
	Surrat	East	4620

If the *Quadrant* of *Altitude* will not reach both Places, take their Diſtance with a Pair of *Compaſſes*, and apply that Extent to the *Equator* or *Ecliptick*, and you'll have their Diſtance in *Degrees*, as before.

PROB. 3. *To find what Hour it is in any Part of the Earth, when 'tis Noon with us at* London.

By reaſon of the *Diurnal Motion* of the *Sun* (or *Earth*) once in 24 Hours (the *Sun* never Enlightening but half of the *Earth* at one time) it comes to paſs, that when 'tis *Morning* in one Place, 'tis *Noon* in a ſecond, Night in a third, and *Midnight* in a fourth, according to their ſeveral Situations, in reſpect of Eaſt and Weſt one from another, and this *Temporary Difference* is known by the *Degrees* in the *Equator* between the *Meridian* of any two Places converted into Time, allowing 15 *Degrees* to an Hour. And further note, that if the Place lies Eaſt from the Place whoſe *Noontide* was given, then 'twill be *Noon* at that, before 'tis at the other Place; if it lies Weſt, then their *Noon* will be after the *Noon* of the Place whoſe *Noontide* is given. As if *London* be brought under the *Braſs Meridian*, then all Places in the Eaſtern *Hemiſphere* are ſaid to lie Eaſt of *London*, and thoſe in the Weſtern *Hemiſphere* Weſt of *London*. This being premiſ'd, ſuppoſe at 12 a Clock at *London* I would know what 'tis at *Port-Royal* in *Jamaica*.

Bring

Bring *London* to the *Meridian* and ſet the *Index* to 12, then turn the *Globe* Eaſtward, becauſe *London* is Eaſt of *Jamaica*, till *Port-Royal* comes under the *Braſs Meridian*, then the Time the *Index* points to (which is 6 *Hours* 37 *Min.*) ſo that 'tis 37 *Minutes* paſt 6 in the *Afternoon* at *Port-Royal* when 'tis 12 a Clock at *London*.

If you would know what Hour 'tis at any Place when 'tis any other Hour at *London*; take the Difference of the *Noontides* of the two Places from the Hour propoſed, if the Place lies Weſtward of that you enquire; but if it lies Eaſtward, add the Difference to the Hour propoſed, the Remainder or Sum is the Hour of the Day in that other Place.

EXAMP. *When 'tis* 1 *a Clock at* London, *what is it at* Jeruſalem, *their Difference of Noon-tides being* 2 ½ *Hours?*

So that 'tis 30 *Min.* paſt 3 in the *Afternoon* at *Jeruſalem* when 'tis 1 at *London*.

1 at *London*,
2 ½ add
3 ½ at *Jeruſalem*.

Again, when 'tis 2 at *London*, what is it at *Bermudas*, their Difference of *Noon-tides* being 4 ¼ *Hours?*

2 at *London*,
4 ¼ Subſtract
9 ¼ a Clock at *Bermudas*.

So that 'tis 9 ¼ a Clock at *Bermudas* when 'tis 2 at *London*.

Note: When you cannot take the Difference from the Hour propoſed, add 12 to it.

PROB. 4. *To know what Parts of the Earth have* Noon *when 'tis any Hour with us.*

Note: If the Hour given at *London* be before *Noon*, the Places that have then *Noon* lie Eaſt

Eaſt of *London*, if the given Hour be *Afternoon* they lie Weſt of *London*.

The *Globe* rectified, and the *Index* ſet at the Given Hour, turn the *Globe* about till the *Index* points to 12, then all Places under the South Part of the *Braſs Meridian* have *Noon* at the Hour propoſed, thus proceed, and you will find that when 'tis 9 a Clock in the *Morning* at *London*, 'tis *Noon* at *Babylon* in *Caldea*; and when 'tis ½ paſt 4 in the *Afternoon* at *London*, 'tis *Noon* at *Bermudas*.

But *note*, when the *Sun* is in one *Degree* of *Cancer*, he Enlightens 23° 30′ of the *Meridian* on the other Side of the *North Pole*, and conſequently 23° 30′ Dark on this Side the *South Pole*, underſtand the contrary when he is in the firſt Point of *Capricorn*, from which Conſideration may eaſily bee ſeen the Limitation in the preceding *Caſe*.

PROB. 5. *To find, at any time of the Year and Hour of the Day, what Part of the Earth has the* Sun *in their* Zenith.

No Part of the *Earth* has this Property but ſuch as are in the *Torrid Zone*, Find therefore the *Sun*'s *Declination* for the Day propoſed, then rectify the *Globe* to the *Latitude* of the Place in which you propoſe the Queſtion, and bring the ſaid Place to the *Braſs Meridian*, ſetting the *Index* to point at the Hour propoſed, this done, turn the *Globe* Weſtward, if before *Noon*, elſe Eaſtward, till the *Index* points at 12: then if you look for the *Degree* of *Declination* on the *Braſs Meridian*, you will have juſt under it the Place which on that Day

and

and at that Hour has the *Sun* in their *Zenith.*

Thus proceed, and you ſhall find that on *Apr.* 10th and *Aug.* 17th, both which Days the *Sun* has 10° 00′ of *North Declination* at 30 *Min.* paſt { 5 *Afternoon* } *Nombre di dios* on the *Iſth-* { 6 *Morning* } *mus* of *Panama*; alſo ſome ſmall Iſlands on the Weſt of *Molucca* in *Eaſt India* to lie under the *Braſs Meridian*, ſo that both thoſe Places at the time propos'd have the *Sun* in their *Zenith.*

PROB. 6. *To find what Parts of the Earth have the Sun Riſing, what Setting, what have Noon, what have Midnight, alſo where Twilight is beginning, where ending, any time of the Year and Hour of the Day.*

Find by the laſt *Problem* what Part of the *Earth* has the *Sun* in their *Zenith* at the time propos'd. Then Elevate the *Globe* to that Place's *Latitude*, after which bring it under the *Braſs Meridian*, ſo ſhall all Places cut by the Eaſtern Part of the *Horizon* have the *Sun-riſing*, thoſe cut by the Weſtern Part of the *Horizon* will have him *ſetting*, thoſe under the Elevated Half of the *Meridian* will have *Noon*, thoſe under the depreſt Half will have *Midnight*, thoſe 18° below the Eaſt Part of the *Horizon* will have *Twilight* beginning, and thoſe Places 18° below the Weſt Side of the *Horizon* will have *Twilight* ending.

PROB. 7. *To find the* Antæci, Periæci, *and* Antipodes *of any Place.*

Antæcians are thoſe that lie in the ſame *Longitude* but have *Different Latitudes*, that is, they

they have as much *South Latitude* as we have *North*, hence they have the Hours with us, but contrary Seaſons, their *Summer* being our *Winter*.

Periæcians are thoſe that lie in the ſame *Degrees* of *Latitude* but 180° Difference of *Longitude*, they have the ſame Seaſons with us but contrary Hours, *viz. Noon* for *Midnight*.

Antipodes are thoſe that are wholly oppoſite, going with their Feet to ours, having 180° Difference of *Longitude*; and 51° ½ of contrary *Latitude*, and ſo have both Seaſons and Hours contrary.

To find them by the *Globe*, bring *London* to the *Meridian*, then count from the *Equator* Southward on the *Braſs Meridian* 51° 30′, and you have *London*'s *Antæci*. Secondly, Count from the *North Pole* the Complement of the *Latitude* 38° 30′ downward and you have *London*'s *Periæci*. Thirdly, reckon 38° 30′ from the *South Pole* downward on the *Meridian* and it gives the Point of *London*'s *Antipodes*.

PROB. 8. *Having the* Sun's *Declination and Meridian Altitude, to find the Latitude of the Place.*

Count the *Declination* from the *Equator* on the *Braſs Meridian* either toward the *North* or *South Pole*, as it is, noting the Point; then turn the *Braſs Meridian* up and down in the Notches of the *Horizon*, till the Point on the *Meridian* be ſo far diſtant from the *Horizon*, as is the given *Altitude*; ſo will the *Globe* reſt at the *Latitude*. Thus, if the *Declination* be 11° 30′ North, and the *Meridian Altitude* 50 *Degrees*, the *Latitude* will be 51° 30′.

Note,

Note, If the Obſerv'd *Altitude* be on the South Side of the *Meridian*, the *Latitude* is generally North; but if on the North Side, the *Latitude* is generally South.

PROB. 9. *By the Meridian Altitude of any fixt* Star, *to find the Latitude.*

Bring the *Star* to the *Meridian*, and from its *Center* count downwards the given *Altitude*, and note the Point on the *Braſs Meridian*; then bring the ſaid Point to the South Part of the *Horizon*, if it was obſerv'd on the South Part of the *Meridian*, (but to the contrary Side of it, if it be obſerv'd on the North Part) and the *Globe* will reſt at the *Latitude* of the Place.

Thus if the *Meridian Height* of { *Algol*, *Spica Virginis* } be { 62°, 32° }

the *Latitude* will be { 67° 30′ North., 47° 30′ North. }

PROB. 10. *To find the Latitude by two* Stars, *one being on the Meridian, the other riſing or ſetting.*

Bring the *Star* obſerv'd on the *Meridian* to the *Meridian* and there fix the Body of the *Globe*, then turn the *Meridian* up and down in the *Horizon* till the other *Star* be juſt againſt the Eaſt or Weſt Side of the *Horizon*, then will the *Globe* ſtand at the *Latitude*.

Thus if *Aldebaran* be on the *Meridian*, and *Danebin* the *Lion*'s *Tail* riſing, the *Latitude* will be about 50° 10′ North.

PROB.

PROB. 11. *To find the Latitude by the Height of* 2 Stars *being both on the ſame Azimuth or Point of the Compaſs.*

Lay the *Quadrant* over both *Stars* at the *Degree* of *Altitude* you obſerv'd them, then turn the *Braſs Meridian* up and down in the *Horizon* till the *Quadrant's* Edge reſts at the Given *Azimuth*, ſo ſhall the *Globe* ſtand at the *Latitude*.

Thus if *Capella* and *Scheder* be both on the North Eaſt *Azimuth*, the firſt 20° high and the laſt 66° high, you will find the *Globe* to reſt at 40 *deg.* which is the *Latitude* deſired.

PROB. 12. *Having the* Sun's *or a* Star's *Declination, to find the Amplitude.*

Elevate the *Globe* to the Given *Amplitude*, and count from the *Pole* on the *Meridian* the *Complement* of the *Amplitude* to get the *Zenith*, and thereto ſcrew the *Quadrant* of *Altitude*, and bring the End thereof to the Eaſt or Weſt Side of the *Horizon*, then count the *Co-Declination* of the *Sun* or *Star* from the *Pole* on the *Equinoctial Colure*, and bring the *Degrees* to the *Quadrant* of *Altitude*, then the Diſtance on the *Quadrant* betwixt that Point and the *Zenith* is the *Latitude* you are in.

Proceed thus and you ſhall find that if the *Sun* or a *Star* have 12 *Degrees* of *Declination* and 20 *Degrees* of *Amplitude* from the Eaſt Northward, that the *Latitude* will be about 51 *Degrees* North.

The following *Table* will ſerve as a Proof for moſt of the *Aſtronomical Operations* by the *Globe*.

A TABLE shewing from the first Day of *January*, to every 10th Day afterwards, the Sun's place also his Declination, Right Ascension, Oblique Ascension, Amplitude of Rising and Setting, Time of Rising and Setting, and Time of Twilight's Beginning and Ending.

Month Days.	Sun's place.	Declin.	R. Ascent.	Obli. Ascen.	Amp. of ris. & set from S.	time ☉ s ris. & set.	Begin of twilight.	end
	° ′	S. ° ′	° ′	° ′	° ′	h. m.	h. m.	h.
Jan. 1	♑21.25	21.51	292.42	323.03	050.48	8.01.4	5.54	7
11	♒01.35	19.45	304.16	331.27	056.15	7.48.5	5.43	7
21	11.44	17.14	314.28	337.27	062.20	7.31.5	5.35	7
31	21.51	14.12	324.22	342.58	065.47	7.14.5	5.17	7
Feb. 10	♓01.55	10.47	334.00	347.53	072.21	6.55.6	5.00	7
20	11.57	07.04	343.24	352.25	078.50	6.35.6	4.45	8
Mar. 2	22.55	02.47 N.	353.35	357.06	085.14	6.16.6	4.20	8
12	♈02.50	01.11	002.45	001.14	091.35	5.56.7	3.59	9
22	12.41	05.08	011.57	005.26	097.58	5.36.7	3.34	9
Apr. 1	22.29	08.35	020.19	009.46	104.24	5.17.7	3.05	9
11	♉02.14	12.11	029.48	014.10	109.17	4.48.8	2.38	10
21	11.56	15.28	039.32	019.10	114.17	4.40.8	2.02	10
May 1	21.35	18.18	049.34	024.58	119.24	4.24.8	1.30	11
11	♊01.11	20.24	058.50	030.54	123.45	4.08.8	0.30	12
21	10.46	22.08	069.25	038.35	128.23	3.57.9	Twi-	
31	20.20	23.07	079.06	046.26	129.12	3.50.9	light	
June 10	29.52	23.30	090.00	056.48	130.26	3.47.9	all	
20	♋09.24	23.11	099.47	067.10	129.12	3.49.9	Night	
30	18.27	22.08	110.34	079.46	128.23	3.56.9		
July 10	28.30	20.24	121.09	093.13	123.45	4.07.8	0.42	12
20	♌08.04	18.18	130.25	105.48	119.24	4.21.8	1.22	11
30	17.39	15.28	140.27	120.03	114.17	4.36.8	2.00	10
Aug. 9	27.17	12.32	149.13	132.58	107.37	4.53.8	2.21	10
19	♍ 6.58	08.57	158.43	147.17	104.24	5.12.7	3.00	9
29	16.41	05.08	168.02	161.32	097.53	5.31.7	3.30	9
Sept. 8	26.27	01.35 S.	176.19	174.20	093.11	5.51.7	3.59	9
18	♎ 6.17	02.23	185.30	188.30	089.49	6.11.6	4.19	8
28	16.10	06.18	194.43	202.44	080.27	6.30.6	4.48	8
Octo. 8	26.06	10.04	204.05	217.00	073.49	6.50.6	5.00	7
18	♏ 6.06	13.33	213.40	231.20	067.24	7.09.5	5.18	7
28	16.08	16.40	223.31	245.39	072.20	7.27.5	5.37	7
Nov. 7	26.14	19.18	233.39	259.59	058.51	7.43.5	5.45	7
17	♐ 6.22	21.21	244.06	273.37	055.18	7.57.5	5.54	7
27	16.31	22.51	255.52	287.57	050.48	8.07.4	5.57	7
Dec. 7	26.42	23.27	266.43	299.52	050.48	8.12.4	6.00	8
17	♑ 6.54	23.18	277.37	310.30	050.48	8.12.4	5.58	7
27	17.05	22.24	288.36	319.44	051.37	8.06.4		

The Deſcription and Uſe of the SECTOR.

THE SECTOR is a *Rule* that opens and ſhuts like a common two Foot *Joint-Rule*, they are from 4 Inches and ½ long when ſhut, to 18 Inches. Upon this *Rule* are put ſeveral *Lines*, the principal of which are put *Sector-wiſe*, that is, they are drawn from the Center of the *Sector* upon both Legs to near the Ends of the Legs, the moſt uſeful of which is the Line of *Lines*, the Line of *Chords*, the Line of *Sines*, the Line of *Tangents*, and the Line of *Secants*.

The Line of *Lines* is no more than a Line of *Equal Parts* divided into 100 or 1000 Equal Parts, according as the *Sector* will bear it.

Theſe Lines are drawn from the Center on each Leg, and are numbred with the Figures 1, 2, 3, *&c.* on each Leg to 10, which is near the End of the Leg. Theſe Lines are noted by the Letter L.

And here note, that this 1 may be taken for 10, or for 100, 1000, *&c.* as Occaſion requires, and then 2 will ſignifie 20, 200, 2000, *&c.* and conſequently that 10 at the End of each Leg will be 100, 1000, 10000, *&c.*

The Line of *Chords* is a Line divided after the uſual manner of the Line of *Chords*, from a *Circle* whoſe *Radius* is nearly the Length of one of the Legs; this Line is plac'd upon each Leg of the *Sector*, beginning at the Center and running toward the End thereof. It is numbred with 10, 20, *&c.* to 60, and to this Line on each Leg is ſet the Letter C.

Note: This Line on ſome *Sectors* runs to 90 *Degrees.*

The Line of *Sines* is a Line of common natural *Sines*, divided from a *Circle* of the ſame *Radius* that the Line of *Chords* was; theſe are alſo plac'd on each Leg of the *Sector*, and numberd with the Figures 10, 20, 30, *&c.* to 90, at the End of which on each Leg is ſet the Letter S.

The Line of *Tangents* is a Line of common *Tangents*, divided from a *Circle* of the forementioned *Radius* and is placed upon each Leg, and runs to 45; it has the Numbers 10, 20, *&c.* to 45, placed upon it with the Letter T for *Tangents.*

Beſides this is another ſmall Line of *Tangents*, divided from a *Radius* of about 2 Inches, and is plac'd upon each Leg of the *Sector*; it begins at 45, which ſtands at the Length of the *Radius* from the Center and runs to about 75 *Degrees* or farther, having the Number 45, 50, *&c.* to 75, with the Letter T ſet thereto. The Uſe of this Line (as hereafter ſhall be ſhown) is to ſupply the Defect in the Great Line.

The Line of *Secants* is only a Line of common *Secants*, divided from a *Circle* of about 2 Inches *Radius*; theſe are placed upon each Leg of the *Sector*, beginning not at the Center but at 2 Inches diſtance therefrom, and runs to 75 *Degrees.* To theſe are ſet the Numbers 10, 20, 30, *&c.* to 75, with the Letter S at the End thereof for *Secants.*

Theſe are the principal Lines now put on *Sectors*, and what will ſupply the Places of moſt

moſt of the other Lines, the Uſe of which is as follows.

The Uſe of the Sector.

The Excellency of this Inſtrument above the Common *Scaleſis*, that it may be made to anſwer all Scales within the greateſt opening and cloſeſt ſhutting the *Sector*, for by it you may divide a Line, not exceeding the Diſtance between 10 and 10 when open, nor leſs than the Diſtance between 10 and 10 when ſhut into any Number of Equal Parts.

Alſo from the Lines of *Chords*, *Sines*, *Tangents*, &c. placed on the *Sector* as afore directed, you have a Line of *Chords*, *Sines*, *Tangents*, &c. to any *Radius* betwixt the Breadth of the *Sector* when ſhut, and Length when open'd, by which Contrivance a *Sector* is made one of the Uſefulleſt *Mathematical Inſtruments* in being; As to the Invention and Contrivance of this Inſtrument, it, no doubt, aroſe from a thorow Conſideration of the 4. *Prop.* of *Euclid*'s 6th, which proves, *That Similar Triangles have their correſponding Sides proportional*; from which Conſideration the Reaſon of all the Operations on this Inſtrument will be Evident.

But before we begin with its *Uſe*, 'tis neceſſary to acquaint the *Artiſt*, that the Lines on the *Sector* are uſed two ſorts of ways, *(viz.)* *Laterally* and *Parallelly*. *Laterally* is when any Length is taken from the Center upon one Leg of the *Sector*; *Parallelly*, when you Extend from one Leg to the other, that is, quite croſs the *Sector*.

Note; The *Inmoſt Parallel* is the true Divided Line, and therefore in uſing the *Compaſſes* you muſt always ſet them upon the Inſide Line both in *Lateral* and *Parallel* Entrance.

Secondly; The Lines plac'd upon the *Sector* now made, are after a different way to what they formerly were; for inſtead of putting the ſame Lines at equal Diſtances from the Inward Edge, they now put them at unequal Diſtances, ſo that upon one Side of the *Sector* one Leg has the Line of *Chords* Innermoſt, the other Leg the Line of *Tangents* Innermoſt; that is, the Innermoſt Line of *Chords* and *Tangents* are equally diſtant from the Inward Edge, and ſo are the Outermoſt Line of *Chords* and *Tangents*. The Advantage of this *Contrivance* is this; When you have ſet the *Sector* for any one Line, as the Line of *Lines*, or the Line of *Chords*, it will ſerve alſo for the Line of *Sines* and the Line of *Tangents*, for the *Parallel* betwixt 10 and 10 upon the Line of *Lines*, 60 and 60 upon the Line of *Chords*, 90 and 90 upon the Line of *Sines*, and 45 and 45 upon the Line of *Tangents* are all Equal. Hence the Reaſon is evident why they now make *Chords* run but to 60 upon the Leg of the *Sector*, whereas they uſed to run to 90, but by Practice the Inconveniency is found out.

The Uſe *of the Line of* Lines.

PROB. 1. *To Divine a* Line *into any Number of Equal Parts, ſuppoſe* 9.

Take the Given Line betwixt the *Compaſſes*, and make it a Parallel betwixt 9 and 9 (which you may call 90 and 90) then ſhall the Parallel

rallel Diſtance of 8 and 8 cut off 8 of thoſe Equal Parts, alſo the Parallel Diſtance of 7. and 7; mark out 7 of thoſe Equal Parts, and ſo by taking the Parallel Diſtance of 5 and 5, 4 and 4, 3 and 3, 2 and 2, 1 and 1, you may divide the Line into 9 Equal Parts. If you would divide a Line into 37 Equal Parts; take the Line betwixt your *Compaſſes* and make it a Parallel betwixt 37 and 37 on the Legs of the *Sector*, then the Parallel Diſtance betwixt 36 and 36 will cut from the ſaid Line 36 of thoſe Parts, and leaves a $\frac{1}{37}$ Part remaining, and betwixt 35 and 35; alſo 34 and 34, *&c.* cuts from the ſaid Line 35 and 34 of ſuch equal Parts as the whole Line contains. After the like manner may any other Line, not exceeding the *Sector*'s Length when open'd, be divided into any Number of Equal Parts.

PROB. 2. *To Increaſe or Diminiſh a Line in a given Proportion; that is, to find two Lines in any Proportion, as* 5 *to* 7.

Take the Given Line betwixt your *Compaſſes* and open the *Sector* till that Extent lies betwixt 5 and 5 on the Line of *Lines*, then if you take the Parallel Diſtance betwixt 7 and 7, 'tis the Line requir'd.

If it were to be *Diminiſhed*, ſuppoſe as 7 to 5, then make the given Line a Parallel betwixt 7 and 7, and the Parallel Extent betwixt 5 and 5 is the Line requir'd.

PROB. 3. *Two Lines given, to find a Third in continual Proportion, Increaſing or Diminiſhing.*

Take the Two Lines given, and ſet upon each Leg on the Line of *Lines* from the Center,

marking the Points where they fall, then if the Proportion be Increaſing, take the Longeſt Line betwixt your *Compaſſes*, and apply that Extent Parallel-wiſe in the terms of the ſhorteſt. The *Sector* remaining at this *Angle*, the Parallel Diſtance betwixt the terms of the longeſt, taken and apply'd to the Leg of the *Sector* from the Center, will give the Length of the third Proportional; If it be *Diminiſhing*, then make the ſhorteſt of the two given Lines a Parallel in the terms of the longeſt, then the Parallel Diſtance betwixt the terms of the ſhorteſt apply'd as before, will give the third Proportional; if the Lines be 18 and 24, then the fourth Proportional *Increaſing* is 32, and the fourth *Decreaſing* is 13 $\frac{1}{2}$.

PROB. 4. *Three Lines given, to find a fourth in Proportion, either Increaſing or Diminiſhing.*

If it be Increaſing, ſet the two ſhorteſt Lines upon each Leg of the *Sector*, then open it till the longeſt Line comes betwixt the terms of the ſhorteſt. The *Sector* thus remaining, take the Parallel Diſtance betwixt the terms of the longeſt, and apply that Extent to the Leg of the *Sector*, it will give you the fourth Proportional *Increaſing*.

If it be Diminiſhing, lay the two longeſt Lines on the Legs of the *Sector* from the Center, and make the ſhorteſt of the three a Parallel in the terms of the longeſt, then the Parallel betwixt the terms of the ſhorteſt, is the fourth Proportion *Diminiſhing*.

PROB.

PROB. 5. *To find the Proportion betwixt two given Lines.*

Make the greater a Parallel betwixt 10 and 10 on the Line of *Lines*, then take the leſſer and carry it Parallel to the greater Line till it reſts in like Points on each Leg, the Number where it reſts is the Proportion to 10, 100, 1000, *&c.* requir'd.

So that from this *Prob.* if the Length of one Line be given, the Length of any other Line may be found in ſuch Parts.

PROB. 6. *A Right Line, as* A B, *any how divided, being given; to find how to divide any other Line, as* B D, *into the like Parts.*

Set the Line A B on both Legs of the *Sector*, and in the Points where this Line reaches, apply the Line B D to be divided Parallel-wiſe; this done, lay the Diviſions, A e, A f and A g on both Legs of the *Sector* from the Center, then the Parallel Extent of theſe Diviſions, *(Viz.)* e e, f f, g g, being apply'd to the Line B B, will divide it into the like Parts, as was the Line A B.

The Uſe of the Line of Chords, Sines, Tangents *and* Secants.

The Convenience of putting theſe ſort of Lines on the *Sector*, *Sector-wiſe*, is of vaſt Advantage, for by diſpoſing them after this manner, we can have Lines of *Chords*, *Sines*, *Tangents* and *Secants*, to all *Radius's* not exceeding the *Sector's* Length.

For

For *Example*; If you would have the *Chord*, *Sine* or *Tangent* of 12 *Degrees* to a *Radius* of 4 Inches: Take the 4 Inches betwixt your *Compaſſes*, and make it a Parallel betwixt 60 and 60 on the *Chords*, then, as I told you before, the *Sines*, *Tangents* and *Lines* are ſet to the ſame *Radius*; the *Sector* remaining in this Poſition, take the Parallel Diſtance betwixt 20 and 20 on the *Chords*, is the *Chord* of 20 *Degrees* to that *Radius*, 30 and 30 is the *Chord* of 30 *Degrees* to the ſame *Radius*, &c. Underſtand the ſame by the *Sines* and *Tangents*; that is, Parallel betwixt 20 and 20 on the Line of *Lines*, is the *Sine* of 20 *Degrees* to that *Radius*, and 30 and 30 the *Sine* of 30 *Degrees*, &c.

If you want the *Chord* of 70 *Degrees*, Take the Parallel Diſtance of ½ the *Arch* propoſed, that is, the *Chord* of 35 *Degrees*, and repeat that Diſtance twice on the *Arch* you lay it down on, and you have the *Chord* of 70 *Degrees*.

If you want the *Tangent* of 70 *Degrees* to this *Radius*, you muſt uſe of the ſmall Line of *Tangents* (the great one running to 45 *Degrees*) thus; Take the *Radius* of 4 Inches, and make it a Parallel betwixt 45 and 45 on this ſmall Line of *Tangents*, then the Parallel Extent of 70 *Degrees* on the ſaid Line is the *Tangent* of 70 *Degrees* to the Three-Inch *Radius*.

If the *Secant* of any *Arch* be requir'd; Take the given *Radius* and make it a Parallel betwixt the Beginning of the Line of *Secants*, that is 0 and 0, to the Parallel Diſtance betwixt

twixt 40 and 40, 70 and 70 on the ſaid *Secant Line* will give you the *Secant* of 40 or 70 *Degrees* to the *Radius*.

Note: If the *Radius* be ſo large that it cannot be made a Parallel betwixt 45 and 45 on the ſmall *Tangents*, then no *Tangent* of an *Arch* above 45 *Degrees* can be found, nor the *Secant* of any *Arch* to ſuch a *Radius*, becauſe all *Secants* exceed the *Semi-Diameters* of the *Circle*.

If the *Reverſe* of any of theſe Things be requir'd; as if I require the *Radius* to which a given Line is the *Chord*, *Sine*, *Tangent* or *Secant* of any *Arch*, ſuppoſe of 30 *Degrees*, then 'tis but making that Line (if it be a *Chord*) a Parallel on the Line of *Chords* betwixt 30 and 30, then the Parallel Diſtance betwixt 60 and 60 is the *Radius*. Underſtand the like if it be *Sine*, *Tangent* or *Secant*.

Thus you ſee 'tis very eaſie to find the *Chord*, *Sine* or *Tangent* to any *Radius*, which is the great Advantage of this Inſtrument, and is of great Uſe not only in *Delineating* and *Projecting* on Paper, but alſo on the *Ground* in laying out any Parcel in any *Form* or *Shape*.

To open the Sector *to any Angle, or the* Sector *being open'd, to find the Quantity of the Angle it is open'd at.*

Suppoſe I would open the Lines of *Chords* on the *Sector* to any Angle of 40 *Degrees*. From the Center of the *Sector* take the *Chord* of 40 *Degrees*, and make it a Parallel betwixt 60 and 60 on the Line of *Chords* ſet to an *Angle* of 40 *Degrees*, but alſo the Line of *Lines*,

Line

Line of *Sines* and Line of *Tangents*, they all Equi-diſtant from the Iuward Edges of the *Sector*.

If you would open the two Inward Lines, ſuppoſe *Chords* and *Tangents*, or *Sines*, and *Lines*; 'tis done by making the *Lateral Chord* of 40 *Degrees* (or any other) a Parallel betwixt 60 and 45 on one Side of the *Sector*, or elſe betwixt 90 and 10 on the other Side, and it ſets thoſe Inner Lines to the *Angle* of 40 *Degrees*.

If you would open the very Edges to a given *Angle*, ſhut the *Sector* cloſe, take the Parallel Extent betwixt the Innermoſt *Chords* and *Tangents*, and apply that Extent *Laterally* on the *Chords* from the Center, it ſhews you the *Angle* thoſe Lines make when the *Sector* is ſhut, which ſuppoſe 6, then 'tis *Evident* if I would open the *Sector*'s Edges to an *Angle* of 40 *Degrees*, I muſt open the Inward Lines to an *Angle* of 46 *Degrees*, for then will the *Sector*'s Edges ſtand at an *Angle* of 40 *Degrees*. To open the Line of *Lines* to a *Right Angle*, take the Line of 10 Parts and make it a *Parallel* betwixt 8 and 6 of the ſame, or the Line of *Sines* is open'd to a *Right Angle*, if the *Sine* of 45 *Degrees* be apply'd over in the *Sines* of 30 *Degrees*.

To find the Angle the Sector *is open'd to.*

Take the Parallel Diſtance between the *Chord* of 60, and apply that Extent *Laterally* from the Center on the *Chords*, the *Degrees* it reaches to is the *Angle* thoſe Line are open'd to; *for with a* Sector *of 2 or 3 Feet when open, we can furniſh*

niſh our ſelves with Scales of all ſorts, and fitting for any Purpoſe or Deſign whatſoever.

The Deſcription and Uſe of the Common SEA-QUADRANT.

THIS Inſtrument was firſt Invented by Capt. *Davis*, an *Engliſh-man*, and is the moſt Convenient and Commodious Inſtrument uſed at *Sea*, for obſerving the *Sun*'s *Meridian Altitude*.

The *Form* is ſooner ſeen in an *Inſtrument-maker*'s *Shop* than here deſcrib'd; however, the *Component Parts* are Two *Arches*, Three ſtrait *Pieces* and Three *Vanes*, they are now commonly made about 26 Inches long.

One *Arch* is called the *60th Arch*, becauſe formerly it contained but 60 *Degrees*, tho' now it is made to contain 65 *Degrees*. This *Arch* is but of ſmall *Radius*, being ſo deſign'd, that the *Shade* coming from the *Vane* uſed on this *Arch* might appear very plain on the *Horizon Vane* to the Eye of the Obſerver; this *Arch* is divided into no leſs than Whole *Degrees*, and is numbred with the *Figures* 5, 10, 15, 20, 25, *&c.* Increaſing by 5 to 65, which 65 ſtands juſt upon the long ſtraight *Piece*.

The *Great Arch* is called the 30*th Arch*, tho' now it contains but 35 *Degrees*, viz. the *Complement* of 65; It's of a large *Radius* that it might contain the *Sub-diviſions* of a *Degree*. This *Arch* is ſo broad, that it hath 10 *Concentrick Circles* deſcribed upon it, which *Circles* are

are Interſected by Three *Diagonal Lines*, by means of which each *Degree* is actually divided into 2 *Minutes*, the outer Limb of this *Arch* is actually divided into every 10 *Minutes* by *long Strokes*, and are numbred with the *Figures* 5, 10, 15, 20, 25, *&c.* which ſtands on the long ſtraight *Piece*.

One of the Three *Vanes* is called the *Horizon Vane*, becauſe it reſpects the *Horizon* in time of obſerving; this is fixt upon the End of the long ſtraight *Piece*. The *Vane* which gives the Shadow is called the *Shade Vane*, and is placed on the *60th Arch*, and the *Angle* thro' which you look is called the *Sight Vane*, and its Place is upon the *30th Arch*, ſometimes the *Shade Vane* has a *Glaſs* in it, that Obſervations might be taken in *Hazy Weather*.

The Uſe of this Inſtrument is to take the *Sun*'s *Meridian Altitude*, which is done thus; The *Vanes* being put on in their proper Places, *viz.* the *Sight Vane* on the *30th Arch*, the *Shade Vane* on the *60th Arch*, and the *Horizon Vane* on the End of the long ſtraight *Piece*; ſet the upper Edge of the *Shade Vane* to a Number of *Degrees* leſs than 15 or 20 than you judge the *Complement* of the *Latitude* to be: This done, turn your Back to the *Sun*, and taking the *Quadrant* in your Hand, look thro' the *Sight Vane*, and cauſe the upper Edge of the *Shade Vane* to fall on the upper Edge of the *Slit* in the *Horizon Vane*; if then at the ſame time the *Horizon* appears thro' the Slit in the *Horizon Vane*, the *Vanes* on the *Quadrant* ſtand at the *Complement* of the preſent *Altitude* of the *Sun*, but if the *Sky* appear inſtead of the *Horizon*,

Horizon, ſlide the *Sight Vane* a little higher; but if the *Sea* appears, then bring the *Vane* lower, thus muſt you do till thro' the *Horizon Vane* the *Horizon* appears. This done, ſee what the *Vanes* cut upon their *Arches*; ſuppoſe the Inſide of the *Sight Vane* cuts 10 *Degrees*, and the upper Edge of the *Shade Vane* cuts the Number 42 *Degrees*, theſe added together give 52 *Degrees*, the *Complement* of the *Sun's Altitude*, for this Inſtrument always takes the *Complement*, and not the *Altitude*, that is, his *Zenith* Diſtance.

Note Firſt: When you would obtain the *Meridian* (which is always the greateſt) *Altitude* the *Sun* will have that Day, begin to obſerve a little before Noon, and continue obſerving, as the *Sun* riſes the *Sea* will appear thro' the *Horizon Vane*, ſo the *Sight Vane* muſt be ſlid lower.

Secondly: When the *Sun* begins to fall the *Sky* will appear thro' the *Horizon*, inſtead of the *Horizon*, upon which leave obſerving that Day.

Having found the *Co-Altitude*, or *Zenith* Diſtance as before directed, the *Altitude* is got by taking it from 90, from which *Altitude* you muſt take 16 *Minutes*, (becauſe the upper Edge of the *Sun* ſhows the Shade, and that is diſtant from the *Sun's* Center 16 *Minutes*) then there will remain the *Sun's Meridian Altitude* near enough for finding the *Latitue* at *Sea*.

The

The Deſcription and Uſe of the FORESTAFF, *by ſome called the* Croſs-Staff.

THIS Inſtrument is ſo called from the *Poſiture* of the *Obſerver* in uſing it, whoſe Face is generally towards the thing obſerved; but for the *Sun* 'tis contriv'd to be uſed backward, for the preſerving the *Eyes* of the *Obſerver*. It's called a *Croſs-Staff* from its Form, being a *Square Staff* with 3 or 4 Pieces a-croſs it; this *Staff* is commonly 2 Feet ½ or 3 Feet long, add ½ of an Inch *Square*, having 4 Sides, and each of them a *Diſtinct Croſs*, but generally they make the ſhorteſt *Croſs* ſerve two Sides of the *Staff*, that is, the Breadth is the 10 *Croſs* and the Length the 30 *Croſs*.

The *Staff* being Square there are 4 Sides; One is called the 10 *Side*, becauſe the Gradations begin at 3 *Degrees* of *Altitude* and end at 10 *Degrees*, increaſing by 10 *Minutes*; to this Side belongs the leaſt *Croſs*. The next is the *Second Side*, where the Diviſions begin at 10 *Degrees* and end at 30 *Degrees*; to this Side belongs the *Second Croſs*; the *Sixty Side* where the Diviſions begin at 20 *Degrees* and end at 60, to which Side belongs the next biggeſt or *Sixty Croſs*. Laſtly the *Ninety Side*, where the *Degrees* of *Altitude* begin at 30 *Degrees* and end at 90; to this Side belongs the biggeſt or *Ninety Croſs*. At the End of theſe Diviſions is writ the Word *Alt.* for *Altitude*, and generally under the *Altitudes* are ſet the *Complements*, and the Word *Com.* writ thereto. If the Diviſions of the *Staff* be true and the Length of the *Croſſes* right, there will be this Agree-

ment

ment betwixt them; Half the 10 *Croſs* being laid on the *Ten Side*, will reach from 10 *Degrees*, to 9 *Degrees* 12 *Minutes*, and the whole from 10 *Degrees* to 8 *Degrees* 31 *Minutes*. *Secondly*; Half the *Thirty Croſs* laid on the *Thirty Side*, will reach from 30 *Degrees* to 23 *Degrees* 52 *Minutes*, and the whole from 30 *Degrees* to 19 *Degrees* 47 *Minutes*. *Thirdly*; Half the 60 *Croſs* laid on the Side proper, will reach from 60 *Degrees* to 40 *Degrees* 13 *Minutes*, and the whole from 60 *Degrees* to 30 *Degrees*. *Laſtly*; Half the *Ninety Croſs* meaſur'd on the *Ninety Side* will reach from 90 *Degrees* to 53 *Degrees* 7 *Minutes*, and the whole from 90 *Degrees* to 36 *Degrees* 52 *Minutes*.

The *Uſe* of this Inſtrument is to find the *Altitude* of the *Sun* or *Stars*, which may be done both by a fore and back Obſervation; it alſo finds the *Diſtance* of two *Stars*.

The *Croſſes* are uſed but One at a time, according as the *Altitude* is. If the *Altitude* be leſs than 10 *Degrees*, uſe the leaſt *Croſs* and its proper Side; if above 10 *Degrees*, and under 30 *Degrees*, uſe the *Thirty Croſs* and *Side* proper to it: So if it be above 30 and under 60 *Degrees*, uſe the 60 *Croſs* and *Side* belonging; if above 60 *Degrees*, uſe the biggeſt *Croſs* and *Side*, but the laſt is not ſo exact as other *Inſtruments*.

To uſe this *Inſtrument* you muſt firſt Conſider, that is, gueſs at the *Sun's Altitude*, and then uſe the *Croſs* and *Side* proper thereto; thus, put the flat Side of the *Croſs* toward you, then taking the *Staff* in your Hand, place the flat End, that is, the End of the *Staff* where the Diviſions end, to the Corner of your Eye

and there hold it ſteady, ſliding the *Croſs* backward and forward till you ſee the Center of the *Sun* or *Star* by the upper End of the *Croſs*, and the *Horizon* by the lower End, then the *Degree* and *Minute* cut by the Inner Edge of the *Croſs*, upon the Side peculiar to the *Croſs*, you uſe, is the *Altitude*.

Note: 'Twill be difficult to cut exactly the Center of the *Sun*, unleſs in *Hazy Weather*; or that you break his Rays by a *Colour'd Glaſs* fixt on the Top of the *Croſs* to defend the *Eye* from the *Sun*'s *Force*: But you may obſerve the upper Limb of the *Sun*, and then muſt take 16 *Minutes* from the *Altitude* ſo found: Or you may take the *Sun*'s lower Limb, and add 16 *Minutes* to the *Altitude* found.

To obſerve the Diſtance of two Stars, Place the *Staff* to your *Eye*, as before directed, look to both Ends of your *Croſs*, moving it farther or nearer till you ſee one *Star* at one End of the *Croſs* and the other at the other, the *Degrees* and *Minutes* cut by the *Croſs* (as aforeſaid) is the true Diſtance *ſought*.

To uſe this Inſtrument for a Back Obſervation, which is chiefly for the Sun. *'Tis made fit for the Purpoſe thus*; There is a Piece of *Wood* or *Ivory* put a-croſs the *Ten Croſs*, ſo that the lower End of this Piece of *Wood* or *Ivory* lies even with the middle of the Square Hole in the *Ten Croſs*, which alſo anſwers to the middle of the *Staff*'s Thickneſs. There is alſo a Piece of *Braſs* with a Slit in it, ſo fitted that it will ſlide off and on the Ends of the *Croſſes*. This done, conſider the *Altitude* you are going to take, and accordingly uſe the *Croſs* proper for it, putting it on the *Staff*, *the flat Side of it even with the*

flat

flat End of the Staff, where skrew it faſt, and at one End of the *Croſs* put on the forementioned Piece of *Braſs*, ſo as to leave a Slit at the lower End of the *Croſs*, then put the *Ten Croſs* (with the Piece a-croſs it) on the *Staff*, the flat Side of it towards the other *Croſs*; This done, turn your Back to the *Sun*, and look thro' the Slit at the lower End of the *Croſs*, till the *Horizon* lies even with the Shade on the *Croſs-Piece* in the *Ten Croſs*, at which time the Shade of the upper End of the *Croſs* muſt fall on the Middle Line in the *Ten Croſs*. If inſtead of the *Horizon* you ſee the *Water*, draw the *Ten Croſs* nearer; but if you ſee the *Sky*, you muſt put it further from you, till at laſt you make the *Horizon* and *Shadow* meet, ſo the Place where the *Ten Croſs* now reſts, on that Side of the *Staff* belonging to the *Croſs* fixt at the End, in which is the *Braſs Sight*, is the *Altitude* or *Complement* as before, in a Forward Obſervation.

The Deſcription and Uſe of the NOCTURNAL.

ON the Fore-ſide of the great Piece of this *Inſtrument* are Two *Circles*, the innermoſt is divided into twice 12, to repreſent the *Hours* in a *Natural Day*; the outermoſt is a *Circle* of *Time*, and contains the *Days* in each *Month* and the *Months* in the *Year*, each Diviſion being 2 *Days*. The next Piece contains Two *Circles*; the innermoſt is divided into 29 equal Parts to ſhow the *Age* of the *Moon*, the outermoſt contains twice 12, for the *Hours* in a *Natural Day*.

The Nature of this *Inſtrument* is only to find the *Right Aſcenſion* of a *Star*, which it evidently doth, if you conſider the *Pole* is the Center of the *Diurnal Motion*, and the *Index* of the *Inſtrument* a *Meridian* paſſing thro' the *Pole* and Two *Pointers*, and conſequently cuts the *Equator* at *Right Angles*.

Upon the Middle Piece is a *Tooth* that comes out, and has the Letters G B for the *Great Bear*, againſt which is the Number 12, this *Tooth* ſet to the *Day* of the *Month*, and the *Nocturnal* held up *Perpendicular* before you, till thro' the Hole you ſee the *Pole-Star*, and by the *Index* the *Pointers*, then will the *Index* ſtand at the *Hour* of the *Night*, which is only the Difference betwixt the *Right Aſcenſion* of the *Sun*, and the *Right Aſcenſion* of the *Pointers*.

The *Right Aſcenſion* of the *Sun* changes every Day about a *Degree*, but the *Right Aſcenſion* of any fixt *Star* is the ſame *inſenſibly* for ſome Years.

The Deſcription and Uſe of the Common Four Foot GAUGING-ROD, *in Gauging of Caſks and other Veſſels.*

FROM the End not *Bevell'd*, on one Side is a Line of *Inches*, running from 1 to 48, each *Inch* being divided into *Tenths*; adjoining to this is another Line which gives the *Area* of any *Circle* in *Ale-Gallons*, and Hundredth Part of a *Gallon* upon one *Inch* deep, having the *Diameter* in *Inches* and *Tenths* of an *Inch*; on this Line is writ *Area in Ale-Gallons*, and is called *Oughtred's Line*.

Upon

Upon another Side of the *Rule* are Two *Diagonals*, one for *Ale* and another for *Wine Gallons*, by which is ſhewn the Number of *Ale* or *Wine Gallons* any Cask contains. The Method is thus; Put the Bevell'd End of the *Rule* in at the *Bung-hole*, and run it down to the Head *Diagonal-wiſe*, then the Number on either of theſe Lines againſt the Middle of the *Bung*, is the Content of the Cask either in *Ale* or *Wine Gallons*.

Note: You muſt take care that the *Bung-hole* be in the Middle of the Cask, and that the *Rule* be put to the Middle of the *Bung-hole*.

This Line will ſerve very well for a Cask of 30 or 40 Gallons, and indifferent well for a Cask of 150 Gallons.

On a Third Side are Three Lines, beginning at the Bevel End with 1, 2, 3, 4, *&c.* and running half way the *Rule*; the firſt of them running to 9, and has *Firkins* writ upon it; the ſecond has the Word *Kilderkin* writ upon it, and runs to 18; the third runs to 36, and has the Word *Barrel* upon it.

Theſe Lines are term'd *Ullage Lines*, and their Uſe is to find what a Cask has in it when it is not full; thus, Put the Bevel End down into the *Bung-hole* to the oppoſite Stave of the Cask, and ſo far as the *Rule* is wet, on the proper Line for the Veſſel, ſo many Gallons is in the Veſſel.

Or the Number from the Inſide of the *Bung-hole* to the Surface of the Liquor, ſhews how many Gallons will fill the Veſſel.

Upon the ſame Flat on the other Half of the *Rule* are Two Lines, the one is *Numbers*, the other is *Segments*; they both begin together

with the Number 100 juſt at the middle Joint, and run to the blunt End of the *Rule* where the Words *Numbers* and *Segments* are writ to them.

Their Uſe is to find the remaining Quantity in any Cask whoſe *Axis* lies parallel to the *Horizon*, and this is done, not only more exact than by the *Ullage Lines*, which ſerve only for a *Firkin*, *Kilderkin* or *Barrel*, but alſo for all ſorts of Casks; *the Method is thus.*

Firſt find the *Content* of the whole Cask, which may be done near enough by the *Diagonal Line*; then find how many Inches are wet of the *Bung Diameter*, as alſo the Length of the *Bung Diameter* it ſelf, by putting the *Rule* Perpendicular in at the *Bunghole*.

Then the Extent on the *Line of Numbers*, from the *Bung Diameter* to the Inches wet, will reach, being laid the ſame way on the Line of *Segments*, from 100 to a fourth *Number*, which remember; Next the Extent on the *Line of Numbers* from 100 to the Veſſel's whole *Content* in *Wine* or *Ale Gallons* will reach, laid the ſame way on the *Line of Numbers*, from the *Number* remember'd to the Gallons in it, or elſe wanting to fill the *Cask*.

EXAMP. *The Content of a Cask is* 37 *Gallons, the Inches wet are* 17, *the whole Diameter* 25, *how many Gallons will fill the Cask?*

Anſw. 9 Gallons 7 Tenths.

Upon the Fourth Side of the *Rule* are Three Lines, beginning at the *Bevel* End with 5, and runs towards the other End; the firſt goes better than half way, and has the Word *Hogſhead* writ upon it; the ſecond runs almoſt ¾ its Length, and has the Word *Butt* lying writ upon

upon it, and the third runs near to the other End and has the Words *Buts Standing* writ upon it; theſe alſo are *Ullage Lines* for theſe Casks ſo poſited, and their Uſe is to find what a Cask has in it when 'tis but partly full. The manner is thus; When the *Hogſhead* or *Butt* is lying, put the Bevell'd End of the *Rule* downright into the *Bung-hole*, to the oppoſite Stave of the Cask, and ſo far as the *Rule* is wet on the proper Line for the Veſſel, it ſhows how many Gallons are in the Veſſel.

Or the Number from the Inſide of the *Bung-hole* to the Surface of the Liquor, ſhews how many Gallons will fill the Cask.

In Caſe the *Butt* be *Standing* the *Rule* muſt be put in at the *Head*, and ſo far of it as is wet upon the *Butt Standing Line*, ſo much is the Number of Gallons in the Cask.

So alſo the Dry Inches betwixt the *Butt*'s Head and Surface of the Liquor, ſhews the Gallons wanting to fill the Cask.

Theſe and the former *Ullage Lines* are thus put upon the *Rule*; Cauſe any of thoſe Casks to be filled, and then draw out the Liquor Gallon by Gallon, and ſo mark the *Rule* as the Liquor ſinks in the Veſſel.

The Deſcription and Uſe of Coggerſhal's-Rule, *in the Meaſurement of* Board *and* Timber.

THE Principal Contrivance of this *Rule* is for the *Practical Meaſuring* of *Timber*, which it doth the beſt of all others. It conſiſts of Two Pieces of a Foot long, ſliding one by another like a *Glazier*'s *Rule*. On each Flat of the *Rule* are Four Lines, Two next

the Outer Edges, and Two next the Inner; thoſe on one Flat next the Outer Edges are *Lines* of *Inches* divided into Halves and Quarters; they are Figured from 1 to 12 on one Piece, and from 12 to 24 on the other.

Thoſe next the Inward Edges of the ſame Side are *Lines* of *Numbers* in two Lengths both alike: They are Figured from 1 at the Beginning to 1 in the Middle, and then from 1 in the Middle to 10 at the End.

On t'other flat Side, the Lines next the Outer Edges are *Lines* of *Meaſure*, one being a *Line* of *Inches*, divided into 10 equal Parts for *Gauging*; the other a *Foot* divided into 100 equal Parts, called the Line of *Foot-Meaſure*.

On one of the Inward Edges of this Flat is a *Line* of *Numbers*, Number'd and Figur'd as before deſcribed.

On the other Inward Edge is a Line called the *Square* or *Girt-Line*, Figured from the Beginning with 4, 5, 6, 7, 8, 9, 10, 20, 30, and 40 which ſtands at the End; this is a *Line* of *Numbers* at a double *Radius*.

Firſt; *To Meaſure Board, Glaſs*, &c. *A Piece of Board is* 1 ¼ *Foot broad*, 24 ½ *long, What's the Content in Square Feet?*

Set 1 to 1 ¼, the Breadth on the *Line* of *Numbers*, then againſt 24 ½ Feet is 30 ½ Feet, the Content *near*.

More Examples.

		Anſw.	Feet	Inch.	Parts
Length 37 ½ Feet,	Breadth ¼ Foot		9	4	6
—— 16 ¼ Feet,	—— 3 ½ Feet		56	10	6
—— 60 ¾ Feet,	—— 2 ¼ Feet		136	8	3
—— 15 ½ Feet,	—— 8 ½ Feet		131	9	0
—— 29 Feet,	—— 7 ¼ Feet		210	3	0
—— 35 ¼ Feet,	—— 9 ¾ Feet		343	8	3

Next

Next for *Painting*, *Plaiſtering*, *Paving* and *Wainſcotting*, which is Meaſured by the *Square Yard*.

A Piece of Paving is 22 ½ *Feet long, and* 14 *Feet broad, how many Square Yards is the Content?*

Set 9 to 14 on the *Line* of *Numbers*, then againſt 22 ½ is 35 *Square Yards*, the Content.

More Examples.

		Yards.
Length 34 ¾ Feet,	Breadth 13 ½ Feet	*Anſw.* 52 ¼
—— 15 ¼ Feet,	—— 11 ½ Feet	19 ½
—— 19 ½ Feet,	—— 9 ¼ Feet	20
—— 52 ¼ Feet,	—— 20 ½ Feet	119
—— 36 ½ Feet,	—— 30 Feet	122
—— 27 ¾ Feet,	—— 24 ½ Feet	76

To Meaſure Square Timber. A Piece of Timber is 1 *Foot* 2 *Inches Square, and* 20 *Feet long, what's the Content?*

Note: Inſtead of 1 Foot 2 Inches, take 14 Inches.

Set 12 on the *Girt-Line*, to 20 the Length of the Piece on the *Line* of *Numbers*, then right againſt the Side of the *Square*, or ¼ of the *Girt* being 14 reckoned in Inches (which muſt always be done) on the *Girt-Line*, is 27 Feet, the Content required.

Note: If the Piece be not Square, but broader than 'tis thick, you may reduce it near to a Square, by adding the Breadth and Thickneſs together, and taking half the Sum.

To Meaſure round Timber. Girt the Tree in the Middle, and take ¼ of that Girt for the Side of the Square.

Then ſuppoſe the Girt of a Tree be 64 *Inches, and the Length* 30 *Feet, what's the Content?*

Then

Then as before: *Set* 12 *on the Girt-Line to* 30 *Feet the Length, then againſt* 16, *the* ¼ *of the Girt, is* 53 ½ *Feet the Content.*

If the Tree be much *tapering*, take the Side of the *Square*, or ¼ of the *Girt*, about ⅓ from the greater End.

If you have *Wainy Timber*, that is, ſuch as is neither round nor ſquare, take one *Wain* in and leave the other out.

More Examples.

¼ of the *Girt*, or the Side of the *Square*.			Feet	Parts.
	10 In.	Length 40 ½ Feet	*Anſw.* 28	¼
	15 In.	—— 39, ½ Feet	61	¾
	13 ½ In.	—— 50 Feet	63	½
	20 In.	—— 48 Feet	130	½
	7 ¼ In.	—— 36 ¼ Feet	15	¼
	30 In.	—— 24	150	

Theſe Dimenſions are actually caſt up by a Rule, *and therefore a ſmall Errour muſt not be heeded.*

The Deſcription and Uſe of the EQUINOCTIAL, *commonly called the* Ring-Dial.

Deſ. THIS *Dial* conſiſts of Five Pieces, *viz.* Two *Rings* or *Circles*, One *Bridge* or *Piece* going croſs the *Outermoſt Ring*, and Two *Sliding Bits* of *Braſs* that move in the *Grooves*, one on the *Outermoſt Ring*, the other on the *Bridge* or *Croſs Piece*.

The *Outermoſt Ring* repreſents the *Meridian* of the Place where-ever you are, having on one Side grav'd twice 90 *Degrees*, with N and S for *North* and *South Latitude*; on the other Side is a *Line* of *Altitudes*, beginning at 1 and ending at 90 *Degrees*. Upon this *Ring*, in a *Groove* on the outer Edge, moves one of the *Sliding Bits*

Bits, upon one Side of which is a *Line* or *Stroke* drawn to ſhew the Center or Middle of thoſe moving *Bits*. In this *Bit* is a little *Ring* to hold it up by in time of *Obſervation*.

There is alſo in this *Outer Ring* a *Pin-hole* to put a *Pin* in when the *Altitude* is obſerv'd.

The *Innermoſt Ring* denotes the *Equinoctial*, and is to turn out upon a Joint at *Right Angles* to the *Meridian*, on one Face is nothing, on the other the 24 Hours of the *Day* and *Night*, numbe'rd by twice 12, which is continu'd to the Edge and mark'd alſo upon the Inner Rim of the *Circle*, each *Hour* is divided into *Quarters* on ſome of theſe *Dials*, upon others into every 5 *Minutes*, according as the Largeneſs is.

The *Bridge*, or *Croſs Piece*, denotes the *Axis* of the *World*, in the middle of which is a *Slit* cut quite through for a *Sliding Piece* to move in, which *Sliding Piece* has a ſmall Hole in it, and has a Line or Stroke on both *Faces* croſſing the middle of the ſaid Hole.

On one *Face* of the *Bridge* is the *Months* and *Days* of the *Month*, on the other *Face* oppoſite againſt each particular *Day*, is the *Sun*'s *Decliclination* proper for the *Day*.

PROB. 1. *To find the Hour of the Day.*

Turn out the *Equinoctial* as far as it will go, for then it ſtands at *Right Angles* to the *Meridian*, and ſet the Streak in the *Sliding Piece* on the *Outer Ring* to the *Latitude* of the Place; alſo the Stroke croſſing the little Hole in the other *Sliding Bit* to the *Day* of the *Month* in the *Bridge*; then hold up the *Dial* by the little *Ring* in the *Outer Sliding Bit* and turn the *Bridge* toward the *Sun*, then move the whole *Dial* this and that way, till the *Sun* ſhining through the little

little Hole in the *Bridge* you ſee the Spot of Light to fall on the middle of the Inſide of the *Equinoctial* among the *Hours*, ſo the *Hour-Line* it falleth upon is the *Hour* of the *Day*. *Note*, for 2 or 3 *Days* when the *Sun* is upon the *Equinoctial*, that is, about the 10*th* of *March* and 12*th* of *September*, this *Dial* is uſeleſs.

PROB. 2. *To find the Sun's Height.*

Set the Stroke in the *Sliding Bit* with the *Ring* to the Beginning of the *Degrees*, that is, to 00 *Degrees*, betwixt N and S on the *Latitude Side*, and put a *Pin* ſtiff in the Hole on the Back-ſide of the *Ring*, and the Shade of the *Pin* will ſhew the *Altitude* in the large *Quadrant* on the Back-ſide of the *Meridian*.

PROB. 3. *To find the Sun's Declination.*

Set the Stroak croſſing the Hole of the *Sliding Bit* in the *Bridge* to the *Day* of the *Month* as before, and againſt it on the Back-ſide is the *Sun's Declination*.

PROB. 4. *To find the Latitude of the Place.*

Firſt find the *Sun's Meridian*, or greateſt *Altitude* for the *Day* as before, then find the *Sun's Declination* for the ſame *Day*; this done, take Notice if you make your *Obſervation* betwixt the 10*th* of *March* and 12*th* of *September*, you muſt Subſtract the *Declination* from the *Altitude*; but if you obſerve betwixt the 12*th* of *September* and 10*th* of *March*, then you muſt add the *Declination* to the *Altitude*, and either of thoſe *Numbers* is the *Height* of the *Equator*, which Subſtracted from 90 *Degrees*, leaves the *Latitude* of the Place.

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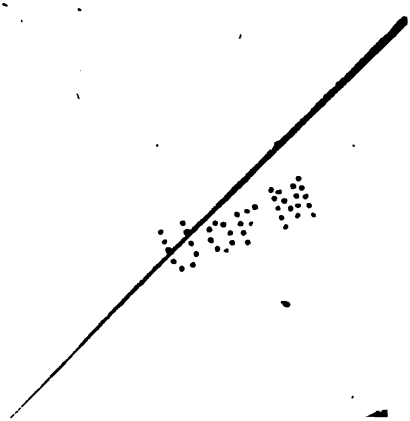

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TABLE

OF

Logarithm Numbers,

From One to Ten Thouſand:

Whereby the

LOGARITHM

OF ANY

NUMBER

Under Four Hundred Thouſand
may be readily diſcovered.

LONDON,

Printed by *J. Leake*, for W. FREEMAN, at the *Bible* againſt the *Middle-Temple-Gate* in *Fleetſtreet*. MDCCVII.

N	Log.	N	Log.	N	Log.
1	0. 000000	34	1.531479	67	1. 826075
2	0. 301030	35	1.544068	68	1. 832509
3	0. 477121	36	1.556303	69	1. 838849
4	0. 602060	37	1.568202	70	1. 845098
5	0. 698970	38	1.579783	71	1. 851258
6	0. 778151	39	1.591064	72	1. 857332
7	0. 845098	40	1.602060	73	1. 863323
8	0. 903090	41	1.612784	74	1. 869232
9	0. 954242	42	1.623249	75	1. 875061
10	1. 000000	43	1.633468	76	1. 880813
11	1. 041393	44	1.643452	77	1. 886491
12	1. 079181	45	1.653212	78	1. 892094
13	1. 113943	46	1.662758	79	1. 897627
14	1. 146128	47	1.672098	80	1. 903090
15	1. 176091	48	1.681241	81	1. 908485
16	1. 204120	49	1.690196	82	1. 913814
17	1. 230449	50	1.698970	83	1. 919078
18	1. 255272	51	1.707570	84	1. 924279
19	1. 278753	52	1.716003	85	1. 929419
20	1. 301030	53	1.724276	86	1. 934498
21	1. 322219	54	1.732394	87	1. 939519
22	1. 342422	55	1.740362	88	1. 944482
23	1. 361728	56	1.748188	89	1. 949390
24	1. 380211	57	1.755875	90	1. 954242
25	1. 397940	58	1.763428	91	1. 959041
26	1. 414973	59	1.770852	92	1. 963788
27	1. 431364	60	1.778151	93	1. 968483
28	1. 447158	61	1.785330	94	1. 973128
29	1. 462398	62	1.792391	95	1. 977723
30	1. 477121	63	1.799340	96	1. 982271
31	1. 491361	64	1.806180	97	1. 986772
32	1. 505150	65	1.812913	98	1. 991226
33	1. 518514	66	1.819544	99	1. 995635

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104	017033	017451	017898	018284	018700
105	021189	021603	022016	022428	022841
106	025306	025715	026125	026533	026942
107	029384	029789	030195	030599	031004
108	033424	033826	034227	034628	035029
109	037426	037825	038223	038620	039017
110	041393	041787	042182	042576	042969
111	045323	045714	046105	046495	046885
112	049218	049603	049993	050379	050766
113	053078	053463	053846	054229	054613
114	056905	057286	057666	058046	058426
115	060698	061075	061452	061829	062206
116	064458	064832	065206	065579	065953
117	068186	068557	068928	069298	065668
118	071882	072249	072617	072985	073352
119	075547	075912	076276	076640	077004
120	079181	079543	079904	080266	080626
121	082785	083144	083503	083861	084219
122	086359	086716	087071	087426	087781
123	089905	090258	090610	090563	091315
124	093422	093772	094122	094471	094820
125	096910	097257	097604	097951	098298
126	100371	100715	101059	101403	101747
127	103804	104146	104487	104828	105169
128	107209	107549	107888	108227	108565
129	100586	100926	111263	111599	111934

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006466	006894	007321	007748	008174	428
010724	011147	011570	011993	012415	424
014940	015359	015779	016107	016616	416
019116	019532	019947	020361	020775	416
023252	023664	024075	024486	024896	412
027349	027757	028164	028371	028978	408
031408	031812	032216	032619	033021	404
035429	035829	036229	036629	037028	400
039414	039811	040207	040602	040998	396
043362	043755	044148	044539	044932	393
047275	047664	048053	048442	048830	389
051153	051538	051924	052309	052694	386
054956	055378	055760	056142	056524	382
058805	059185	059563	059942	060320	379
062582	062958	063333	063709	064083	376
066326	066699	067071	067443	067815	372
070038	070407	070776	071145	071514	369
073718	074085	074451	074816	075182	366
077368	077731	078094	078457	078819	363
080987	081347	081707	082067	082426	360
084576	084934	085291	085647	086004	357
088136	088490	088845	089198	089552	355
091667	092018	092369	092721	093071	351
095169	095518	095866	096215	096562	349
098644	098989	099335	099681	100026	346
102091	102434	102777	103119	103462	343
105510	105851	106191	106531	106871	340
108903	109241	109579	109916	110253	338
112269	112605	112939	113275	113609	335

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131	117271	117603	117934	118265	118595
132	120574	120903	121231	121559	121888
133	123852	124178	124504	124830	125156
134	127105	127429	127753	128076	128399
135	130334	130655	130977	131298	131619
136	133539	133858	134177	134496	134814
137	136721	137037	137354	137671	137987
138	139879	140194	140508	140822	141136
139	143015	143327	143639	143951	144263
140	146128	146438	146748	147058	147367
141	149219	149527	149835	150142	150449
142	152288	152594	152899	153205	153509
143	155336	155639	155943	156246	156549
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145	161368	161667	161967	162266	162564
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147	167317	167613	167908	168203	168497
148	170262	170555	170848	171141	171434
149	173186	173478	173769	174059	174351
150	176091	176381	176669	176959	177248
151	178977	179264	179552	179839	180126
152	181844	182129	182415	182699	182985
153	184691	184975	185259	185542	185825
154	187521	187803	188084	188316	188647
155	190332	190612	190892	191171	191451
156	193125	193403	193681	193959	194237
157	195899	196176	196453	196729	197005
158	198657	198932	199206	199481	199755
159	201397	201670	201943	202216	202488

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118926	119256	119586	119915	120245	330
122216	122544	122871	123198	123525	328
125481	125806	126131	126456	126781	325
128722	129045	129368	129689	130012	323
131939	132259	132579	132899	133219	321
135133	135451	135769	136086	136403	318
138303	138618	138934	139249	139564	315
141449	141763	142076	142389	142702	314
144574	144885	145196	145507	145818	311
147676	147985	148294	148603	148911	309
150756	151063	151369	151676	151982	307
153815	154119	154423	154728	155032	305
156852	157154	157457	157759	158061	303
159868	160168	160469	160769	161068	301
162863	163161	163459	163758	164055	299
165838	166134	166430	166726	167022	297
168792	169086	169380	169674	169968	295
171726	172019	172311	172603	172895	293
174641	174932	175222	175512	175802	291
177536	177825	178113	178401	178689	289
180413	180699	180986	181272	181558	287
183269	183555	183839	184123	184407	285
186108	186391	186674	186956	187239	283
188928	189209	189490	189771	190051	281
191730	192009	192289	192567	192846	279
194514	194792	195069	195346	195623	278
197281	197556	197832	198107	198382	276
200029	200303	200577	200850	201124	274
202761	203033	203303	203577	203848	272

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161	206826	207096	207365	207364	207904
162	209515	209783	210051	210319	210586
163	212187	212454	212720	212986	213252
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958325	958373	958421	658468	658516	48
958803	958850	958898	958946	958994	48

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910	959041	959089	959137	959185	959231
911	959518	959566	959614	959661	959709
912	959995	960042	960090	960138	960185
913	960471	960518	960566	960613	960661
914	960946	960994	961041	961089	961136
915	961421	961469	961516	961563	961611
916	961895	961943	961990	962038	962085
917	962369	962417	962464	962511	962559
918	962842	962886	962937	962985	963032
919	963315	963363	963410	963457	963504
920	963788	963835	963882	963929	963977
921	964259	964307	964354	964401	964448
922	964731	964778	964825	964872	964919
923	965202	965249	965256	965343	965389
924	965672	965719	965766	965813	965859
925	966142	966189	966239	966283	966329
926	966611	966658	966705	966752	966799
927	967079	967127	967173	967220	967267
928	967548	967595	967642	967688	967735
929	968016	968062	968109	968156	968202
930	968483	968529	968576	968623	968669
931	968949	968996	969043	969089	969136
932	969416	969463	969509	969556	969602
933	969882	969928	969975	970021	970068
934	970347	970393	970439	970486	970533
935	970812	970858	970904	970951	970997
936	971286	971322	971369	971415	971461
937	971739	971786	971832	971879	971925
938	972203	972249	972295	972342	972388
939	972666	972712	972758	972804	972851

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959279	959328	959375	959423	959471	48
959757	959804	959852	959895	959947	48
960233	960280	960328	960376	960423	48
960709	960756	960804	960851	960899	48
961184	961231	961279	961326	961374	47
961658	961706	961753	961801	961848	47
962132	962179	962227	962275	962322	47
962606	962653	962701	962748	962795	47
963079	963126	963174	963221	963268	47
963552	963599	963646	963693	963741	47
964024	964071	964118	964165	964212	47
964495	964542	964589	964637	964684	47
964966	965012	965061	965108	965155	47
965437	965484	965531	965578	965624	47
965906	965954	966001	966048	966095	47
966376	966423	966470	966517	966564	47
966845	966892	966939	966986	967033	47
967314	967361	967408	967454	967501	47
967782	967829	967875	967922	967969	47
968249	968296	968343	968389	968436	47
968716	968763	968809	968856	968902	47
969183	969229	969276	969323	969369	47
969649	969695	969741	969789	969835	47
970114	970161	970207	970254	970300	47
970579	970626	970672	970719	970765	46
971044	971090	971137	971183	971229	46
971508	971554	971601	971647	971693	46
971971	972018	972064	972110	972157	46
972434	972481	972527	972573	972619	46
972897	972943	972989	973035	973082	46

N	0	1	2	3	4
940	973128	973174	973220	973266	973313
941	973589	973636	973682	973728	973774
942	974050	974097	974143	974189	974235
943	974512	974558	974604	974649	974695
944	974972	975018	975064	975109	975156
945	975432	975478	975524	975569	975616
946	975891	975937	975983	976029	976075
947	976349	976396	976442	976488	976533
948	976808	976854	976899	976946	976992
949	977266	977312	977358	977403	977449
950	977724	977769	977815	977861	977906
951	978181	978226	978272	978317	978363
952	978637	978683	978728	978774	978819
953	979093	979138	979184	979229	979275
954	979548	979594	979639	979685	979730
955	980003	980049	980094	980139	980185
956	980458	980503	980549	980594	980639
957	980912	980957	981003	981048	981093
958	981366	981411	981456	981501	981547
959	981819	981864	981909	981954	981999
960	982271	982316	982362	982407	982452
961	982723	982769	982814	982859	982904
962	983175	983220	983265	983310	983356
963	983626	983671	983716	983762	983807
964	984077	984122	984167	984212	984257
965	984527	984572	984617	984662	984707
966	984977	985022	985067	985112	985157
967	985426	985471	985516	985561	985606
968	985875	985920	985965	986009	986055
969	986324	986369	986413	986458	986504

5	6	7	8.	9	D
973359	973405	973451	973497	973543	46
973820	973866	973913	973959	974005	46
974281	974327	974374	974419	974466	46
974742	974788	974834	974819	974926	46
975202	975248	975294	975339	975386	46
975662	975707	975753	975799	975845	46
976121	976167	976212	976258	976304	46
976579	976625	976671	976717	976763	46
977037	977083	977129	977175	977220	46
977495	977541	977586	977632	977678	46
977952	977998	978042	978089	978135	46
978409	978454	978500	978546	978591	46
978865	978911	978956	979002	979047	46
979321	979366	979412	979457	979503	46
979776	979821	979867	979912	979958	46
980231	980276	980322	980367	980412	45
980685	980730	980776	980821	980867	45
981139	981184	981229	981275	981320	45
981592	981637	981683	981728	981773	45
982045	982090	982135	982181	982226	45
982497	982543	982588	982633	982678	45
982949	982994	983039	983085	983129	45
983401	983446	983490	983536	983581	45
983852	983897	983942	983987	984032	45
984302	984347	984392	984437	984482	45
984752	984797	984842	984887	984932	45
985202	985247	985292	985337	985382	45
985651	985696	985741	985786	985830	45
986099	986144	986189	986234	986279	45
986548	986593	986637	986682	986727	45

N	0	1	2	3	4
970	986772	986817	986861	986906	986951
971	987219	987264	987309	987353	987398
972	987666	987711	987756	987800	987845
973	988113	988157	988202	988247	988291
974	988559	988604	988748	988693	988737
975	989005	989049	989094	989138	989183
976	989449	989494	989539	989584	989628
977	989895	989939	989983	990028	990072
978	990339	990383	990428	990472	990516
979	990783	990827	990871	990916	990960
980	991226	991270	991315	991359	991403
981	991669	991713	991758	991802	991846
982	992111	992156	992199	992244	992288
983	992554	992598	992642	992686	992730
984	992995	993039	993083	993127	993172
985	993436	993480	993524	993568	993613
986	993877	993921	993965	994009	994053
987	994317	994361	994405	994449	994493
988	994756	994801	994845	994889	994933
989	995196	995240	995284	995328	895372
990	995635	995679	995723	995764	995811
991	996074	996117	996161	996205	996249
992	996512	996555	996599	996643	996687
993	996949	996993	997037	997080	997124
994	997386	997430	997474	997517	997561
995	997823	997867	997910	997954	997998
996	998259	998303	998347	998390	998434
997	998695	998739	998783	998826	998869
998	999133	999174	999218	995261	999305
999	999565	999609	999652	995696	999739

5	6	7	8	9	D
986996	987040	987085	987129	987175	45
987443	987488	987532	987577	987622	45
987889	987934	987979	988024	988068	45
988336	988381	988425	988469	988514	45
988782	988826	988871	988916	988960	45
989227	989272	989316	989361	989405	45
989672	989717	989761	989806	989850	44
990117	990161	990206	990150	990294	44
990561	990605	990649	990694	990738	44
991004	991049	991093	991137	991182	44
991448	991492	991536	991580	991625	44
991890	991935	991979	992023	992067	44
992333	992377	992421	992465	992509	44
992774	992819	992863	992907	992951	44
993216	993259	993304	993348	993392	44
993657	993701	993745	993789	993833	44
994097	994141	994185	994229	994273	44
994537	994581	994625	994669	994713	44
994977	995021	995065	995108	995152	44
995416	995459	995504	995547	995591	44
995854	995898	995942	995986	996029	44
996293	996337	996380	996424	996468	44
996731	996774	996818	996862	996906	44
997168	997212	997255	997299	997343	44
997605	997648	997692	997736	997779	44
998041	998085	998129	998170	998216	44
998477	998521	998564	998608	998652	44
998913	998956	998999	999043	999087	44
999348	999392	999435	999479	999522	44
999783	999826	999869	999913	999957	43

A

TABLE

OF

Proportional Parts.

D	1	2	3	4	5	6	7	8	9
43	4	8	12	17	21	25	30	34	38
44	4	8	13	17	22	26	30	35	39
45	4	9	13	18	22	27	31	36	40
46	4	9	13	18	23	27	32	36	41
47	4	9	14	18	23	28	32	37	42
48	4	9	14	19	24	28	33	38	43
49	4	9	14	19	24	29	34	39	44
50	5	10	15	20	25	30	35	40	45
51	5	10	15	20	25	30	35	40	45
52	5	10	15	20	26	31	36	41	46
53	5	10	15	21	26	31	37	42	47
54	5	10	16	21	27	32	37	43	48
55	5	11	16	22	27	33	38	44	49
56	5	11	16	22	28	33	39	44	50
57	5	11	17	22	28	34	39	45	51
58	5	11	17	23	29	34	40	46	52
59	5	11	17	23	29	35	41	47	53
60	6	12	18	24	30	36	42	48	54
61	6	12	18	24	30	36	42	48	54
62	6	12	18	24	31	37	43	49	55

The Table of Proportional Parts.

D	1	2	3	4	5	6	7	8	9
63	6	12	18	25	31	37	44	50	56
64	6	12	19	25	32	38	44	51	57
65	6	13	19	26	32	39	45	52	58
66	6	13	19	26	33	39	46	52	59
67	6	13	20	26	33	40	46	53	60
68	6	13	20	27	34	40	47	54	61
69	6	13	20	27	34	41	48	55	62
70	7	14	21	28	35	42	49	56	63
71	7	14	21	28	35	42	49	56	63
72	7	14	21	28	36	43	50	57	64
73	7	14	21	29	36	43	51	58	65
74	7	14	22	29	37	44	51	59	66
75	7	15	22	30	37	45	52	60	67
76	7	15	22	30	38	45	53	60	68
77	7	15	23	30	38	46	53	61	69
78	7	15	23	31	39	46	54	62	70
79	7	15	23	31	39	47	55	63	71
80	8	16	24	32	40	48	56	64	72
81	8	16	24	32	40	48	56	64	72
82	8	16	24	32	41	49	57	65	73
83	8	16	24	33	41	49	58	66	74
84	8	16	25	33	42	50	58	67	75
85	8	17	25	34	42	51	59	68	76
86	8	17	25	34	43	51	60	68	77
87	8	17	26	34	43	52	60	69	78
88	8	17	26	35	44	52	61	70	79
89	8	17	26	35	44	53	62	71	80
90	9	18	27	36	45	54	63	72	81
91	9	18	27	36	45	54	63	72	81
92	9	18	27	36	46	55	64	73	82

The Table of Proportional Parts.

D	1	2	3	4	5	6	7	8	9
93	9	18	27	37	46	55	65	74	83
94	9	18	28	37	47	56	65	75	84
95	9	19	28	38	47	57	66	76	85
96	9	19	28	38	48	57	67	76	86
97	9	19	29	38	48	58	67	77	87
98	9	19	29	39	49	58	68	78	88
99	9	19	29	39	49	59	69	79	89
100	10	20	30	40	50	60	70	80	90
101	10	20	30	40	50	60	70	80	90
102	10	20	30	40	51	61	71	81	91
103	10	20	30	41	51	61	72	82	92
104	10	20	31	41	52	62	72	83	93
105	10	21	31	42	52	63	73	84	94
106	10	21	31	42	53	63	74	84	95
107	10	21	32	42	53	64	74	85	96
108	10	21	32	43	54	64	75	86	97
109	10	21	32	43	54	65	76	87	98
110	11	22	33	44	55	66	77	88	99
111	11	22	33	44	55	66	77	88	99
112	11	22	33	44	56	67	78	89	100
113	11	22	33	45	57	67	78	90	101
114	11	22	34	45	57	68	79	91	102
115	11	23	34	46	57	69	80	92	103
116	11	23	34	46	58	69	81	92	104
117	11	23	35	46	58	70	81	93	105
118	11	23	35	47	59	70	82	94	106
119	11	23	35	47	59	71	83	95	107
120	12	24	36	48	60	72	84	96	108
121	12	24	36	48	60	72	84	96	108
122	12	24	36	48	61	73	85	97	109

The Table of Proportional Parts.

D	1	2	3	4	5	6	7	8	9
123	12	24	36	48	61	73	86	98	110
124	12	24	37	49	62	74	86	99	111
125	12	25	37	50	62	75	87	100	112
126	12	25	37	50	63	75	88	100	113
127	12	25	38	50	63	76	88	101	114
128	12	25	38	51	64	76	89	102	115
129	12	25	38	51	64	77	90	103	116
130	13	26	39	52	65	78	91	104	117
131	13	26	39	52	65	78	91	104	117
132	13	26	39	52	66	79	92	105	118
133	13	26	39	53	66	79	93	106	119
134	13	26	40	53	67	80	93	107	120
135	13	27	40	54	67	81	94	108	121
136	13	27	40	54	68	81	95	108	122
137	13	27	41	54	68	82	95	109	123
138	13	27	41	55	69	82	96	110	124
139	13	27	41	55	69	83	97	111	125
140	14	28	42	56	70	84	98	112	126
141	14	28	42	56	70	84	99	112	126
142	14	28	42	56	71	85	99	113	127
143	14	28	42	57	71	85	100	114	128
144	14	28	43	57	72	86	100	115	129
145	14	28	43	58	72	87	101	116	130
146	14	29	43	58	73	87	102	116	131
147	14	29	44	58	73	88	102	117	132
148	14	29	44	59	74	88	103	118	133
149	14	29	44	59	74	89	104	119	134
150	15	30	45	60	75	90	105	120	135
151	15	30	45	60	75	90	105	120	135
152	15	30	45	60	76	91	106	121	136

D	1	2	3	4	5	6	7	8	9
153	15	30	45	60	76	91	107	122	137
154	15	30	46	61	77	92	107	123	138
155	15	31	46	62	77	93	108	124	139
156	15	31	46	62	78	93	109	124	140
157	15	31	47	62	78	94	109	125	141
158	15	31	47	63	79	94	110	126	142
159	15	31	47	63	79	95	111	127	143
160	16	32	48	64	80	96	112	128	144
161	16	32	48	64	80	96	112	128	144
162	16	32	48	64	81	97	113	129	145
163	16	32	48	65	82	98	114	130	146
164	16	32	49	66	82	98	114	131	147
165	16	33	49	66	82	99	115	132	148
166	16	33	49	66	83	99	116	132	149
167	16	33	50	66	83	100	116	133	150
168	16	33	50	67	84	100	117	134	151
169	16	33	50	67	84	101	118	135	152
170	17	34	51	68	85	102	119	136	153
171	17	34	51	68	85	102	119	136	153
172	17	34	51	68	86	103	120	137	154
173	17	34	51	69	86	103	121	138	155
174	17	34	52	69	87	104	121	139	156
175	17	34	52	70	87	105	122	140	157
176	17	35	52	70	88	105	123	140	158
177	17	35	53	70	88	106	123	141	159
178	17	35	53	71	89	106	124	142	160
179	17	35	53	71	89	107	125	143	161
180	18	36	54	72	90	108	126	144	162
181	18	36	54	72	90	108	126	144	162
182	18	36	54	72	91	109	127	145	163

The Table of Proportional Parts.

D	1	2	3	4	5	6	7	8	9
183	18	36	54	73	91	109	128	146	164
184	18	36	55	73	92	110	128	147	165
185	18	37	55	74	92	111	129	148	166
186	18	37	55	74	93	111	130	148	167
187	18	37	56	74	93	112	130	149	168
188	18	37	56	75	94	112	131	150	169
189	18	37	56	75	94	113	132	151	170
190	19	38	57	76	95	114	133	152	171
191	19	38	57	76	95	114	133	152	171
192	19	38	57	76	96	115	134	153	172
193	19	38	57	77	96	115	135	154	173
194	19	38	58	77	97	116	135	155	174
195	19	39	58	78	97	117	136	156	175
196	19	39	59	78	98	117	136	156	176
197	19	39	59	78	98	118	137	157	177
198	19	39	59	79	99	118	138	158	178
199	19	39	59	79	99	119	139	159	179
200	20	40	60	80	100	120	140	160	180
201	20	40	60	80	100	120	140	160	180
202	20	40	60	80	101	121	141	161	181
203	20	40	60	81	101	121	142	162	182
204	20	40	61	81	102	122	142	163	183
205	20	41	61	82	102	123	143	164	184
206	20	41	61	82	103	123	144	164	185
207	20	41	62	82	103	124	144	165	186
208	20	41	62	83	104	124	145	166	187
209	20	41	62	83	104	125	146	167	188
210	21	42	63	84	105	126	147	168	189
211	21	42	63	84	105	126	147	168	189
212	21	42	63	84	106	127	148	169	190

D	1	2	3	4	5	6	7	8	9
213	21	42	63	85	106	127	149	170	191
214	21	42	64	85	107	128	149	171	192
215	21	43	64	86	107	129	150	172	193
216	21	43	64	86	108	129	151	172	194
217	21	43	65	86	108	130	151	173	195
218	21	43	65	87	109	130	152	174	196
219	21	43	65	87	109	131	153	175	197
220	22	44	66	88	110	132	154	176	198
221	22	44	66	88	110	132	154	176	198
222	22	44	66	88	111	133	155	177	199
223	22	44	66	89	111	133	156	178	200
224	22	44	67	89	112	134	156	179	201
225	22	45	67	90	112	135	157	180	202
226	22	45	67	90	113	135	158	180	203
227	22	45	68	90	113	136	158	181	204
228	22	45	68	91	114	136	159	182	205
229	22	45	68	91	114	137	160	183	206
230	23	46	69	92	115	138	161	184	207
231	23	46	69	92	115	138	161	184	207
232	23	46	69	92	116	139	162	185	208
233	23	46	69	93	116	139	163	186	209
234	23	46	70	93	117	140	163	187	210
235	23	47	70	94	117	141	164	188	211
236	23	47	70	94	118	141	165	188	212
237	23	47	71	94	118	142	165	189	213
238	23	47	71	95	119	142	166	190	214
239	23	47	71	95	119	143	167	191	215
240	24	48	72	96	120	144	168	192	216
241	24	48	72	96	120	144	168	192	216
242	24	48	72	96	121	145	169	193	217

The Table of Proportional Parts.

D	1	2	3	4	5	6	7	8	9
243	24	48	72	97	121	145	170	194	218
244	24	48	73	97	122	146	170	195	219
245	24	49	73	98	122	147	171	196	220
246	24	49	73	98	123	147	172	196	221
247	24	49	74	98	123	148	172	197	222
248	24	49	74	99	124	148	173	198	223
249	24	49	74	99	124	149	174	199	224
250	25	50	75	100	125	150	175	200	225
251	25	50	75	100	125	150	175	200	225
252	25	50	75	100	126	151	176	201	226
253	25	50	75	101	126	151	177	202	227
254	25	50	76	101	127	152	177	203	228
255	25	50	76	102	127	153	178	204	229
256	25	51	76	102	128	153	179	204	230
257	25	51	77	102	128	154	179	205	231
258	25	51	77	103	129	154	180	206	232
259	25	51	77	103	129	155	181	207	233
260	26	52	78	104	130	156	182	208	234
261	26	52	78	104	130	156	182	208	234
262	26	52	78	104	131	156	183	209	235
263	26	52	78	105	131	157	184	210	236
264	26	52	79	105	132	158	184	211	237
265	26	53	79	106	132	159	185	212	238
266	26	53	79	106	133	159	186	212	239
267	26	53	80	106	133	160	186	213	240
268	26	53	80	107	134	160	187	214	241
269	26	53	80	107	134	161	188	215	242
270	27	54	81	108	135	162	189	216	243
271	27	54	81	108	135	162	189	216	243
272	27	54	81	108	136	163	190	217	244

The Table of Proportional Parts.

D	1	2	3	4	5	6	7	8	9
273	27	54	81	109	136	163	191	218	245
274	27	54	82	109	137	164	191	219	246
275	27	55	82	110	137	165	192	220	247
276	27	55	82	110	138	165	193	220	248
277	27	55	83	110	138	166	193	221	249
278	27	55	83	111	139	166	194	222	250
276	27	55	83	111	139	167	195	223	251
280	28	56	84	112	140	168	196	224	252
281	28	56	84	112	140	168	196	224	252
282	28	56	84	112	141	169	197	225	253
283	28	56	84	113	141	169	198	226	254
284	28	56	85	113	142	170	198	227	255
285	28	57	85	114	142	171	199	228	256
286	28	57	85	114	143	171	200	228	257
287	28	57	86	114	143	172	200	229	258
288	28	57	86	115	144	172	201	230	259
289	28	57	86	115	144	173	202	231	260
290	29	58	87	116	145	174	203	232	261
291	29	58	87	116	145	174	203	232	261
292	29	58	87	116	146	175	204	233	262
293	29	58	87	117	146	175	205	234	263
294	29	58	88	117	147	176	205	235	264
295	29	59	88	118	147	177	206	236	265
296	29	59	88	118	148	177	207	236	266
297	29	59	88	118	148	178	207	237	267
298	29	59	89	119	149	178	208	238	268
299	29	59	89	119	149	179	209	239	269
300	30	60	90	120	150	180	210	240	270
301	30	60	90	120	150	180	210	240	270
302	30	60	90	120	151	181	211	241	271

The Table of Proportional Parts.

D	1	2	3	4	5	6	7	8	9
303	30	60	90	121	151	181	212	242	272
304	30	60	91	121	152	182	212	243	273
305	30	61	91	122	152	183	213	244	274
306	30	61	91	122	153	183	214	244	275
307	30	61	92	122	153	184	214	245	276
308	30	61	92	123	154	184	215	246	277
309	30	61	92	123	154	185	216	247	278
310	31	62	93	124	155	186	217	248	279
311	31	62	93	124	155	186	217	248	279
312	31	62	93	124	156	187	218	249	280
313	31	62	93	125	156	187	219	250	281
314	31	62	94	125	157	188	219	251	282
315	31	63	94	126	157	189	220	252	283
316	31	63	94	126	158	189	221	252	284
317	31	63	95	126	158	190	221	253	285
318	31	63	95	127	159	190	222	254	286
319	31	63	95	127	159	191	223	255	287
320	32	64	96	128	160	192	224	256	288
321	32	64	96	128	160	192	224	256	288
322	32	64	96	128	161	193	225	257	289
323	32	64	96	129	161	193	226	258	290
324	32	64	97	129	162	194	226	259	291
325	32	65	97	130	162	195	227	260	292
326	32	65	97	130	163	195	228	260	293
327	32	65	98	130	163	196	228	261	294
328	32	65	98	131	163	196	229	262	295
329	32	65	98	131	164	197	230	263	296
330	33	66	99	132	165	198	231	264	297
331	33	66	99	132	165	198	231	264	297
332	33	66	99	132	166	199	232	265	298

The Table of Proportional Parts.

D	1	2	3	4	5	6	7	8	9
333	33	66	99	133	166	199	233	266	299
334	33	66	100	133	167	200	233	267	300
335	33	67	100	134	167	201	234	268	301
336	33	67	100	134	168	201	235	268	302
337	33	67	101	134	168	202	235	269	303
338	33	67	101	135	169	202	236	270	304
339	33	67	101	135	169	203	237	271	305
340	34	68	102	136	170	204	238	272	306
341	34	68	102	136	170	204	238	272	306
342	34	68	102	136	171	205	239	273	307
343	34	68	102	137	171	205	240	274	308
344	34	68	103	137	172	206	240	275	309
345	34	69	103	138	172	207	241	276	310
346	34	69	103	138	173	207	242	276	311
347	34	69	104	138	173	208	242	277	312
348	34	69	104	139	174	208	243	278	313
349	34	69	104	139	174	209	244	279	314
350	35	70	105	140	175	210	245	280	315
351	35	70	105	140	175	210	245	280	315
352	35	70	105	140	176	211	246	281	316
353	35	70	105	141	176	211	247	282	317
354	35	70	106	141	177	212	247	283	318
355	35	71	106	142	177	213	248	284	319
356	35	71	106	142	178	213	249	284	320
357	35	71	107	142	178	214	249	285	321
358	35	71	107	143	179	214	250	286	322
359	35	71	107	143	179	215	251	287	323
360	36	72	108	144	180	216	252	288	324
361	36	72	108	144	180	216	252	288	324
362	36	72	108	144	181	217	253	289	325

The Table of Proportional Parts.

D	1	2	3	4	5	6	7	8	9
363	36	72	108	145	181	217	254	290	326
364	36	72	109	145	182	218	254	291	327
365	36	73	109	146	182	219	255	292	328
366	36	73	109	146	182	219	256	292	329
367	36	73	110	146	183	220	256	293	330
368	36	73	110	147	184	220	257	294	331
369	36	73	110	147	184	221	258	295	332
370	37	74	111	148	185	222	259	296	333
371	37	74	111	148	185	222	259	296	333
372	37	74	111	148	186	223	260	297	334
373	37	74	111	149	186	223	261	298	335
374	37	74	112	149	187	224	261	299	336
375	37	75	112	150	187	225	262	300	337
376	37	75	112	150	188	225	263	300	338
377	37	75	113	150	188	226	263	301	339
378	37	75	113	151	189	226	264	302	340
379	37	75	113	151	189	227	265	303	341
380	38	76	114	152	190	228	266	304	342
381	38	76	114	152	190	228	266	304	342
382	38	76	114	152	191	225	267	305	343
383	38	76	114	153	191	225	268	306	344
384	38	76	115	153	192	230	268	307	345
385	38	77	115	154	192	231	269	308	346
386	38	77	115	154	193	231	270	308	347
387	38	77	116	154	193	232	270	305	348
388	38	77	116	155	194	232	271	310	349
389	38	77	116	155	194	233	272	311	350
390	39	78	117	156	195	233	273	312	351
391	39	78	117	156	195	233	273	312	351
392	39	78	117	156	156	234	274	313	352

The Table of Proportional Parts.

D	1	2	3	4	5	6	7	8	9
393	39	78	117	157	196	235	275	314	353
394	39	78	118	157	197	236	275	315	354
395	39	79	118	158	197	237	276	316	355
396	39	79	118	158	198	237	277	316	356
397	39	79	119	158	198	238	277	317	357
398	39	79	119	159	199	238	278	318	358
399	39	79	119	159	199	239	279	319	359
400	40	80	120	160	200	240	280	320	360
401	40	80	120	160	200	240	280	320	360
402	40	80	120	160	201	241	281	321	361
403	40	80	120	161	201	241	282	322	362
404	40	80	121	161	202	242	282	323	363
405	40	81	121	162	202	243	283	324	364
406	40	81	121	162	203	243	284	324	365
407	40	81	122	162	203	244	284	325	366
408	40	81	122	163	204	244	285	326	367
409	40	81	122	163	204	245	286	327	368
410	41	82	123	164	205	246	287	328	369
411	41	82	123	164	205	246	287	328	369
412	41	82	123	164	206	247	288	329	370
413	41	82	123	165	206	247	289	330	371
414	41	82	124	165	207	248	289	331	372
415	41	83	124	166	207	249	290	332	373
416	41	83	124	166	208	249	291	332	374
417	41	83	125	166	208	250	291	333	375
418	41	83	125	167	209	250	292	334	376
419	41	83	125	167	209	251	293	335	377
420	42	84	126	168	210	252	294	336	378
421	42	84	126	168	210	252	294	336	378
422	42	84	126	168	211	253	295	337	379

The Table of Proportional Parts.

D	1	2	3	4	5	6	7	8	9
423	42	84	126	169	211	253	296	338	380
424	42	84	127	169	212	254	296	339	381
425	42	85	127	170	212	255	297	340	382
426	42	85	127	170	213	255	298	340	383
427	42	85	128	170	213	256	298	341	384
428	42	85	128	171	214	256	299	342	385
429	42	85	128	171	214	257	300	343	386
430	43	86	129	172	215	258	301	344	387
431	43	86	129	172	215	258	301	344	387
432	43	86	129	172	216	259	302	345	388
433	43	86	129	173	216	259	302	346	389
434	43	86	130	173	217	260	304	347	390
435	43	87	130	174	217	261	304	348	391

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TABLE

OF

ARTIFICIAL SINES

AND

TANGENTS

To every

DEGREE and MINUTE

OF THE

QUADRANT.

LONDON,

Printed by *J. Leake*, for W. FREEMAN, at the *Bible* againſt the *Middle-Temple-Gate* in *Fleetſtreet*. MDCCVII.

Degree 0.

M	Sine		Tangent		
0	0.000000	0.000000	0.000000	*Infinita.*	60
1	6.463726	9.999999	6.463726	13.536274	59
2	6.764756	9.999999	6.764756	13.235244	58
3	6.940847	9.999999	6.940847	13.059153	57
4	7.065786	9.999999	7.065786	12.934214	56
5	7.162696	9.999999	7.162696	12.837304	55
6	7.241877	9.999999	7.241878	12.758122	54
7	7.308824	9.999999	7.308825	12.691175	53
8	7.366816	9.999999	7.366817	12.633183	52
9	7.417568	9.999999	7.417970	12.582030	51
10	7.463726	9.999998	7.463727	12.536273	50
11	7.505118	9.999998	7.505120	12.494880	49
12	7.542906	9.999997	7.542909	12.457091	48
13	7.577668	9.999997	7.577272	12.422328	47
14	7.609853	9.999996	7.609857	12.390143	46
15	7.639816	9.999996	7.639826	12.360180	45
16	7.667844	9.999995	7.667849	12.332151	44
17	7.694173	9.999995	7.694179	12.305821	43
18	7.718977	9.999994	7.719003	12.281997	42
19	7.742477	9.999993	7.742484	12.257516	41
20	7.764754	9.999993	7.764761	12.235239	40
21	7.785943	9.999992	7.785951	12.214049	39
22	7.806146	9.999991	7.806145	12.193845	38
23	7.825451	9.999990	7.826460	12.174540	37
24	7.843034	9.999989	7.842944	12.156056	36
25	7.861662	9.999989	7.861674	12.138326	35
26	7.878695	9.999988	7.878708	12.121292	34
27	7.895085	9.999987	7.895099	12.104901	33
28	7.910879	9.999986	7.910894	12.089106	32
29	7.926119	9.999985	7.926134	12.073886	31
30	7.940842	9.999983	7.940858	12.059142	30
		Sine		Tangent	

Degree 89.

Degree 0.

M	Sine		Tangent		
30	7.940842	9.999983	7.940858	12.059142	30
31	7.955082	9.999982	7.955100	12.044900	29
32	7.968870	9.999981	7.968889	12.031111	28
33	7.982233	9.999980	7.982253	12.017747	27
34	7.995198	9.999978	7.995215	12.004781	26
35	8.007787	9.999978	8.007810	11.992191	25
36	8.020021	9.999976	8.020044	11.979956	24
37	8.031919	9.999975	8.031945	11.968055	23
38	8.043601	9.999973	8.043527	11.956473	22
39	8.054781	9.999972	8.054809	11.945181	21
40	8.065776	9.999971	8.065806	11.934194	20
41	8.076500	9.999969	8.076531	11.923469	19
42	8.086965	9.999968	8.086997	11.913003	18
43	8.097183	9.999966	8.097217	11.902783	17
44	8.107167	9.999964	8.107203	11.892797	16
45	8.116926	9.999963	8 116963	11.883037	15
46	8.126471	9.999961	8.126510	11.873490	14
47	8.135810	9.999959	8.135851	11.864149	13
48	8.144953	9.999958	8.144996	11.855004	12
49	8.153907	9.999956	8.153952	11.846048	11
50	8.162681	9.999954	8.162737	11.837273	10
51	8.171280	9.999952	8.171328	11.828672	9
52	8.179713	9.999950	8.179763	11.820237	8
53	8.187985	9.999948	8.188056	11.811964	7
54	8.196102	9.999946	8.196156	11.803844	9
55	8.204070	9.999944	8.204126	11.795674	5
56	8.211895	9.999942	8.211953	11.788047	4
57	8.819581	9.999940	8.219641	11.780359	3
58	8.227134	9.999938	8.227195	11.772805	2
59	8.234557	9.999936	8.234621	11.765379	1
60	8.241855	9.999934	8.241921	11.758079	0
		Sine.		Tangent	M

Degree 89.

Degree 1.

M	Sine		Tangent		
0	8.241855	9.999934	8.241921	11.758079	60
1	8.249033	9.999932	8.249102	11.750898	59
2	8.256094	9.999929	8.256165	11.743835	58
3	8.263042	9.999927	8.263115	11.736885	57
4	8.269881	9.999925	8.269956	11.730044	56
5	8.276614	9.999922	8.276691	11.723309	55
6	8.283243	9.999920	8.283323	11.716677	54
7	8.289773	9.999918	8.289856	11.716144	53
8	8.296207	9.999915	8.296292	11.703708	52
9	8.302546	9.999913	8.302634	11.697366	51
10	8.308794	9.999910	8.308884	11.691116	50
11	8.314954	9.999907	8.315046	11.684954	49
12	8.321027	9.999905	8.321122	11.678878	48
13	8.327016	9.999902	8.327114	11.672886	47
14	8.332924	9.999899	8.333025	11.666975	46
15	8.338753	9.999897	8.338856	11.661144	45
16	8.344504	9.999894	8.344610	11.655390	44
17	8.350180	9.999891	8.350289	11.649711	43
18	8.355783	9.999888	8.355895	11.644105	42
19	8.361315	9.999885	8.361430	11.638570	41
20	8.366777	9.999882	8.366895	11.633105	40
21	8.372171	9.999879	8.372292	11.627708	39
22	8.377499	9.999876	8.377622	11.622378	38
23	8.382762	9.999873	8.382889	11.617111	37
24	8.387962	9.999870	8.388092	11.611908	36
25	8.393101	9.999867	8.393234	11.606766	35
26	8.398179	9.999864	8.398315	11.601685	34
27	8.403199	9.999861	8.403338	11.596662	33
28	8.408161	9.999858	8.408304	11.591696	32
29	8.413068	9.999854	8.413213	11.586787	31
30	8.417919	9.999851	8.418068	11.581932	30
		Sine		Tangent	M

Degree 88.

Degree 1.

M	Sine		Tangent		
30	8.417919	9.999851	8.418068	11.581932	30
31	8.422717	9.999848	8.422869	11.577131	29
32	8.427462	9.999844	8.427618	11.572382	28
33	8.432156	9.999841	8.432315	11.567685	27
34	8.436800	9.999838	8.436962	11.563038	26
35	8.441394	9.999834	8.441560	11.558440	25
36	8.445941	9.999831	8.446110	11.553990	24
37	8.450440	9.999827	8.450613	11.549387	23
38	8.454893	9.999824	8.455070	11.544930	22
39	8.459301	9.999820	8.459481	11.540519	21
40	8.463665	9.999816	8.463849	11.536151	20
41	8.467985	9.999812	8.468172	11.531828	19
42	8.472263	9.999809	8.472454	11.527546	18
43	8.476498	9.999805	8.476693	11.523307	17
44	8.480693	9.999801	8.480892	11.519108	16
45	8.484848	9.999797	8.485050	11.514950	15
46	8.488963	9.999794	8.486170	11.510830	14
47	8.493040	9.999790	8.483250	11.506750	13
48	8.497078	9.999786	8.497293	11.502707	12
49	8.501080	9.999782	8.501298	11.498702	11
50	8.505045	9.999778	8.505267	11.494733	10
51	8.508974	9.999774	8.509200	11.490800	9
52	8.512867	9.999769	8.513098	11.486902	8
53	8.516726	9.999765	8.516961	11.483039	7
54	8.520551	9.999761	8.520790	11.479210	6
55	8.524343	9.999756	8.524586	11.475414	5
56	8.528102	9.999753	8.528349	11.471651	4
57	8.531828	9.999748	8.532080	11.467620	3
58	8.535523	9.999744	8.535779	11.464221	2
59	8.539186	9.999740	8.539447	11.460553	1
60	8.542819	9.999735	8.543084	11.456916	0
		Sine		Tangent	M

Degree 88.

Degree 2.

M	Sine		Tangent		
0	8.542819	9.999735	8.543084	11.456916	60
1	8.546422	9.999731	8.546691	11.453309	59
2	8.549995	9.999726	9.550268	11.449732	58
3	8.553558	9.999722	8.553817	11.446183	57
4	8.557054	9.999717	8.557336	11.442664	56
5	8.560540	9.999713	8.560827	11.439172	55
6	8.563999	9.999708	8.564291	11.435709	54
7	8.567431	9.999703	8.567727	11.432272	53
8	8.570836	9.999699	8.571137	11.428863	52
9	8.574214	9.999694	8.574520	11.425480	51
10	8.577566	9.999689	8.577877	11.422123	50
11	8.580892	9.999685	8.581208	11.418792	49
12	8.584193	9.999680	8.584514	11.415486	48
13	8.587469	9.999675	8.587795	11.412205	47
14	8.590721	9.999670	8.591051	11.408949	46
15	8.593948	9.999665	8.594283	11.405717	45
16	8.597152	9.999660	8.597492	11.402508	44
17	8.600332	9.999655	8.600677	11.399323	43
18	8.603488	9.999650	8.603838	11.356161	42
19	8.606622	9.999645	8.606978	11.393022	41
20	8.609734	9.999640	8.610094	11.389906	40
21	8.612823	9.999635	8.613189	11.386811	39
22	8.615891	9.999629	8.616262	11.383738	38
23	8.618937	9.999624	8.619313	11.380687	37
24	8.621967	9.999619	8.622343	11.377657	36
25	8.624965	9.999614	8.625352	11.374648	35
26	8.627948	9.999608	8.628340	11.371660	34
27	8.630911	9.999603	8.631308	11.368692	33
28	8.633854	9.999597	8.634456	11.365744	32
29	8.636776	9.999592	8.637184	11.362816	31
30	8.639679	9.999586	8.640093	11.359907	30
M		Sine		Tangent	M

Degree 87.

Degree 2.

M	Sine		Tangent		M
30	8.639679	9.999586	8.640093	11.359907	30
31	8.642563	9.999581	8.642982	11.357017	29
32	8.645428	9.999575	8.645853	11.354147	28
33	8.648274	9.999570	8.648704	11.351296	27
34	8.651102	9.999564	8.651538	11.348463	26
35	8.653911	9.999558	8.654352	11.345648	25
36	8.656702	9.999553	8.657149	11.342851	24
37	8.659475	9.999547	8.659928	11.340072	23
38	8.662230	9.999541	8.662689	11.337311	22
39	8.664968	9.999535	8.665433	11.334567	21
40	8.667689	9.999529	8.668160	11.331840	20
41	8.670393	9.999523	8.670869	11.329130	19
42	8.673080	9.999518	8.673563	11.326437	18
43	8.675751	9.999512	8.676239	11.323761	17
44	8.678405	9.999506	8.678899	11.321100	16
45	8.681043	9.999499	8.681544	11.318456	15
46	8.683665	9.999493	8.684172	11.315828	14
47	8.686272	9.999487	8.686784	11.313216	13
48	8.688892	9.999481	8.689381	11.310619	12
49	8.691438	9.999475	8.691963	11.308037	11
50	8.693998	9.999469	8.694529	11.305471	10
51	8.696543	9.999462	8.697081	11.302919	9
52	8.699073	9.999456	8.699617	11.300383	8
53	8.701589	9.999450	8.702139	11.297861	7
54	8.704090	9.956443	8.704646	11.295354	6
55	8.706576	9.999437	8.707130	11.292860	5
56	8.709049	9.999431	8.709618	11.290381	4
57	8.711507	9.999424	8.712083	11.287917	3
58	8.713952	9.999418	8.714534	11.285466	2
59	8.716383	9.999411	8.716972	11.283028	1
60	8.718800	9.999404	8.719396	11.280604	0
		Sine		Tangent	M

Degree 87.

Degree 3.

M	Sine		Tangent		
0	8.718800	9.999404	8.719396	11.280604	60
1	8.721204	9.999398	8.721806	11.278194	59
2	8.723595	9.999391	8.724254	11.275796	58
3	8.725972	9.999384	8.726588	11.273412	57
4	8.728336	9.999378	8.728959	11.271041	56
5	8.730688	9.999371	8.731317	11.268683	55
6	8.733027	9.999364	8.733663	11.266337	54
7	8.735354	9.999357	8.735996	11.264004	53
8	8.737667	9.999350	8.738317	11.261682	52
9	8.739969	9.999343	8.740626	11.259374	51
10	8.742259	9.999336	8.742922	11.257078	50
11	8.744536	9.999329	8.745007	11.254793	49
12	8.746801	9.999322	8.747479	11.252521	48
13	8.745955	9.999315	8.749740	11.250240	47
14	8.751297	9.999308	8.751989	11.248011	46
15	8.753528	9.999301	8.754227	11.245773	45
16	8.755747	9.999294	8.756453	11.243542	44
17	8.757955	9.999286	8.758668	11.241332	43
18	8.760151	9.999279	8.760872	11.239128	42
19	8.762337	9.999272	8.763065	11.236935	41
20	8.764511	9.999265	8.765246	11.234754	40
21	8.766675	9.999257	8.767417	11.232583	39
22	8.768828	9.999250	8.769578	11.230422	38
23	8.770970	9.999243	8.771727	11.228273	37
24	8.773101	9.999235	8.773866	11.229134	36
25	8.775223	9.999227	8.775995	11.224005	35
26	8.777333	9.999220	8.778114	11.221886	34
27	8.779434	9.999212	8.783222	11.219778	33
28	8.781524	9.999204	8.782320	11.217680	32
29	8.783605	9.999197	8.784404	11.215592	31
30	8.785675	9.999189	8.786486	11.213514	30
M		Sine		Tangent	M

Degree 86.

Degree 3.

M	Sine		Tangent		
30	8.785675	9.999189	8.786486	11.213514	30
31	8.787736	9.999181	8.788554	11.211446	29
32	8.789787	9.999174	8.790613	11.209387	28
33	8.791828	9.999166	8.792662	11.207338	27
34	8.793859	9.999158	8.794701	11.205299	26
35	8.795881	9.999150	8.796731	11.203269	25
36	8.797894	9.999142	8.798752	11.201248	24
37	8.799897	9.999134	8.800763	11.199237	23
38	8.801891	9.999126	8.802765	11.197235	22
39	8.803876	9.999118	8.804758	11.195242	21
40	8.805852	9.999110	8.806742	11.193258	20
41	8.807819	9.999102	8.808717	11.191283	19
42	8.809777	9.999094	8.812683	11.189317	18
43	8.811726	9.999086	8.812641	11.187359	17
44	8.813667	9.999077	8.814589	11.185411	16
45	8.815598	9.999069	8.816529	11.183471	15
46	8.817522	9.999061	8.818461	11.181539	14
47	8.819436	9.999052	8.820384	11.179616	13
48	8.821342	9.999044	8.822298	11.177702	12
49	8.823240	9.999036	8.824205	11.175795	11
50	8.825130	9.999027	8.826103	11.173897	10
51	8.827011	9.999019	8.827992	11.172008	9
52	8.828884	9.999010	8.829874	11.170126	8
53	8.830749	9.999002	8.831748	11.168252	7
54	8.832106	9.998993	8.833613	11.166387	6
55	8.834456	9.998984	8.835471	11.164529	5
56	8.836297	9.998976	8.837321	11.162679	4
57	8.838130	9.998967	8.839163	11.160837	3
58	8.839956	9.998958	8.840998	11.159002	2
59	8.841774	9.998940	8.842825	11.157175	1
60	8.843585	9.998941	8.844644	11.155356	0
		Sine		Tangent	M

Degree 86.

Degree 4.

M	Sine		Tangent		
0	8.843584	9.998941	8.844644	11.155356	60
1	8.845387	9.998931	8.846455	11.153545	59
2	8.847183	9.998923	8.848240	11.151740	58
3	8.848971	9.998914	8.850057	11.149943	57
4	8.850751	9.998905	8.851846	11.148154	56
5	8.852525	9.998896	8.853628	11.146372	55
6	8.854291	9.998887	8.855403	11.144597	54
7	8.856049	9.998878	8.857171	11.142829	53
8	8.857801	9.998869	8.858932	11.141068	52
9	8.859546	9.998860	8.860686	11.139314	51
10	8.861283	9.998851	8.862433	11.137567	50
11	8.863014	9.998841	8.864173	11.135827	49
12	8.864738	9.998832	8.865906	11.134094	48
13	8.866454	9.998823	8.867632	11.132368	47
14	8.868165	9.998813	8.869351	11.130649	46
15	8.869868	9.998804	8.871064	11.128936	45
16	8.871565	9.998795	8.872750	11.127230	44
17	8.873255	9.998785	8.874469	11.125531	43
18	8.874938	9.998776	8.876162	11.123838	42
19	8.876615	9.998766	8.877849	11.122151	41
20	8.878285	9.998757	8.879529	11.120471	40
21	8.879949	9.998747	8.881202	11.118798	39
22	8.881607	9.998738	8.882869	11.117131	38
23	8.883258	9.998728	8.884530	11.115470	37
24	8.884903	9.998718	8.886185	11.113815	36
25	8.886542	9.998708	8.887833	11.112167	35
26	8.888174	9.998699	8.889476	11.110524	34
27	8.889801	9.998689	8.891112	11.108888	33
28	8.891421	9.998679	8.892742	11.107258	32
29	8.893035	9.998669	8.894366	11.105634	31
30	8.894643	9.998659	8.845984	11.104016	30
M		Sine		Tangent	M

Degree 85.

Degree 4.

M	Sine		Tangent		M
30	8.894643	9.998659	8.895984	11.104016	30
31	8.896246	9.998649	8.897596	11.102404	29
32	8.897842	9.998639	8.899203	11.100797	28
33	8.899432	9.998629	8.900803	11.099197	27
34	8.901017	9.998619	8.902398	11.097602	26
35	8.902596	9.998609	8.903987	11.096013	25
36	8.904169	9.998599	8.905570	11.094430	24
37	8.905736	9.998589	8.907147	11.092853	23
38	8.907297	9.998577	8.908719	11.091281	22
39	8.908853	9.998568	8.910285	11.089715	21
40	8.910404	9.998558	8.911846	11.088154	20
41	8.911949	9.998548	8.913401	11.086599	19
42	8.913488	9.998537	8.914951	11.085049	18
43	8.915022	9.998527	8.916495	11.083505	17
44	8.916550	9.998516	8.918034	11.081966	16
45	8.918073	9.998506	8.919568	11.080432	15
46	8.919591	9.998495	8.921096	11.078904	14
47	8.921103	9.998485	8.922619	11.077381	13
48	8.922610	9.998474	8.924136	11.075864	12
49	8.924112	9.998464	8.925649	11.074351	11
50	8.925609	9.998453	8.927156	11.072844	10
51	8.927100	9.998442	8.928658	11.071342	9
52	8.928587	9.998431	8.930155	11.069845	8
53	8.930068	9.998421	8.931647	11.068353	7
54	8.931544	9.998410	8.933134	11.096866	6
55	8.933015	9.998399	8.934616	11.065384	5
56	8.934481	9.998388	8.936093	11.063907	4
57	8.935942	9.998377	8.937565	11.062435	3
58	8.937398	9.998366	8.939032	11.060968	2
59	8.938850	9.998355	8.940494	11.059506	1
60	8.940296	9.998344	8.941952	11.058048	0
		Sine		Tangent	M

Degree 85

Degree 5.

M	Sine		Tangent		
0	8.940296	9.998344	8.941952	11.058048	60
1	8.941738	9.998333	8.943404	11.056596	59
2	8.943174	9.998322	8.944852	11.055148	58
3	8.944606	9.998311	8.946295	11.053705	57
4	8.946034	9.998300	8.947734	11.052266	56
5	8.957456	9.998289	8.949168	11.050832	55
6	8.958814	9.998277	8.950597	11.049403	54
7	8.950287	9.998266	8.952021	11.047979	53
8	8.951656	9.998255	8.953441	11.046559	52
9	8.953099	9.998243	8.954856	11.045144	51
10	8.954499	9.998232	8.956267	11.043733	50
11	8.955894	9.998220	8.957674	11.042326	49
12	8.957284	9.998209	8.959075	11.040925	48
13	8.958670	9.998197	8.960473	11.039527	47
14	8.960052	9.998186	8.961866	11.038134	46
15	8.961429	9.998174	8.963254	11.036746	45
16	8.962801	9.998163	8.964639	11.035361	44
17	8.964170	9.998151	8.966019	11.033981	43
18	8.965534	9.998139	8.967394	11.032606	42
19	8.966893	9.998128	8.968766	11.031234	41
20	8.968249	9.998106	8.970133	11.029867	40
21	8.969600	9.998104	8.971495	11.028505	39
22	8.970947	9.998092	8.972855	11.027145	38
23	8.972289	9.998080	8.974209	11.025791	37
24	8.973626	9.998068	8.975560	11.024440	36
25	0.974962	9.998056	8.976906	11.023094	35
26	8.976293	9.998044	8.978248	11.021752	34
27	8.977619	9.998032	8.979586	11.020414	33
28	8.978941	9.998020	8.980921	11.019079	32
29	8.980259	9.998008	8.982251	11.017749	31
30	8.981573	9.997996	8.983577	11.016423	30
		Sine		Tangent	M

Degree 84.

Degree 5.

M	Sine		Tangent		
30	8.981572	9.997996	8.983577	11.016423	30
31	8.982883	9.997984	8.984899	11.015101	29
32	8.984189	9.997971	8.986217	11.013783	28
33	8.985491	9.997959	8.987532	11.012468	27
34	8.986789	9.997947	8.988842	11.011158	26
35	8.988083	9.997935	8.990149	11.009851	25
36	8.989374	9.997922	8.991451	11.008549	24
37	8.990660	9.997910	8.992750	11.007250	23
38	8.991943	9.997897	8.994045	11.005955	22
39	8.993228	9.997885	8.995337	11.004663	21
40	8.994497	9.997873	8.996624	11.003376	20
41	8.995768	9.997860	8.997908	11.002092	19
42	8.997036	9.997847	8.999188	11.000812	18
43	8.998299	9.997835	9.000465	10.999535	17
44	8.999560	9.997822	9.001738	10.998262	16
45	9.000816	9.997809	9.003007	10.996993	15
46	9.002069	9.997797	9.004272	10.995728	14
47	9.003318	9.997784	9.005534	10.994466	13
48	9.004563	9.997771	9.006792	10.993208	12
49	9.005805	9.997758	9.008047	10.991953	11
50	9.007044	9.997730	9.009298	10.990708	10
51	9.008278	9.997732	9.010546	10.989454	9
52	9.009510	9.997719	9.011790	10.988210	8
53	9.010737	9.997706	9.013031	10.986969	7
54	9.011962	9.997693	9.014268	10.985732	6
55	9.013182	9.997680	9.015502	10.984498	5
56	9.014399	9.997667	9.016732	10.983268	4
57	9.015613	9.997654	9.017959	10.982041	3
58	9.016824	9.997641	9.019183	10.980817	2
59	9.018031	9.997628	9.020403	10.979597	1
60	9.019235	9.997612	9.021620	10.978380	0
		Sine		Tangent	

Degree 84.

Degree 6.

M	Sine		Tangent		
0	9.019235	9.997614	9.021620	10.978380	60
1	9.020435	9.997601	9.022834	10.977166	59
2	9.021632	9.997588	9.024044	10.975956	58
3	9.022825	9.997574	9.025251	10.974749	57
4	9.024016	9.997561	9.026455	10.973545	56
5	9.025203	9.997548	9.027655	10.972345	55
6	9.026386	9.997534	9.028852	10.971148	54
7	9.027567	9.997520	9.030046	10.969954	53
8	9.028744	9.997507	9.031237	10.968763	52
9	9.029918	9.997493	9.032425	10.967575	51
10	9.031089	9.997480	9.033609	10.966391	50
11	9.032257	9.997466	9.034791	10.965209	49
12	9.033421	9.997452	9.035969	10.964031	48
13	9.034582	9.997439	9.036144	10.962856	47
14	9.035741	9.997425	9.038316	10.961684	46
15	9.036896	9.997411	9.039485	10.960515	45
16	9.038048	9.997397	9.040651	10.959349	44
17	9.039197	9.997383	9.041813	10.958187	43
18	9.040342	9.997369	9.042973	10.957027	42
19	9.041485	9.997355	9.044130	10.955870	41
20	9.042625	9.997341	9.045284	10.954716	40
21	9.043762	9.997327	9.046434	10.953566	39
22	9.044895	9.997313	9.047582	10.952418	38
23	9.046026	9.997299	9.048727	10.951273	37
24	9.047154	9.997285	9.049869	10.950131	36
25	9.049279	9.997271	9.051008	10.948992	35
26	9.049400	9.997256	9.052144	10.947856	34
27	9.050519	9.997242	9.043277	10.946723	33
28	9.051635	9.997228	9.054408	10.945592	32
29	9.052749	9.997214	9.055535	10.944465	31
30	9.053859	9.997199	9.056640	10.943340	30
		Sine		Tangent	M

Degree 83.

Degree 6.

M	Sine		Tangent		M
30	9.053859	9.997199	9.056640	10.943340	30
31	9.054566	9.997185	9.057781	10.942219	29
32	9.056071	9.997170	9.058900	10.941100	28
33	9.057172	9.997156	9.060016	10.939984	27
34	9.058271	9.997141	9.061130	10.938870	26
35	9.059367	9.997127	9.062240	10.937760	25
36	9.060460	9.997112	9.063348	10.936652	24
37	9.061551	9.997098	9.064453	10.935547	23
38	9.062638	9.997083	9.065556	10.934444	22
39	9.063723	9.997068	9.066655	10.933345	21
40	9.064806	9.997053	9.067752	10.932248	20
41	9.065885	9.997039	9.068847	10.931153	19
42	9.066962	9.997024	9.069938	10.930062	18
43	9.068036	9.997009	9.071027	10.928973	17
44	9.069107	9.996994	9.072113	10.927887	16
45	9.070176	9.996979	9.073197	10.926803	15
46	9.071242	9.996964	9.074278	10.925722	14
47	9.072306	9.996949	9.075356	10.924644	13
48	9.073366	9.996934	9.076432	10.923568	12
49	9.074424	9.996919	9.077505	10.922495	11
50	9.075480	9.996904	9.078576	10.921424	10
51	9.076533	9.996889	9.079644	10.920356	9
52	9.077583	9.996874	9.080710	10.919290	8
53	9.078631	9.996858	9.081773	10.918227	7
54	9.079676	9.996843	9.082833	10.917167	6
55	9.080719	9.996828	9.083891	10.916109	5
56	9.081759	9.996812	9.084947	10.915053	4
57	9.082797	9.996797	9.085999	10.914100	3
58	9.083832	9.996782	9.087050	10.912950	2
59	9.084864	9.996766	9.088098	10.911902	1
60	9.085894	9.996751	9.089144	10.910856	0
M		Sine		Tangent	M

Degree 83.

Degree 7.

M	Sine		Tangent		
0	9.085894	9.996751	9.089144	10.910856	60
1	9.086922	9.996735	9.090187	10.909813	59
2	9.087947	9.996720	9.091228	10.908772	58
3	9.088970	9.996704	9.092266	10.907734	57
4	9.089990	9.996688	9.093302	10.906698	56
5	9.091088	9.996673	9.094336	10.905664	55
6	9.092024	9.996657	9.095367	10.904633	54
7	9.093037	9.996641	9.096395	10.903604	53
8	9.094047	9.996625	9.097422	10.902578	52
9	9.095056	9.996610	9.098446	10.901554	51
10	9.096062	9.996594	9.099468	10.900532	50
11	9.097065	9.996578	9.100487	10.899513	49
12	9.098066	9.996562	9.101504	10.898496	48
13	9.099065	9.996546	9.102519	10.897481	47
14	9.100062	9.996530	9.103532	10.896468	46
15	9.101056	9.996514	9.104542	10.895458	45
16	9.102048	9.996498	9.105550	10.894450	44
17	9.103037	9.996482	9.106556	10.893444	43
18	9.104025	9.996465	9.107559	10.892441	42
19	9.105010	9.996449	9.108560	10.891440	41
20	9.105992	9.996433	9.109559	10.890441	40
21	9.106973	9.996417	9.110556	10.889444	39
22	9.107951	9.996400	9.111551	10.888449	38
23	9.108927	9.996384	9.112543	10.887457	37
24	9.109901	9.996368	9.113533	10.886467	36
25	9.110873	9.996351	9.114521	10.885478	35
26	9.111842	9.996335	9.115507	10.884493	34
27	9.112809	9.996318	9.116491	10.883500	33
28	9.113774	9.996302	9.117472	10.882528	32
29	9.114737	9.996285	9.118452	10.881548	31
30	9.115698	9.996269	9.119429	10.880571	30
		Sine		Tangent	M

Degree 82.

Degree 7.

M	Sine		Tangent		
30	9.115698	9.996269	9.119429	10.880571	30
31	9.116656	9.996252	9.120404	10.879596	29
32	9.117612	9.996235	9.121377	10.878623	28
33	9.118567	9.996218	9.122348	10.877652	27
34	9.119519	9.996202	9.123317	10.876683	26
35	9.120469	9.996185	9.124284	10.875716	25
36	9.121417	9.996168	9.125248	10.874751	24
37	9.122362	9.996151	9.126211	10.873789	23
38	9.123306	9.996134	9.127172	10.872828	22
39	9.124248	9.996117	9.128130	10.871870	21
40	9.125187	9.996100	9.129087	10.870913	20
41	9.126125	9.996083	9.130041	10.869959	19
42	9.127060	9.996066	9.130994	10.869006	18
43	9.127993	9.996049	9.131944	10.868056	17
44	9.128925	9.996032	9.132893	10.867107	16
45	9.129854	9.996015	9.133839	10.869161	15
46	9.130781	9.995998	9.134784	10.865216	14
47	9.131706	9.995980	9.135726	10.864274	13
48	9.132630	9.995963	9.136666	10.863334	12
49	9.133551	9.995946	9.137605	10.862395	11
50	9.134470	9.995928	9.138542	10.861458	10
51	9.135387	9.995911	9.139476	10.860524	9
52	9.136303	9.995894	9.140409	10.859591	8
53	9.137216	9.995876	9.141340	10.858660	7
54	9.138127	9.995859	9.142269	10.857731	6
55	9.139037	9.995841	9.143196	10.856804	5
56	9.139944	9.995825	9.144121	10.855879	4
57	9.140850	9.995806	9.145044	10.854956	3
58	9.141754	9.995788	9.145965	10.854035	2
59	9.142655	9.995770	9.146885	10.853115	1
60	9.142555	9.995753	9.147803	10.852197	0
		Sine		Tangent	

Degree 82.

Degree 8.

M	Sine		Tangent		
0	9.143555	9.995753	9.147803	10.852197	60
1	9.144453	9.995735	9.148718	10.851282	59
2	9.145349	9.995717	9.149632	10.850368	58
3	9.146243	9.995699	9.150544	10.849456	57
4	9.147136	9.995681	9.151454	10.848546	56
5	9.148026	9.995664	9.152363	10.847637	55
6	9.148919	9.995646	9.153269	10.846731	54
7	9.149881	9.995628	9.154174	10.845825	53
8	9.150686	9.995610	9.155077	10.844923	52
9	9.151569	9.995591	9.155978	10.844022	51
10	9.152451	9.995573	9.156877	10.843123	50
11	9.153330	9.995555	9.157775	10.842225	49
12	9.154208	9.995537	9.158671	10.841329	48
13	9.155082	9.995519	9.159565	10.840435	47
14	9.155957	9.995501	9.160457	10.839543	46
15	9.156830	9.995482	9.161347	10.838653	45
16	9.157700	9.995464	9.162236	10.837764	44
17	9.158569	9.995446	9.163123	10.836877	43
18	9.159436	9.995427	9.164008	10.835992	42
19	9.160301	9.995409	9.164892	10.835108	41
20	9.161164	9.995390	9.165773	10.834226	40
21	9.162025	9.995372	9.166654	10.833346	39
22	9.162885	9.995353	9.167532	10.832468	38
23	9.163743	9.995334	9.168409	10.831591	37
24	9.164600	9.995316	9.169284	10.830716	36
25	9.165454	9.995297	9.160157	10.829843	35
26	9.166307	9.995278	9.171029	10.828971	34
27	9.167158	9.995260	9.171899	10.828101	33
28	9.168008	9.995241	9.172767	10.827233	32
29	9.168856	9.995222	9.173634	10.826366	31
30	9.169702	9.995203	9.174499	10.825501	30
		Sine		Tangent	M

Degree 81.

Degree 8.

M	Sine		Tangent		
30	9.169702	9.995203	9.174499	10.825501	30
31	9.170546	9.995184	9.175362	10.824638	29
32	9.171389	9.995165	9.176224	10.823776	28
33	9.172230	9.995146	9.177084	10.822916	27
34	9.173070	9.995127	9.177942	10.822057	26
35	9.173908	9.995108	9.178799	10.821201	25
36	9.174744	9.995089	9.179655	10.820345	24
37	9.175578	9.995070	9.180508	10.819492	23
38	9.176411	9.995061	9.181360	10.818640	22
39	9.177242	9.995032	9.182211	10.817789	21
40	9.178072	9.995012	9.183060	10.816940	20
41	9.178900	9.994993	9.183907	10.816093	19
42	9.179726	9.994974	9.184752	10.815248	18
43	9.180551	9.994955	9.185597	10.814403	17
44	9.181374	9.994935	9.186439	10.813561	16
45	9.182196	9.994916	9.187280	10.812720	15
46	9.183016	9.994896	9.188120	10.811880	14
47	9.183834	9.994876	9.188957	10.811042	13
48	9.184651	9.994857	9.189794	10.810206	12
49	9.185466	9.994838	9.190629	10.809371	11
50	9.186280	9.994818	9.191462	10.808538	10
51	9.187092	9.994798	9.192294	10.807706	9
52	9.187903	9.994779	9.193124	10.806876	8
53	9.188712	9.994759	9.193953	10.806047	7
54	9.189519	9.994739	9.194780	10.805220	6
55	9.190325	9.994719	9.195606	10.804394	5
56	9.191130	9.994699	9.196440	10.803569	4
57	9.191933	9.994680	9.197253	10.802747	3
58	9.192734	9.994660	9.198674	10.801926	2
59	9.193534	9.994640	9.198894	10.801106	1
60	9.194332	9.994620	9.199712	10.800287	0
M		Sine		Tangent	M

Degree 81.

Degree 9.

M	Sine		Tangent		
0	9.194332	9.994620	9.199712	10.800287	60
1	9.195129	9.994600	9.200529	10.799470	59
2	9.195925	9.994580	9.201345	10.798655	58
3	9.196718	9.994560	9.202159	10.797841	57
4	9.197511	9.994540	9.202971	10.797029	56
5	9.198302	9.994519	9.203782	10.796218	55
6	9.199091	9.994499	9.204592	10.795408	54
7	9.199879	9.994479	9.205400	10.794600	53
8	9.200666	9.994459	9.206207	10.793793	52
9	9.201451	9.994438	9.207013	10.792987	51
10	9.202234	9.994418	9.207817	10.792183	50
11	9.203017	9.994398	9.208619	10.791381	49
12	9.203797	9.994377	9.209420	10.790580	48
13	9.204577	9.994357	9.210220	10.789780	47
14	9.205354	9.994336	9.211018	10.788982	46
15	9.206131	9.994316	9.211815	10.788185	45
16	9.206906	9.994295	9.212611	10.787385	44
17	9.207679	9.994274	9.213405	10.786595	43
18	9.208452	9.994254	9.214198	10.785802	42
19	9.209222	9.994233	9.214989	10.785011	41
20	9.209992	9.994212	9.215780	10.784220	40
21	9.210760	9.994191	9.216568	10.783432	39
22	9.211526	9.994171	9.217356	10.782644	38
23	9.212291	9.994150	9.218142	10.781858	37
24	9.213055	9.994129	9.218926	10.781070	36
25	9.213818	9.994108	9.219710	10.780294	35
26	9.214579	9.994087	9.220491	10.779508	34
27	9.215338	9.994066	9.221272	10.778728	33
28	9.216097	9.994044	9.222052	10.777948	32
29	9.216854	9.994024	9.222830	10.777170	31
30	9.217609	9.994003	9.223607	10.776393	30
		Sine		Tangent	M

Degree 80.

Degree 9.

M	Sine		Tangent		M
30	9.217609	9.994003	9.223607	10.776393	30
31	9.218363	9.993982	9.224382	10.775618	29
32	9.219116	9.993960	9.225156	10.774844	28
33	9.219868	9.993939	9.225929	10.774071	27
34	9.220618	9.993918	9.226704	10.773300	26
35	9.221367	9.993897	9.227471	10.772529	25
36	9.222115	9.993875	9.228240	10.771760	24
37	9.222861	9.993854	9.229007	10.770993	23
38	9.223606	9.993832	9.229774	10.770226	22
39	9.224349	9.993811	9.230539	10.769461	21
40	9.225092	9.993789	9.231302	10.768698	20
41	9.225833	9.993768	9.232065	10.767935	19
42	9.226573	9.993746	9.232826	10.767174	18
43	9.227311	9.993725	9.233586	10.766414	17
44	9.228048	9.993703	9.234345	10.765655	16
45	9.228784	9.993681	9.235103	10.764897	15
46	9.239518	9.993660	9.235859	10.764141	14
47	9.230252	9.993638	9.236614	10.763386	13
48	9.230984	9.993616	9.237368	10.762632	12
49	9.231715	9.993594	9.238120	10.761880	11
50	9.232444	9.993572	9.238872	10.761128	10
51	9.233172	9.993550	9.239622	10.760378	9
52	9.233899	9.993528	9.240371	10.759629	8
53	9.234625	9.993506	9.241118	10.758882	7
54	9.235349	9.993484	9.241865	10.758135	6
55	9.236073	9.993462	9.242610	10.757390	5
56	9.236795	9.993440	9.243354	10.756646	4
57	9.237515	9.993418	9.244097	10.755903	3
58	9.238235	9.993396	9.244839	10.755161	2
59	9.238952	9.993374	9.245579	10.754421	1
60	9.239670	9.993351	9.246319	10.753681	0
M		Sine		Tangent	M

Degree 80.

Degree 10.

M	Sine		Tangent		
0	9.239670	9.993351	9.246319	10.753681	60
1	9.240386	9.993329	9.247057	10.752943	59
2	9.241101	9.993307	9.247794	10.752206	58
3	9.241814	9.993284	9.248530	10.751470	57
4	9.242526	9.993262	9.249264	17.750736	56
5	9.243237	9.993240	9.249998	10.750002	55
6	9.243947	9.993217	9.250730	10.749270	54
7	9.244656	9.993195	9.251461	10.748539	53
8	9.245363	9.993172	9.252191	10.747809	52
9	9.246070	9.993149	9.252920	10.747080	51
10	9.246775	9.993127	9.253648	10.746352	50
11	9.247478	9.993104	9.254374	10.745626	49
12	9.248181	9.993011	9.255200	10.744900	48
13	9.248883	9.993059	9.255824	10.744176	47
14	9.249583	9.993036	9.256547	10.743453	46
15	9.250282	9.993013	9.257269	10.742731	45
16	9.250980	9.992990	9.257990	10.742010	44
17	9.251677	9.992967	9.258710	10.741290	43
18	9.252373	9.992944	9.259429	10.740571	42
19	9.253067	9.992921	9.260146	10.739854	41
20	9.253761	9.992898	9.260863	10.739137	40
21	9.254453	9.992875	9.261578	10.738422	39
22	9.255144	9.992852	9.262292	10.737708	38
23	9.255834	9.992829	9.263005	10.736995	37
24	9.256523	9.992806	9.263717	10.736283	36
25	9.257211	9.992783	9.264428	10.735572	35
26	9.257898	9.992759	9.265138	10.734862	34
27	9.258583	9.992736	9.265847	10.734153	33
28	9.259268	9.992713	9.266555	10.733445	32
29	9.259951	9.992690	9.267261	10.732739	31
30	9.260633	9.992666	9.267967	10.732033	30
		Sine		Tangent	M

Degree 79.

Degree 10.

M	Sine		Tangent		
30	9.260633	9.992666	9.267967	10.732033	30
31	9.261314	9.992643	9.268671	10.731329	29
32	9.261994	9.992619	9.269375	10.730625	28
33	9.262673	9.992596	9.270078	10.729923	27
34	9.263351	9.992572	9.271479	10.729221	26
35	9.264027	9.992549	9.271470	10.728521	25
36	9.264703	9.992525	9.272178	10.727822	24
37	9.265378	9.992501	9.272876	10.727124	23
38	9.266051	9.992478	9.273573	10.726427	22
39	9.266723	9.992454	9.274269	10.725731	21
40	9.267395	9.992430	9.274964	10.725036	20
41	9.268065	9.992406	9.275658	10.724342	19
42	9.268734	9.992382	9.276351	10.723649	18
43	9.269402	9.992362	9.277043	10.722957	17
44	9.270069	9.992335	9.277734	10.722267	16
45	9.270735	9.992311	0.278424	10.721576	15
46	9.271400	9.992287	9.279113	10.720887	14
47	9.272063	9.992263	9.279801	10.720199	13
48	9.272726	9.992239	9.280488	10.719512	12
49	9.273388	9.992214	9,281174	10.718826	11
50	9.274049	9.992190	9.281858	10.718142	10
51	9.274708	9.992166	9.282542	10.717458	9
52	9.275367	9.992142	9.283225	10.716775	8
53	9.276025	9.992118	9.283907	10.716093	7
54	9.276681	9.992093	9.284588	10.715412	6
55	9.277337	9.992069	9.285268	10.714732	5
56	9.277991	9.992045	9.285946	10.714053	4
57	9.278685	9.992020	9.286624	10.713376	3
58	9.279297	9.991996	9.287301	10.712699	2
59	9.279948	9.991971	9.287977	10.712023	1
60	9.280599	9.991947	9.288652	10.711348	0
		Sine		Tangent	M

Degree 79.

Degree 11.

M	Sine		Tangent		
0	9.280599	9.991947	9.288652	10.711348	60
1	9.281229	9.991922	9.289326	10.710674	59
2	9.281897	9.991897	9.289999	10.710001	58
3	9.282544	9.991873	9.290671	10.709329	57
4	9.283190	9.991848	9.291342	10.708658	56
5	9.283836	9.991823	9.292013	10.707987	55
6	9.284480	9.991799	9.292682	10.707318	54
7	9.285124	9.991774	9.293350	10.706650	53
8	9.285766	9.991749	6.294017	10.705983	52
9	9.286408	9.991724	9.294684	10.705316	51
10	9.287048	9.991699	9.295349	10.704651	50
11	9.287688	9.991674	9.296013	10.703987	49
12	9.288326	9.991649	9.296677	10.703323	48
13	9.288964	9.991624	9.297339	10.702661	47
14	9.289600	9.991599	9.298001	10.701999	46
15	9.290236	8.991574	9.298662	10.701338	45
16	9.290870	9.991549	9.299322	10.700678	44
17	9.291504	9.991524	9.299980	10.700020	43
18	9.292137	9.991498	9.300638	10.699362	42
19	9.292768	9.991473	9.301295	10.698705	41
20	9.293399	9.991448	9.301951	10.698049	40
21	9.294029	9.991422	9.302607	10.697393	39
22	9.294658	9.991397	9.303261	10.696739	38
23	9.295286	9.961372	9.303914	10.696086	37
24	9.295913	9.991346	9.304567	10.695433	36
25	9.296539	9.991321	9.305218	10.694782	35
26	9.297164	9.991295	9.305867	10.694131	34
27	9.297788	9.991270	9.306519	10.693481	33
28	9.298412	9.991244	9.307163	10.692832	32
29	9.299034	9.991218	9.307816	10.692184	31
30	9.299655	9.991193	9.308463	10.691537	30
		Sine		Tangent	M

Degree 78.

Degree 11.

M	Sine		Tangent		
30	9.296655	9.991193	9.308463	10.691537	30
31	9.300276	9.991167	9.309109	10.690891	29
32	9.300895	9.991141	9.309754	10.690246	28
33	9.301514	9.991115	9.310399	10.689601	27
34	9.302132	9.991090	9.311042	10.688958	26
35	9.302749	9.991064	9.311685	10.688315	25
36	9.303364	9.991038	9.312327	19.687673	24
37	9.303979	9.991012	9.312968	10.687032	23
38	9.304593	9.990986	9.313608	10.686392	22
39	9.305207	9.990960	9.314247	10.685753	21
40	9.305819	9.990934	9.314885	10.685115	20
41	9.306430	9.990908	9.315523	10.684477	19
42	9.307041	9.990882	9.316159	10.683841	18
43	9.307650	9.990855	9.316795	10.683205	17
44	9.308259	9.990829	9.317430	10.682570	16
45	9.308867	9.990803	9.318064	10.681936	15
46	9.309474	9.990777	9.318647	10.681303	14
47	9.310080	9.990750	9.319330	10.680670	13
48	9.310685	9.990724	9.319961	10.680039	12
49	9.311289	9.990697	9.320592	10.679408	11
50	9.311899	9.990671	9.321222	10.678778	10
51	9.312495	9.990645	9.321851	10.678149	9
52	9.313097	9.990618	9.322479	10.677521	8
53	9.313698	9.990591	9.323106	10.676894	7
54	9.314297	9.990565	9.323733	10.676267	6
55	9.314897	9.990538	9.324358	10.675642	5
56	9.315495	9.990512	9.324983	10.675017	4
57	9.316092	9.990485	9.325607	10.674393	3
58	9.316689	9.990458	9.326231	10.673769	2
59	9.317284	9.990431	9.326853	10.673147	1
60	9.317879	9.990404	9.327475	10.672525	0
		Sine		Tangent	M

Degree 78.

Degree 12.

M	Sine		Tangent		
0	9.317879	9.990404	9.327475	10.672525	60
1	9.318473	9.990377	9.328095	10.671905	59
2	9.319066	9.990351	9.328715	10.671285	58
3	9.319658	9.990324	9.329334	10.670666	57
4	9.320250	9.990297	9.329953	10.670047	56
5	9.320840	9.990270	9.330570	10.669430	55
6	9.321430	9.990242	9.331187	10.668813	54
7	9.322019	9.990215	9.331803	10.668197	53
8	9.322607	9.990188	9.332418	10.667582	52
9	9.323194	9.990161	9.333033	10.666967	51
10	9.323780	9.990134	9.333646	10.666354	50
11	9.324366	9.990107	9.334259	10.665741	49
12	9.324950	9.990079	9.334871	10.665129	48
13	9.325534	9.990052	9.335482	10.664518	47
14	9.326117	9.990025	9.336093	10.663907	46
15	9.326699	9.989997	9.336700	10.663298	45
16	9.327281	9.989970	9.337311	10.662689	44
17	9.327862	9.989942	9.337919	10.662081	43
18	9.328441	9.989915	9.338527	10.661473	42
19	9.329020	9.989887	9.339133	10.660867	41
20	9.329599	9.989860	9.339739	10.660261	40
21	9.330176	9.989832	9.340344	10.659656	39
22	9.330753	9.989804	9.340948	10.659052	38
23	9.331328	9.989777	9.341552	10.658448	37
24	9.331903	9.989749	9.342155	10.657845	36
25	9.332478	9.989721	9.342757	10.657243	35
26	9.333051	9.989693	9.343358	10.656642	34
27	9.333624	9.989665	9.343958	10.656042	33
28	9.334195	9.989637	9.344558	10.655442	32
29	9.334766	9.989609	9.345157	10.654843	31
30	9.335337	9.989581	9.345755	10.654245	30
		Sine		Tangent	M

Degree 77.

Degree 12.

M	Sine		Tangent		
30	9.335337	9.989581	9.345755	10.654245	30
31	9.335906	9.989553	9.346353	10.653647	29
32	9.336475	9.989525	9.346949	10.653051	28
33	9.337043	9.989497	9.347545	10.652455	27
34	9.337610	9.989469	9.348141	10.651859	26
35	9.338176	9.989441	9.348735	10.651265	25
36	9.338742	9.989413	9.349329	10.650671	24
37	9.339306	9.989384	9.349922	10.650078	23
38	9.339870	9.989356	9.350514	10.649486	22
39	9.340434	9.989328	9.351106	10.648894	21
40	9.340996	9.989299	9.351697	10.648303	20
41	9.341558	9.989271	9.352287	10.647713	19
42	9.342119	9.989243	9.352876	10.647124	18
43	9.342679	9.989214	9.353465	10.646535	17
44	9.343239	9.989186	9.354053	10.645947	16
45	9.343797	9.989157	9.354640	10.645360	15
46	9.344355	9.989128	9.355227	10.644773	14
47	9.344912	9.989100	9.355812	10.644187	13
48	9.345469	9.989071	9.356398	10.643602	12
49	9.346024	9.989042	9.356982	10.643018	11
50	9.346579	9.989014	9.357566	10.642434	10
51	9.347134	9.988985	9.358149	10.641851	9
52	9.347687	9.988956	9.358731	10.641269	8
53	9.348240	9.988927	9.359313	10.640687	7
54	9.348792	9.988898	9.359893	10.640107	6
55	9.349343	9.988869	9.360474	10.639526	5
56	9.349893	9.988840	9.361053	10.638947	4
57	9.350443	9.988811	9.361632	10.638368	3
58	9.350992	9.988782	9.362210	10.637790	2
59	9.351540	9.988754	9.362787	10.637213	1
60	9.352088	9.988724	9.363364	10.636636	0
		Sine		Tangent	M

Degree 77.

Degree 13.

M	Sine		Tangent		
0	9.352088	9.988724	9.363364	10.636636	60
1	9.352635	9.988695	9.363940	10.636060	59
2	9.353181	9.988666	9.364515	10.635485	58
3	9.353726	9.988636	9.365090	10.634910	57
4	9.354271	9.988607	9.365664	10.634336	56
5	9.354815	9.988578	9.366237	10.633763	55
6	9.355358	9.988548	9.366810	10.633190	54
7	9.355901	9.988519	9.367382	10.632618	53
8	9.356443	9.988489	9.367953	10.632047	52
9	9.356984	9.988460	9.368524	10.631476	51
10	9.357524	9.988430	9.369094	10.630906	50
11	9.358064	9.988401	9.369663	10.630337	49
12	9.358603	9.988371	9.370232	10.629768	48
13	9.359141	9.988341	9.370799	10.629201	47
14	9.359679	9.988312	9.371367	10.628633	46
15	9.360215	9.988282	9.371933	10.628067	45
16	9.360752	9.988252	9.372499	10.627501	44
17	9.361287	9.988223	9.373064	10.626936	43
18	9.361822	9.988193	9.373629	10.626371	42
19	9.362356	9.988163	9.374193	10.625807	41
20	9.362889	9.988133	9.374756	10.625244	40
21	9.363422	9.988103	9.375319	10.624681	39
22	9.363954	9.988073	9.375881	10.624119	38
23	9.364485	9.988043	9.376442	10.623558	37
24	9.365016	9.988013	9.377003	10.622997	36
25	9.365546	9.987983	9.377563	10.622437	35
26	9.366075	9.987953	9.378122	10.621878	34
27	9.366604	9.987922	9.378681	10.621319	33
28	9.367132	9.987892	9.379239	10.620761	32
29	9.367659	9.987862	9.379797	10.620203	31
30	9.368185	9.987832	9.380354	10.619646	30
		Sine		Tangent	M

Degree 76.

Degree 13.

M	Sine		Tangent		
30	9.368185	9.987832	9.380354	10.619646	30
31	9.368711	9.987801	9.380910	10.619090	29
32	9.369236	9.987771	9.381466	10.618534	28
33	9.369761	9.987740	9.382021	10.617980	27
34	9.370285	9.987710	9.382575	10.617425	26
35	9.370808	9.987679	9.383129	10.616871	25
36	9.371330	9.987649	9.383682	10.616318	24
37	9.371852	9.987618	9.384234	10.615766	23
38	9.372373	9.987588	9.384786	10.615214	22
39	9.372894	9.987557	9.385337	10.614663	21
40	9.373414	9.987526	9.385888	10.614112	20
41	9.373933	9.987496	9.386438	10.613562	19
42	9.374452	9.987465	9.386987	10,613013	18
43	9.374970	9.987434	9.387536	10.612464	17
44	9.375487	9.987403	9.388084	10.611916	16
45	9.376003	9.987372	9.388631	10.611369	15
46	9.376519	9.987341	8.389178	10.610822	14
47	9.377035	9.987310	9.389724	10.610276	13
48	9.377549	9.987279	9.390270	10.609730	12
49	9.378063	9.987248	9 390815	10.609185	11
50	9.378577	9.987217	9.391360	10.608640	10
51	9.379089	9.987186	9.391907	10.608097	9
52	9.379601	8.987155	9.392467	10.607553	8
53	9.380113	9.987124	9.392989	10.607011	7
54	9.380624	9.987092	9.393531	10.606469	6
55	9.381134	9.987061	9.394074	10.605927	5
56	9.381643	9.987030	9.394614	10.605386	4
57	9.382152	9.986998	9.395154	10.604846	3
58	9.382661	9.986967	9.395694	10.604306	2
59	9.383168	9.986936	9.396233	10.603767	1
60	9.383675	9.986904	9.396770	10.603229	0
		Sine		Tangent	M

Degree 76.

Degree 14.

M	Sine		Tangent		
0	9.383675	9.986904	9.396771	10.603229	60
1	9.384181	9.986873	9.397309	10.602694	59
2	9.384687	9.986841	9.397846	10.602154	58
3	9.385192	9.986809	9.398383	10.601617	57
4	9.385697	9.986778	9.398919	10.601081	56
5	9.386201	9.986746	9.399455	10.600545	55
6	9.386704	9.986714	9.399990	10.600010	54
7	9.387207	9.986683	9.400524	10.599476	53
8	9.387709	9.986651	9.401058	10.598942	52
9	9.388210	9.986619	9.401591	10.598409	51
10	9.388711	9.986587	9.402124	10.597876	50
11	9.389211	9.986555	9.402656	10.597344	49
12	9.389711	9.986523	9.403187	10.596813	48
13	9.390210	9.986491	9.403718	10.596282	47
14	9.390708	9.986459	9.404249	10.595751	46
15	9.391206	9.986427	9.404778	10.595222	45
16	9.391703	9.986395	9.405306	10.594692	44
17	9.392199	9.986363	9.405836	10.594164	43
18	9.392695	9.986331	9.406364	10.593636	42
19	9.393190	9.986299	9.406892	10.593608	41
20	9.393685	9.986266	9.407419	10.592581	40
21	9.394179	9.986234	9.407945	10.592055	39
22	9.394673	9.986201	9.408471	10.591529	38
23	9.395166	9.986169	9.408996	10.591001	37
24	9.395654	9.986137	9.409521	10.590479	36
25	9.396150	9.986104	9.410045	10.589954	35
26	9.396641	9.986072	9.410569	10.589431	34
27	9.397131	9.986039	9.411097	10.588908	33
28	9.397621	9.986007	9.411615	10.588385	32
29	9.398111	9.985974	9.412137	10.587863	31
30	9.398600	9.985942	9.412658	10.587342	30
		Sine		Tangent	M

Degree 75.

Degree 14.

M	Sine		Tangent		M
30	9.398600	9.985942	9.412658	10.587342	30
31	9.399087	9.985909	9.413179	10.586821	29
32	9.399575	9.985876	9.413699	10.586301	28
33	9.400062	9.985843	9.414219	10.585781	27
34	9.400549	9.985811	9.414738	10.585262	26
35	9.401035	9.985778	9.415257	10.584742	25
36	9.401520	9.985745	9.415775	10.584225	24
37	9.402005	9.985712	9.416293	10.583707	23
38	9.402489	9.985679	9.416810	10.583190	22
39	9.402972	9.985646	9.417326	10.582674	21
40	9.403455	9.985613	9.417842	10.582157	20
41	9.403938	9.985580	9.418357	10.581642	19
42	9.404420	9.985547	9.418873	10.581127	18
43	9.404901	9.985513	9.419387	10.580613	17
44	9.405382	9.985480	9.419901	10.580099	16
45	9.405862	9.985447	9.420415	10.579585	15
46	9.406341	9.985414	9.420927	10.579072	14
47	9.406820	9.985380	9.421440	10.578560	13
48	9.407299	9.985347	9.421951	10.578048	12
49	9.407776	9.985314	9.422463	10.577537	11
50	9.408254	9.985280	9.422973	10.577026	10
51	9.408731	9.985247	9.423484	10.576516	9
52	9.409207	9.985213	9.423993	10.576007	8
53	9.409682	9.985180	9.424503	10.575497	7
54	9.410157	9.985146	9.425011	10.574989	6
55	9.410632	9.985112	9.425518	10.574480	5
56	9.411106	9.985079	9.426027	10.573973	4
57	9.411579	9.985045	9.426534	10.573466	3
58	9.412052	9.985011	9.427041	10.572959	2
59	9.412524	9.984977	9.427547	10.572453	1
60	9.412996	9.984943	9.428052	10.571947	0
		Sine		Tangent	M

Degree 75.

Degree 15.

M	Sine		Tangent		
0	9.412956	9.984944	9.428052	10.571947	60
1	9.413467	9.984910	9.428557	10.571442	59
2	9.413938	9.984876	9.429067	10.570938	58
3	9.414408	9.984842	9.429566	10.570434	57
4	9.414878	9.984808	9.430070	10.569930	56
5	9.415347	9.984774	9.430573	10.569427	55
6	9.415815	9.984740	9.431075	10.568925	54
7	9.416283	9.984706	9.431577	10.568423	53
8	9.416850	9.984672	9.432079	10.567921	52
9	9.417217	9.984637	9.432580	10.567420	51
10	9.417684	9.984603	9.433080	10.566920	50
11	9.418149	9.984569	9.433580	10.566419	49
12	9.418615	9.984535	9.434080	10.565920	48
13	9.419079	9.984500	9.434579	10.565421	47
14	9.419544	9.984466	9.435078	10.564922	46
15	9.420007	9.984431	9.435576	10.564424	45
16	9.420470	9.984397	9.436073	10.563927	44
17	9.420933	9.984363	9.436570	10.563430	43
18	9.421395	9.984328	9.437067	10.562933	42
19	9.421856	9.984293	9.437563	10.562437	41
20	9.422317	9.984259	9.438059	10.561941	40
21	9.422778	9.984224	9.438554	10.561446	39
22	9.423238	9.984189	9.439548	10.560952	38
23	9.423697	9.984155	9.439543	10.560457	37
24	9.424156	9.984120	9.440036	10.559964	36
25	9.424615	9.984085	9.440529	10.559471	35
26	9.425072	9.984050	9.441022	10.558978	34
27	9.425530	9.984015	9.441514	10.558486	33
28	9.425987	9.983980	9.442006	10.557994	32
29	9.426443	9.983945	9.442497	10.557503	31
30	9.426899	9.983910	9.442988	10.557011	30
		Sine		Tangent	M

Degree 74.

Degree 15.

M	Sine		Tangent		
30	9.426899	9.983910	9.442988	10.557011	30
31	9.427354	9.983875	9.443479	10.556521	29
32	9.427809	9.983840	9.443968	10.556031	28
33	9.428264	9.983805	9.444458	10.555542	27
34	9.428717	9.983770	9.444947	10.555035	26
35	9.429170	9.983735	9.445435	10.554565	25
36	9.429623	9.983699	9.445923	10.554077	24
37	9.430075	9.983664	9.446411	10.553589	23
38	9.430507	9.983629	9.446898	10.553102	22
39	9.430978	9.983593	9.447384	10.552616	21
40	9.431429	9.983558	9.447870	10.552129	20
41	9.431879	9.983523	9.448356	10.551644	19
42	9.432328	9.983487	9.448841	10.551159	18
43	9.432778	9.983452	9.449326	10.550674	17
44	9.433206	9.983416	9.449810	10.550181	16
45	9.433674	9.983380	9.450294	10.549706	15
46	9.434122	9.983345	9.450777	10.549223	14
47	9.434569	9.983309	9.451260	10.548740	13
48	9.435016	9.983273	9.451743	10.548257	12
49	9.435462	9.983238	9.452225	10.547775	11
50	9.435918	9.983202	9.452706	10.547294	10
51	9.436353	9.983166	9.453187	10.546813	9
52	9.436798	9.983130	9.453668	10.546332	8
53	9.437242	9.983094	9.454148	10.545852	7
54	9.437686	9.983058	9.454629	10.545372	6
55	9.438129	9.983022	9.455107	10.544893	5
56	9.438572	9.982986	9.455586	10.544414	4
57	9.439014	9.982950	9.456064	10.543936	3
58	9.439456	9.982914	9.456542	10.543458	2
59	9.439897	9.982878	9.457019	10.542980	1
60	9.440338	9.982842	9.457496	10.542503	0
		Sine		Tangent	M

Degree 74.

Degree 16.

M	Sine		Tangent		M
0	9.440338	9.982842	9.457496	10.542503	60
1	9.440778	9.982805	9.457973	10.542027	59
2	9.441218	9.982769	9.458449	10.541551	58
3	9.441658	9.982733	9.458925	10.541075	57
4	9.442096	9.982696	9.459400	10.540600	56
5	9.442535	9.982660	9.459875	10.540125	55
6	9.442973	9.982623	9.460349	10.539651	54
7	9.443416	9.982587	9.460829	10.539177	53
8	9.443848	9.982550	9.461297	10.538703	52
9	9.444284	9.982514	9.461770	10.538230	51
10	9.444720	9.982477	9.462242	10.537758	50
11	9.445155	9.982441	9.462714	10.537285	49
12	9.445590	9.982404	9.463186	10.536814	48
13	9.446025	9.982367	9.463658	10.536342	47
14	9.446459	9.982330	4.464129	10.535871	46
15	9.446893	9.982294	9.464599	10.535401	45
16	9.447326	9.982257	9.465069	10.534931	44
17	9.447759	9.982220	9.465539	10.534461	43
18	9.448191	9.982183	9.466008	10.533992	42
19	9.448623	9.982146	9.466476	10.533523	41
20	9.449054	9.982109	9.466945	10.533055	40
21	9.449485	9.982072	9.467413	10.532587	39
22	9.449915	9.982035	9.467880	10.532120	38
23	9.450345	9.981998	9.468347	10.531653	37
24	9.450775	9.981961	9.468814	16.531186	36
25	9.451203	9.981923	9.469280	10.530720	35
26	9.451632	9.981886	9.469746	10.530254	34
27	9.452060	9.981849	9.470211	10.529789	33
28	9.452488	9.981812	9.470676	10.529324	32
29	9.452915	9.981774	9.471141	10.528859	31
30	9.453342	9.981737	9.471605	10.528295	30
		Sine		Tangent	M

Degree 73.

Degree 16.

M	Sine		Tangent		
30	9.453342	9.981737	9.471605	10.528395	30
31	9.253768	9.981699	9.472068	10.527931	29
32	9.254194	9.981662	9.472532	10.527468	28
33	9.454619	9.981624	9.472995	10.527005	27
34	9.455044	9.981587	9.473457	10.526543	26
35	9.455469	9.981549	9.473919	10.526081	25
36	9.455892	9.981512	9.474381	10.525619	24
37	9.456316	9.981474	9.474842	10.525158	23
38	9.456739	9.981436	9.475303	10.524695	22
39	9.457162	9.981398	9.475763	19.524237	21
40	9.457584	9.981361	9.476223	10.523777	20
41	9.458006	9.981323	9.476683	10.523317	19
42	9.458427	9.981285	9.477142	10.522858	18
43	9.458848	9.981247	9.477601	10.522399	17
44	9.459268	9.981209	9.478059	10.521941	16
45	9.459684	9.981171	9.478517	10.521483	15
46	9.460108	9.781133	9.478975	10.521025	14
47	9.460527	9.981095	9.479432	10.520168	13
48	9.460946	9.981057	9.479886	10.520111	12
49	9.461364	9.981019	9.480345	10.519655	11
50	9.461782	9.980980	9.480801	10.519199	10
51	9.462199	9.980942	9.481257	10.518743	9
52	9.462616	9.980904	9.481712	10.518288	8
53	9.463032	9.980866	9.482167	10.517833	7
54	9.463448	9.980827	9.482621	10.517379	6
55	9.463864	9.980789	9.483075	10.516925	5
56	9.464279	9.980750	9.483528	10.516471	4
57	9.464694	9.980712	9.483982	10.516018	3
58	9.465108	9.980672	9.484434	10.515585	2
59	9.465522	9.980635	9.484887	10.515113	1
60	9.465935	9.980596	9.485339	10.514661	0
		Sine		Tangent	M

Degree 73.

Degree 17.

M	Sine		Tangent		
0	9.465935	9.980596	9.485339	10.514661	60
1	9.466348	9.980558	9.485791	10.514209	59
2	9.466761	9.980519	9.486272	10.513758	58
3	9.467173	9.980480	9.486693	10.513307	57
4	9.467585	9.980441	9.487143	10.512857	56
5	9.467996	9.980403	9.487593	10.512407	55
6	9.468407	9.980364	9.488043	10.511957	54
7	9.468817	9.980325	9.488493	10.511507	53
8	9.469227	9.980286	9.488944	10.511059	52
9	9.469637	9.980247	9.489390	10.510610	51
10	9.460446	9.980208	9.489838	10.510162	50
11	9.470455	9.990169	9.490286	10.509714	49
12	9.471863	9.980130	9.490733	10.509267	48
13	9.471071	9.980091	9.491180	10.508820	47
14	9.471678	9.980052	9.491627	10.508373	46
15	9.472086	9.980012	9.492073	10.507928	45
16	9.472492	9.979573	9.492519	10.507481	44
17	9.472898	9.979934	9.492964	10.507035	43
18	9.473304	9.979894	9.493410	10.506590	42
19	9.473710	9.979855	9.493854	10.506145	41
20	9.474115	9.979816	9.494299	10.505701	40
21	9.474519	9.979776	9.494743	10.505257	39
22	9.474923	9.979737	9.495186	10.504813	38
23	9.475327	9.979697	9.495630	10.504370	37
24	9.475730	9.979658	9.496073	10.503928	36
25	9.476133	9.979618	9.496515	10.503485	35
26	9.476539	9.979578	9.496957	10.503043	34
27	9.476938	9.979539	9.497399	10.502601	33
28	9.477340	9.979499	9.497840	10.502160	32
29	9.477741	9.979459	9.498282	10.510718	31
30	9.478142	9.979419	9.498722	10.501278	30
		Sine		Tangent	M

Degree 72.

Degree 17,

M	Sine		Tangent		
30	9.478142	9.979419	9.498722	10.501278	30
31	9.478542	9.979380	9.499163	10.500837	29
32	9.478942	9.979340	9.499602	10.500398	28
33	9.479342	9.979300	9.500042	10.499958	27
34	9.479741	9.979260	9.500481	10.499519	26
35	9.480140	9.979220	9.500920	10.499080	25
36	9.480538	9.979180	9.501359	10.498641	24
37	9.480936	9.979140	9.501797	10.498203	23
38	9.481334	9.979099	9.502234	10.497765	22
39	9.481731	9.979059	9.502672	10.497328	21
40	9.482128	9.979019	9.503109	10.496891	20
41	9.482525	9.978980	9.503546	10.456454	19
42	9.482921	9.978939	9.503982	10.496018	18
43	9.483316	9.978898	9.504418	10.495582	17
44	9.483711	9.978858	9.504854	10.495146	16
45	9.484106	9.978817	9.505289	10.494711	15
46	9.484501	9.978777	9.505724	10.494216	14
47	9.484895	9.978736	9.506158	10.493841	13
48	9.485289	9.978696	9.506593	10.493407	12
49	9.485682	9.978655	9.507026	10.492973	11
50	9.486075	9.978615	9.507459	10.492540	10
51	9.486467	9.978574	9.507892	10.492107	9
52	9.486859	9.978533	9.508326	10.491674	8
53	9.487251	9.978493	9.508759	10.491241	7
54	9.487642	9.978452	9.509181	10.490809	6
55	9.488033	9.978411	9.509622	10.490377	5
56	9.488424	9.978370	9.510044	10.489916	4
57	9.488814	9.978329	9.510486	10.489515	3
58	9.489204	9.978288	9.510916	10.489084	2
59	9.489593	9.978247	9.511346	10.488654	1
60	9.489982	9.978206	9.511776	10.488225	0
		Sine		Tangent	M

Degree 72.

Degree 18.

M	Sine		Tangent		
0	9.489982	9.978206	9.511776	10.488224	60
1	9.490371	9.978165	9.512206	10.487794	59
2	9.490759	9.978124	9.512635	10.487365	58
3	9.491147	9.978083	9.513064	10.486936	57
4	9.491534	9.978042	9.513493	10.486507	56
5	9.491922	9.978000	9.513921	10.486079	55
6	9.492308	9.977959	9.514349	10.485651	54
7	9.492695	9.977918	9.514777	10.485223	53
8	9.493080	9.977877	9.515204	10.484756	52
9	9.493466	9.977835	9.515631	10.484369	51
10	9.493851	9.977794	9.516057	10.483942	50
11	9.494236	9.977752	9.516484	10.483516	49
12	9.494620	9.977711	9.516910	10.483090	48
13	9.495005	9.977669	9.517335	19.482665	47
14	9.495388	9.977628	9.517761	10.482239	46
15	9.495771	9.977586	9.518185	10.481814	45
16	9.456154	9.977544	9.518610	10.481390	44
17	9.496537	9.977503	9.519034	10.480966	43
18	9.496919	9.977461	9.519458	10.480542	42
19	9.497301	9.977419	9.519882	10.480118	41
20	9.497682	9.977377	9.520305	10.475695	40
21	9.498063	9.977335	9.520728	10.479272	39
22	9.498444	9.977293	9.521151	10.478849	38
23	9.498824	9.977251	9.521573	10.478427	37
24	9.499204	9.977209	9.521995	10.478005	36
25	9.499584	9.977167	9.522417	10.477583	35
26	9.499963	9.977125	9.522838	10.477162	34
27	9.500342	9.977083	9.513259	10.476741	33
28	9.500720	9.977041	9.523679	10.476320	32
29	9.501099	9.976999	9.524109	10.475900	31
30	9.501476	9.976956	9.524520	10.475480	30
		Sine		Tangent	M

Degree 71.

Degree 18.

M	Sine		Tangent		M
30	9.501478	9.976956	9.524520	10.475480	30
31	9.501854	9.976914	9.524939	10.475060	29
32	9.502231	9.976872	9.525359	10.474641	28
33	9.502607	9.976830	9.525778	10.474222	27
34	9.502984	9.976787	9.526197	10.473803	26
35	9.503360	9.976745	9.526615	10.473385	25
36	9.503735	9.976702	9.527033	10.472967	24
37	9.504110	9.976660	9.527451	10.472549	23
38	9.504485	9.976617	9.527868	10.472132	22
39	9.504840	9.976574	9.528285	10.471715	21
40	9.505234	9.976532	9.528702	10.471298	20
41	9.505608	9.976489	9.529118	10.470881	19
42	9.505981	9.976446	9.529535	10.470465	18
43	9.506354	9.976404	9.529950	10.470049	17
44	9.506727	9.976361	9.530366	10.469634	16
45	9.507099	9.976318	9.530781	10.469219	15
46	9.507471	9.976275	9.531196	10.468804	14
47	9.507843	9.976232	9.531611	10.468389	13
48	9.508214	9.976185	9.532025	10.467975	12
49	9.508585	9.976146	9.532436	10.467561	11
50	9.508955	9.976103	9.532852	10.467147	10
51	9.509326	9.976060	9.533266	10.466734	9
52	9.509696	9.976017	9.533679	10.466321	8
53	9.510065	9.975973	9.534092	10.465908	7
54	9.510434	9.975930	9.534504	10.465456	6
55	9.510803	9.975887	9.534916	10.465084	5
56	9.511171	9.975844	9.535328	10.464672	4
57	9.511540	9.975800	9.535739	10.464261	3
58	9.511907	9.975757	9.536150	10.463849	2
59	9.512275	9.975713	9.536561	10.463439	1
60	9.512642	9.975670	9.536972	10.463028	0
		Sine		Tangent	M

Degree 71.

Degree 19.

M	Sine		Tangent		
0	9.512642	9.975670	9.536972	10.463028	60
1	9.513009	9.975626	9.537382	10.462618	59
2	9.513375	9.975583	9.537792	10.462208	58
3	9.513741	9.975539	9.538202	10.461798	57
4	9.514107	9.975496	9.538610	10.461389	56
5	9.514472	9.975452	9.539020	19.460980	55
6	9.514837	9.975408	9.539429	10.460571	54
7	9.515202	9.975364	9.539837	10.460163	53
8	9.515566	9.975321	9.540245	10.459755	52
9	9.515930	9.975277	9.540653	10.459347	51
10	9.516294	9.975233	9.541061	10.458939	50
11	9.516657	9.975189	9.541468	10.458532	49
12	9.517020	9.975145	9.541875	10.458125	48
13	9.517382	9.975101	9.542281	10.457719	47
14	9.517745	9.975057	9.542688	10.457312	46
15	9.518107	9.975013	9.543094	10.456906	45
16	9.518468	9.974969	9.543499	10.456501	44
17	9.518829	9.974925	9.543905	10.456095	43
18	9.519190	9.974880	9.544310	10.455690	42
19	9.519551	9.974836	9.544715	10.455285	41
20	9.519911	9.974792	9.545119	10.454881	40
21	9.520271	9.974747	9.545524	10.454476	39
22	9.520631	9.974703	9.545927	10.454072	38
23	9.520990	9.974659	9.546331	10.453669	37
24	9.521349	9.974614	9.546735	10.453265	36
25	9.521707	9.974570	9.547138	10.452862	35
26	9.522065	9.974525	9.547540	10.452459	34
27	9.522423	9.974480	9.547943	10.452057	33
28	9.522781	9.974436	9.548345	10.451655	32
29	9.523138	9.974391	9.548747	10.451253	31
30	9.523495	9.974346	9.549149	10.450851	30
		Sine		Tangent	M

Degree 70.

Degree 19.

M	Sine		Tangent		M
30	9.523405	9.974346	9.549149	10.450851	30
31	9.523851	9.974302	9.549550	10.450450	29
32	9.524208	9.974257	9.549951	10.450049	28
33	9.524564	9.974212	9.550352	19.449648	27
34	9.524920	9.974167	9.550752	10.449248	26
35	9.525275	9.974122	9.551152	10.448848	25
36	9.525630	9.974077	9.551552	10.448448	24
37	9.525984	9.974032	9.551952	10.448048	23
38	9.526339	9.973987	9.552351	10.447649	22
39	9.526693	9.973942	9.552750	10.447250	21
40	9.527046	9.973897	9.553149	10.446851	20
41	9.527400	9.973852	9.553548	10.446452	19
42	9.527753	9.973807	9.553946	10.446054	18
43	9.528105	9.973761	9.554344	10.445656	17
44	9.528458	9.973716	9.554741	10.445259	16
45	9.528810	9.973671	9.555139	10.444861	15
46	9.529161	9.973615	9.555536	10.444464	14
47	9.529513	9.973580	9.555932	10.444068	13
48	9.529864	9.973535	9.556329	10.443671	12
49	9.530214	9.973489	9.556727	10.443275	11
50	9.530565	9.973443	9.557121	10.442879	10
51	9.530915	9.973398	9.557517	10.442483	9
52	9.531265	9.973352	9.557912	10.442088	8
53	9.531614	9.973307	9.558308	10.441693	7
54	9.531963	9.973261	9.558702	10.441298	6
55	9.532312	9.973215	9.559097	10.440903	5
56	9.532661	9.973169	9.559491	10.440509	4
57	9.533009	9.973123	9.559885	10.440115	3
58	9.533357	9.973078	9.560279	10.439721	2
59	9.533704	9.973032	9.560673	10.439327	1
60	9.534052	9.972986	9.561066	10.438934	0
		Sine		Tangent	M

Degree 70.

Degree 20.

M	Sine		Tangent		
0	9.534052	9.972986	9.561066	10.438934	60
1	9.534399	9.972940	9.561459	10.438541	59
2	9.534746	9.972894	9.561851	10.438148	58
3	9.535091	9.972848	9.562244	10.437756	57
4	9.535437	9.972801	9.562636	10.437364	56
5	9.535782	9.972755	9.563028	10.436972	55
6	9.436129	9.972709	9.563419	10.436580	54
7	9.536474	9.972663	9.563811	10.436189	53
8	9.536818	9.972617	9.564202	10.435798	52
9	9.537163	9.972570	9.564592	10.435407	51
10	9.537507	9.972524	9.564983	10.435017	50
11	9.537851	9.972477	9.565373	10.434627	49
12	9.538194	9.972431	9.565763	10.434237	48
13	9.538537	9.972384	9.566153	10.433847	47
14	9.538880	9.972338	9.566542	10.433457	46
15	9.539222	9.972291	9.566932	10.433068	45
16	9.539565	9.972245	9.567320	10.432679	44
17	9.539907	9.972198	9.567709	10.432291	43
18	9.540249	9.972151	9.568097	10.431902	42
19	9.540590	9.972105	9.568486	10.431514	41
20	9.540931	9.972058	9.568873	10.431126	40
21	9.541272	9.972011	9.569261	10.430739	39
22	9.541612	9.971564	9.569648	10.430351	38
23	9.541953	9.971917	9.570035	10.429964	37
24	9.542292	9.971870	9.570422	10.429578	36
25	9.542632	9.971823	9.570809	10.429191	35
26	9.542971	9.971776	9.571195	10.428805	34
27	9.543310	9.971729	9.571581	10.428419	33
28	9.543649	9.971682	9.571967	10.428033	32
29	9.543997	9.971635	9.572352	10.427648	31
30	9.544325	9.971588	9.572738	10.427262	30
		Sine		Tangent	M

Degree 69.

Degree 20.

M	Sine		Tangent		
30	9.544325	9.971588	9.572738	10.427262	30
31	9.544663	9.971540	9.573123	10.426877	29
32	9.545000	9.971493	9.573507	10.426492	28
33	9.545338	9.971446	9.573892	10.426108	27
34	9.545674	9.971398	9.574276	10.425724	26
35	9.546011	9.971351	9.574660	10.425340	25
36	9.546347	9.971303	9.575044	10.424956	24
37	9.546683	9.971256	9.575427	10.424573	23
38	9.547019	9.971208	9.575810	10.424189	22
39	9.547354	9.971161	9.576193	10.423807	21
40	9.547689	9.971112	9.576576	10.423424	20
41	9.548024	9.971065	9.576958	10.423041	19
42	9.548358	9.971018	9.577341	10.422659	18
43	9.548693	9.970970	9.577723	10.422277	17
44	9.549026	9.970922	9.578104	10.421896	16
45	9.549360	9.970874	9.578486	10.421514	15
46	9.549693	9.970826	9.578867	10.421133	14
47	9.550026	9.970779	9.579248	10.420752	13
48	9.550359	9.970731	9.579628	10.420371	12
49	9.550692	9.970683	9.580009	10.419991	11
50	9.551024	9.970634	9.580389	10.419611	10
51	9.551355	9.970586	9.580769	10.419231	9
52	9.551687	9.970538	9.581149	10.418851	8
53	9.552018	9.970490	9.581528	10.418472	7
54	9.552349	9.970442	9.581907	10.418092	6
55	9.552680	9.970394	9.582286	10.417713	5
56	9.553010	9.970345	9.582665	10.417335	4
57	9.553340	9.970297	9.583043	10.416956	3
58	9.553670	9.970249	9.583422	10.416578	2
59	9.554000	9.970200	9.583800	10.416200	1
60	9.554329	9.970152	9.584177	10.415823	0
		Sine		Tangent	M

Degree 69.

Degree 21.

M	Sine		Tangent		
0	9.554329	9.970152	9.584177	10.415822	60
1	9.554658	9.970103	9.584555	10.415445	59
2	9.554987	9.970055	9.584932	10.415068	58
3	9.555315	9.970006	9.585308	10.414691	57
4	9.555643	9.969957	9.585686	10.414314	56
5	9.555971	9.969909	9.586062	10.413938	55
6	9.556299	9.969860	9.586439	10.413561	54
7	9.556626	9.969811	9.586815	10.413185	53
8	9.556953	9.969762	9.587190	10.412800	52
9	9.557279	9.969713	9.587566	10.412434	51
10	9.557606	9.969665	9.587941	10.412059	50
11	9.557932	9.969616	9.588316	10.411684	49
12	9.558258	9.969567	9.588691	10.411309	48
13	9.558583	9.969518	9.589066	10.410934	47
14	9.558909	9.969469	9.589440	10.410560	46
15	9.559234	9.969419	9.589814	10.410185	45
16	9.559558	9.969370	9.590188	10.409812	44
17	9.559883	9.969321	9.590561	10.409438	43
18	9.560207	9.969272	9.590935	10.409065	42
19	9.560531	9.969223	9.591308	10.408692	41
20	9.560855	9.969173	9.591681	10.408319	40
21	9.161178	9.969124	9.592054	10.407946	39
22	9.561501	9.969075	9.592426	10.407574	38
23	9.561824	9.969025	9.592798	10.407201	37
24	9.562146	9.968976	9.593170	10.406829	36
25	9.562468	9.968926	9.693542	10.406457	35
26	9.562790	9.968877	9.593914	10.406086	34
27	9.563112	9.968827	9.594285	10.405715	33
28	9.563433	9.968777	9.594656	10.405344	32
29	9.563754	9.968728	9.595027	10.405073	31
30	9.564075	9.968678	9.595397	10.404602	30
		Sine		Tangent	M

Degree 68.

Degree 21.

M	Sine		Tangent		
30	9.564075	9.968678	9.595397	10.404602	30
31	9.564396	9.968623	9.595768	10.404232	29
32	9.564716	9.968578	9.596138	10.403862	28
33	9.565036	9.968528	9.596508	10.403492	27
34	9.565356	9.968478	9.596878	10.403122	26
35	9.565675	9.968428	9.597247	10.402753	25
36	9.565995	9.968378	9.597616	10.402384	24
37	9.566314	9.968328	9.597985	10.402015	23
38	9.566632	9.968278	9.598354	10.401646	22
39	9.566951	9.968228	9.598722	10.401277	21
40	9.567269	9.968178	9.599091	10.400909	20
41	9.567587	9.968128	9.599459	10.400541	19
42	9.567904	9.968078	9.599827	10.400173	18
43	9.568222	9.968027	9.600194	10.399806	17
44	9.568539	9.967977	9.600562	10.399438	16
45	9.568855	9.967927	9.600929	10.399071	15
46	9.569172	9.967876	9.601296	10.398704	14
47	9.569488	9.967826	9.601662	10.398337	13
48	9.569804	9.967775	9.602029	10.397971	12
49	9.570120	9.967725	9.602395	10.397605	11
50	9.570435	9.967674	9.602761	10.397239	10
51	9.570751	9.967623	9.603127	10.396873	9
52	9.571065	9.967573	9.603493	10.296507	8
53	9.571380	9.967522	9.603858	10.396142	7
54	9.571695	9.967471	9.604223	10.395777	6
55	9.572009	9.967420	9.604588	10.395412	5
56	9.572322	9.967370	9.604953	10.395047	4
57	9.572636	9.967319	9.605317	10.394683	3
58	9.572949	9.967268	9.605681	10.394318	2
59	9.573263	9.967217	9.606046	10.393954	1
60	9.573575	9.967166	9.606409	10.393590	0
		Sine		Tangent	M

Degree 68.

Degree 22.

M	Sine		Tangent		
0	9.573575	9.967166	9.606409	10.393590	60
1	9.573888	9.967115	9.606773	10.393227	59
2	9.574200	9.967064	9.607136	10.392863	58
3	9.574512	9.967012	9.607500	10.392500	57
4	9.574824	9.966961	9.607862	10.392137	56
5	9.575135	9.966910	9.608225	10.391774	55
6	9.575447	9.966859	9.608588	10.391412	54
7	9.575758	9.966807	9.608950	10.391050	53
8	9.576068	9.966756	9.609312	10.390688	52
9	9.576379	9.966705	9.609674	10.390326	51
10	9.576689	9.966653	9.610036	10.389964	50
11	9.576999	9.966602	9.610397	10.389603	49
12	9.577309	9.966550	9.610758	10.389241	48
13	9.577618	9.966499	9.611119	10.388880	47
14	9.577927	9.966447	9.611480	10.388520	46
15	9.578236	9.966395	9.611841	10.388159	45
16	9.578545	9.966344	9.612201	10.387799	44
17	9.578853	9.966292	9.612561	10.387438	43
18	9.579161	9.966240	9.612921	10.387078	42
19	9.579469	9.966188	9.613281	10.386719	41
20	9.579777	9.966136	9.613641	10.386359	40
21	9.580084	9.966084	9.614000	10.386000	39
22	9.580392	9.966032	9.614359	10.385641	38
23	9.580698	9.965980	9.614718	10.385282	37
24	9.581005	9.965928	9.615077	10.384923	36
25	9.581311	9.965876	9.615435	10.384565	35
26	9.581618	9.965824	9.615793	10.384207	34
27	9.581923	9.965772	9.616151	10.383448	33
28	9.582229	9.965720	9.616509	10.383491	32
29	9.582534	9.965668	9.616867	10.383133	31
30	9.582840	9.965615	9.617224	10.382776	30
		Sine		Tangent	M

Degree 67.

Degree 22.

M	Sine		Tangent		M
30	9.582840	9.965615	9.617224	10.382776	30
31	9.583144	9.965563	9.617581	10.382418	29
32	9.583449	9.965511	9.617938	10.382061	28
33	9.583753	9.965458	9.618295	10.381705	27
34	9.584058	9.965406	9.618652	10.381348	26
35	9.584361	9.965353	9.619008	10.380992	25
36	9.584665	9.965301	9.619364	10.380635	24
37	9.584968	9.965248	9.619720	10.380279	23
38	9.585271	9.965195	9.620076	10.379924	22
39	9.585574	9.965143	9.620432	10.379568	21
40	9.585877	9.965090	9.620787	10.379213	20
41	9.586179	9.995037	9.621142	10.378858	19
42	9.586481	9.964984	9.621497	10.378503	18
43	9.586783	9.964931	9.621852	10.378148	17
44	9.587085	9.964978	9.622206	10.377793	16
45	9.587386	9.964825	9.622561	10.377439	15
46	9.587687	9.964772	9.622915	10.377085	14
47	9.587988	9.964719	9.623269	10.376731	13
48	9.598289	9.964666	9.623623	10.376377	12
49	9.588589	9.964613	9.623976	10.376024	11
50	9.588890	9.964560	9.624330	10.375670	10
51	9.589190	9.964507	9.624683	10.375317	9
52	9.589489	9.964454	9.625036	10.374964	8
53	9.589789	9.964400	9.625388	10.374612	7
54	9.590088	9.964347	9.625741	10.374259	6
55	9.590387	9.964294	9.626093	10.373907	5
56	9.590686	9.964240	9.626445	10.373555	4
57	9.590984	9.964187	9.626797	10.373203	3
58	9.591282	9.964133	9.627149	10.372850	2
59	9.591580	9.964080	9.627501	10.372499	1
60	9.591878	9.964026	9.627852	10.372148	0
		Sine		Tangent	M

Degree 67.

Degree 23.

M	Sine		Tangent		
0	9.591878	9.564026	9.627852	10.372148	60
1	9.592175	9.963972	9.628203	10.371797	59
2	9.592473	9.963919	9.628554	10.371446	58
3	9.592770	9.963865	9.628905	10.371095	57
4	9.593067	9.963811	9.629255	10.370744	56
5	9.593363	9.963757	9.629606	10.370394	55
6	9.593659	9.963703	9.629956	10.370044	54
7	9.593955	9.963650	9.630306	10.369694	53
8	9.594251	9.963596	9.630655	10.369344	52
9	9.594547	9.963542	9.631005	10.368995	51
10	9.594842	9.963488	9.631354	10.368645	50
11	9.595137	9.963433	9.631704	10.368296	49
12	9.595432	9.963379	9.632053	10.367947	48
13	9.595727	9.963325	9.632401	10.367598	47
14	9.596021	9.963271	9.632750	10.367250	46
15	9.596315	9.963217	9.633098	10.366901	45
16	9.596610	9.963102	9.633447	10.366553	44
17	9.596903	9.963108	9.633795	20.366205	43
18	9.597196	9.963054	9.634143	10.365857	42
19	9.597490	9.962999	9.634490	10.365510	41
20	9.597783	9.962945	9.634838	10.365162	40
21	9.598075	9.962892	9.635185	10.364815	39
22	9.598368	9.962836	9.635530	10.364468	38
23	9.598660	9.962781	9.635879	10.364121	37
24	9.598952	9.962726	9.636226	10.363774	36
25	9.599244	9.962672	9.636571	10.363428	35
26	9.599536	9.962617	9.636918	10.363081	34
27	9.599827	9.962562	9.637205	10.362735	33
28	9.600118	9.962507	9.637610	10.362389	32
29	9.600409	9.962453	9.637956	10.362044	31
30	9.600700	9.962398	9.638302	10.361698	30
		Sine		Tangent	M

Degree 66.

Degree 23.

M	Sine		Tangent		
30	9.600700	9.962398	9.638302	10.361698	30
31	9.600990	9.962343	9.638647	10.361353	29
32	9.601280	9.962288	9.638992	10.361007	28
33	9.601570	9.962233	9.639337	10.360662	27
34	9.601860	9.962178	9.639682	10.360318	26
35	9.602149	9.962122	9.640027	10.359973	25
36	9.602439	9.962067	9.640371	10.359629	24
37	9.602728	9.962012	9.640716	10.359284	23
38	9.603017	9.961957	9.641060	10.358940	22
39	9.603305	9.961902	9.641404	10.358596	21
40	9.603594	9.961846	9.641747	10.358253	20
41	9.603882	9.961791	9.642091	10.357909	19
42	9.604170	9.961735	9.642434	10.357566	18
43	9.604457	9.961680	9.642777	10.357223	17
44	9.604745	9.961624	9.643120	10.356980	16
45	9.605032	9.961569	9.643463	10.356537	15
46	9.605319	9.961513	9.643806	10.356194	14
47	9.605606	9.961458	9.644148	10.355852	13
48	9.605892	9.961402	9.644490	10.355510	12
49	9.606179	9.961346	9.644832	10.355168	11
50	9.606465	9.961290	9.645174	10.354826	10
51	9.606750	9.961235	9.645516	10.354484	9
52	9.607036	9.961179	9.645857	10.354142	8
53	9.607322	9.961123	9.646199	10.353801	7
54	9.607607	9.961067	9.646540	10.353460	6
55	9.907892	9.961011	9.646881	10.353119	5
56	9.608176	9.960955	9.647222	10.352778	4
57	9.608461	9.960899	9.647562	10.352438	3
58	9.608745	9.960842	9.647903	10.352097	2
59	9.609029	9.960786	9.648243	10.351757	1
60	9.609313	9.960730	9.648583	10.351417	0
		Sine		Tangent	M

Degree 66.

Degree 24.

M	Sine		Tangent		
0	9.609313	9.960730	9.648583	10.351417	60
1	9.609597	9.960674	9.648923	10.351077	59
2	9.609880	9.960617	9.649263	10.350737	58
3	9.610163	9.960561	9.649602	10.350398	57
4	9.610446	9.960505	9.649942	10.350058	56
5	9.610729	9.960448	9.650281	10.349719	55
6	9.611012	9.960392	9.650620	10.349380	54
7	9.611294	9.960335	9.650959	10.349041	53
8	9.611576	9.960279	9.651297	10.348703	52
9	9.611858	9.960222	9.651636	10.348364	51
10	9.612140	9.960165	9.651974	10.348026	50
11	9.612421	9.960109	9.652312	10.347688	49
12	9.612702	9.960052	9.652650	10.347350	48
13	9.612983	9.959995	9.652988	10.347012	47
14	9.613264	9.959938	9.653326	10.346674	46
15	9.613545	9.959881	9.653663	10.346337	45
16	9.613825	9.959824	9.654000	10.345999	44
17	9.614105	9.959768	9.654337	10.345662	43
18	9.614385	9.959710	9.654674	10.345325	42
19	9.614665	9.959653	9.655011	10.344989	41
20	9.614944	9.959596	9.655348	10.344652	40
21	9.615223	9.959539	9.655684	10.344316	39
22	9.615502	9.959482	9.656020	10.343980	38
23	9.615781	9.959425	9.656356	10.343643	37
24	9.616060	9.959367	9.656692	10.343308	36
25	9.616338	9.959310	9.657028	10.342972	35
26	9.616616	9.959253	9.657363	10.342636	34
27	9.616894	9.959195	9.657699	10.342301	33
28	9.617172	9.959138	9.658034	10.341966	32
29	9.617450	9.959080	9.658369	10.341631	31
30	9.617727	9.959023	9.658704	10.341296	30
		Sine		Tangent	M

Degree 65.

Degree 24.

M	Sine		Tangent		
30	9.617727	9.959023	9.658704	10.341296	30
31	9.618004	9.958965	9.659039	10.340926	29
32	9.618281	9.958908	9.659373	10.340627	28
33	9.618558	9.958850	9.659708	10.340292	27
34	9.618834	9.958792	9.660042	10.339958	26
35	9.619110	9.958734	9.660376	10.339624	25
36	9.619386	9.958677	9.660710	10.339290	24
37	9.619662	9.958619	9.661043	10.338957	23
38	9.619938	9.958561	9.661377	10.338623	22
39	9.620213	9.958503	9.661710	10.338290	21
40	9.620488	9.958445	9.662043	10.337956	20
41	9.620763	9.958387	9.662376	10.337623	19
42	9.621038	9.958329	9.662709	10.337291	18
43	9.621313	9.958271	9.663042	10.336958	17
44	9.621587	9.958212	9.663374	10.336625	16
45	9.621861	9.958154	9.663707	10.336293	15
46	9.622135	9.958096	9.664039	10.335961	14
47	9.622409	9.958038	9.664371	10.335629	13
48	9.622682	9.957979	9.664703	10.335297	12
49	9.622956	9.957921	9.665035	10.334965	11
50	9.623219	9.957862	9.665366	10.334624	10
51	9.623502	9.957804	9.665697	10.334302	9
52	9.623776	9.957745	9.666029	10.333971	8
53	9.624047	9.957687	9.666360	10.333640	7
54	9.624319	9.957628	9.666691	10.333309	6
55	9.624591	9.957570	9.667021	10.332979	5
56	9.624863	9.957511	9.667352	10.332648	4
57	9.625134	9.957452	9.667682	10.332318	3
58	9.625406	9.957393	9.668012	10.331987	2
59	9.625677	9.957334	9.668343	10.331657	1
60	9.625948	9.957276	9.668672	10.331327	0
M		Sine		Tangent	M

Degree 65.

Degree. 25.

M	Sine		Tangent		M
0	9.625948	9.957276	9.668672	10.331327	60
1	9.626219	9.957217	9.669002	10.330998	59
2	9.626490	9.957158	9.669332	10.330668	58
3	9.626760	9.957099	9.669661	10.330339	57
4	9.627030	9.957040	9.669990	10.330009	56
5	9.627300	9.956981	9.670320	10.329680	55
6	9.627570	9.956922	9.670649	10.329351	54
7	9.627840	9.956862	9.670977	10.329022	53
8	9.628109	9.956803	9.671306	10.328694	52
9	9.628378	9.956744	9.671634	10.328365	51
10	9.628647	9.956684	9.671963	10.328037	50
11	9.628916	9.956625	9.672291	10.327709	49
12	9.629184	9.956565	9.672619	10.327381	48
13	9.629453	9.956208	9.672947	10.327053	47
14	9.629721	9.956446	9.673274	10.326725	46
15	9.629989	9.956387	9.673603	10.326398	45
16	9.630257	9.956327	9.673929	10.326070	44
17	9.630524	9.956267	9.674256	10.325743	43
18	9.630792	9.956208	9.674584	10.325416	42
19	9.631059	9.956148	9.674910	10.325089	41
20	9.631326	9.956088	9.675237	10.324763	40
21	9.631592	9.956029	9.675564	10.324436	39
22	9.631859	9.955969	9.675890	10.324110	38
23	9.632125	9.955909	9.676216	10.323783	37
24	9.632392	9.955849	9.676543	10.323457	36
25	9.632657	9.955789	9.676869	10.323131	35
26	9.632923	9.955739	9.677194	10.322805	34
27	9.633189	9.955669	9.677520	10.322480	33
28	9.633454	9.955609	9.677845	10.322154	32
29	9.633719	9.955548	9.678171	10.321829	31
30	9.633984	9.955488	9.678496	10.321504	30
		Sine		Tangent	M

Degree 64.

Degree 25.

M	Sine		Tangent		M
30	9.633984	9.955488	9.678496	10.321504	30
31	9.634249	9.955428	9.678821	10.321179	29
32	9.634514	9.955367	9.679146	10.320854	28
33	9.634778	9.955307	9.679471	10.320529	27
34	9.635042	9.955246	9.679795	10.320205	26
35	9.635306	9.955186	9.680120	10.319880	25
36	9.635570	9.955125	9.680444	10.319556	24
37	9.635833	9.955065	9.680768	10.319232	23
38	9.636097	9.955004	9.681092	10.318908	22
39	9.636360	9.954944	9.681416	10.318584	21
40	9.636623	9.954883	9.681740	10.318260	20
41	9.636886	9.954823	9.682063	10.317937	19
42	9.637148	9.954762	9.682386	10.317613	18
43	9.637411	9.954701	9.682710	10.317290	17
44	9.637673	9.954640	9.683033	10.316967	16
45	9.637935	9.954579	9.683356	10.316644	15
46	9.638197	9.954518	9.683678	10.316321	14
47	9.638458	9.954457	9.684001	10.315999	13
48	9.638720	9.954396	9.684324	10.315676	12
49	9.638981	9.954335	9.684646	10.315354	11
50	9.639242	9.954274	9.684968	10.315032	10
51	9.939503	9.954213	9.685290	10.314710	9
52	9.639764	9.954152	9.685612	10.313388	8
53	9.640024	9.954090	9.685934	10.314066	7
54	9.640284	9.954029	9.686255	10.313745	6
55	9.640544	9.953968	9.686577	10.313423	5
56	9.640804	9.953906	9.686898	10.313102	4
57	9.641064	9.953845	9.687219	10.312781	3
58	9.641323	9.953783	9.687540	10.312460	2
59	9.641583	9.953722	9.687861	10.312138	1
60	9.641842	9.953660	9.688182	10.311818	0
M		Sine		Tangent	M

Degree 64.

Degree 26.

M	Sine		Tangent		
0	9.641842	9.953660	9.688182	10.311818	66
1	9.642101	9.953598	9.688502	10.311498	59
2	9.642360	9.953537	9.688823	10.311177	58
3	9.642618	9.953475	9.689143	10.310857	57
4	9.642876	9.953413	9.689463	10.310537	56
5	9.643135	9.953351	9.689783	10.310237	55
6	9.643393	9.953290	9.690103	10.309897	54
7	9.643650	9.953228	9.690423	10.309577	53
8	9.643908	9.953166	9.690742	10.309258	52
9	9.644165	9.953104	9.691063	10.308938	51
10	9.644423	9.953042	9.691381	10.308619	50
11	9.644680	9.952980	9.691700	10.308300	49
12	9.644936	9.952917	9.692019	10.307981	48
13	9.645193	9.952855	9.692338	10.307662	47
14	9.645449	9.952793	9.692656	10.307343	46
15	9.645706	9.952731	9.692975	10.307025	45
16	9.645962	9.952668	9.693293	10.306706	44
17	9.646218	9.952606	9.693612	10.306388	43
18	9.646473	9.952544	9.693930	10.306070	42
19	9.646729	9.952481	9.694248	10.305752	41
20	9.646984	9.952419	9.694566	10.305434	40
21	9.647239	9.952356	9.694883	10.305117	39
22	9.647494	9.952294	9.695201	10.304799	38
23	9.647749	9.952231	9.695518	10.304482	37
24	9.648004	9.952168	9.695835	10.304164	36
25	9.648258	9.952105	9.696153	10.303847	35
26	9.648512	9.952043	9.696470	10.303530	34
27	9.648766	9.951980	9.696786	10.303213	33
28	9.649020	9.951917	9.697103	10.302897	32
29	9.649274	9.951854	9.697420	10.302580	31
30	9.649527	9.951791	9.697738	10.302264	30
		Sine		Tangent	M

Degree 63.

Degree 26.

M	Sine		Tangent		
30	9.649527	9.951791	9.697738	10.302264	30
31	9.649781	9.951728	9.698052	10.301947	29
32	9.650034	9.951665	9.698369	10.301631	28
33	9.650287	9.951602	9.698685	10.301315	27
34	9.650519	9.951539	9.699001	10.300999	26
35	9.650798	9.951476	9.699316	10.300684	25
36	9.651044	9.951412	9.699632	10.300368	24
37	9.651296	9.951349	9.699947	10.300052	23
38	9.651648	9.951286	9.700263	10.299737	22
39	9.651800	9.951222	9.700578	10.299422	21
40	9.652052	9.951159	9.700893	10.299107	20
41	9.652303	9.951095	9.701208	10.298792	19
42	9.652555	9.951032	9.701522	10.298477	18
43	9.652806	9.950968	9.701837	10.298163	17
44	9.653057	9.950905	9.702152	10.297848	16
45	9.653307	9.950841	9.702466	10.297534	15
46	9.653558	9.950777	9.702780	10.297219	14
47	9.653808	9.950714	9.703095	10.296905	13
48	9.654059	9.950650	9.703409	10.296591	12
49	9.654309	9.950586	9.703722	10.296277	11
50	9.653558	9.950522	9.704036	10.295964	10
51	9.654808	9.950458	9.704350	10.295656	9
52	9.655057	9.950394	9.704663	10.295337	8
53	9.655307	9.950330	9.704976	10.295023	7
54	9.655556	9.950266	9.905290	10.294710	6
55	9.655805	9.950202	9.705603	10.294397	5
56	9.656053	9.950138	9.705915	10.294084	4
57	9.656302	9.950074	9.706228	10.293771	3
58	9.656550	9.950009	9.706541	10.293459	2
59	9.656799	9.949945	9.706853	10.293146	1
60	9.656947	9.949881	9.707166	10.292834	0
		Sine		Tangent	M

Degree 63.

Degree 27.

M	Sine		Tangent		
0	9.657047	9.949880	9.707166	10.292834	60
1	9.657295	9.949816	9.707478	10.292523	59
2	9.657542	9.949752	9.707790	10.292210	58
3	9.657790	9.949687	9.708102	10.291897	57
4	9.658037	9.949623	9.708414	10.291586	56
5	9.658284	9.949598	9.708726	10.291274	55
6	9.658531	9.949494	9.709037	10.290962	54
7	9.658777	9.949429	9.709349	10.290651	53
8	9.659024	9.949364	9.709660	10.290340	52
9	9.659271	9.949300	9.709971	10.290029	51
10	9.659517	9.949235	9.710282	10.289718	50
11	9.659763	9.949170	9.710593	10.289407	49
12	9.660009	9.949105	9.710904	10.289096	48
13	9.660255	9.949040	9.711214	10.288785	47
14	9.660500	9.948976	9.711525	10.288475	46
15	9.660746	9.948910	9.711836	10.288164	45
16	9.660991	9.948845	9.712146	10.287854	44
17	9.661036	9.948760	9.712456	10.287544	43
18	9.661481	9.948715	9.712766	10.287234	42
19	9.661726	9.948650	9.713076	10.286924	41
20	9.661970	9.948584	9.713386	10.286614	40
21	9.662214	9.948519	9.713695	10.286305	39
22	9.662459	9.948453	9.714005	10.285995	38
23	9.662702	9.948388	9.714314	10.285686	37
24	9.662947	9.948323	9.714624	10.285376	36
25	9.663190	9.948257	9.714933	10.285067	35
26	9.663433	9.948191	9.715241	10.284758	34
27	9.663677	9.948126	9.715550	10.284449	33
28	9.663920	9.948060	9.715859	10.284140	32
29	9.664163	9.947995	9.716168	10.283832	31
30	9.664406	9.947929	9.716477	10.283523	30
		Sine		Tangent	M

Degree 62.

Degree 27.

M	Sine		Tangent		
30	9.664406	9.947929	9.716477	10.283523	30
31	9.664648	9.947863	9.716785	10.283215	29
32	9.664891	9.947797	9.717093	10.282907	28
33	9.665133	9.947731	9.717401	10.282598	27
34	9.665375	9.947665	9.717709	10.282290	26
35	9.665617	9.947599	9.718017	10.281983	25
36	9.665858	9.947533	9.718325	10.281675	24
37	9.666100	9.947467	9.718633	10.281367	23
38	9.666341	9.947401	9.718940	10.281060	22
39	9.666583	9.947335	9.719248	10.280752	21
40	9.666824	9.947269	9.719555	10.280445	20
41	9.667065	9.947203	9.719862	10.280138	19
42	9.667305	9.947136	9.720169	10.279831	18
43	9.667546	9.947070	9.720476	10.279524	17
44	9.667786	9.947004	9.720783	10.279217	16
45	9.668026	9.946937	9.721089	10.278911	15
46	9.668266	9.946871	9.721395	10.278604	14
47	9.668506	9.946804	9.721702	10.278298	13
48	9.668746	9.946738	9.722008	10.277991	12
49	9.668986	9.946671	9.722315	10.277685	11
50	9.669225	9.946604	9.722621	10.277379	10
51	9.669464	9.946537	9.722927	10.277073	9
52	9.669703	9.946471	9.723232	10.276768	8
53	9.669942	9.946404	9.723538	10.276462	7
54	9.670181	9.946337	9.723843	10.276156	6
55	9.970419	9.946270	9.724149	10.275851	5
56	9.670657	9.946203	9.724454	10.275546	4
57	9.670896	9.946136	9.724759	10.275240	3
58	9.671134	9.946069	9.725065	10.274935	2
59	9.671372	9.946002	9.725369	10.274630	1
60	9.671609	9.945935	9.725674	10.274326	0
		Sine		Tangent	M

Degree 62.

Degree 28.

M	Sine		Tangent		
0	9.671609	9.945935	9.725674	10.274326	60
1	9.671847	9.945869	9.725979	10.274021	59
2	9.672084	9.945800	9.726284	10.273816	58
3	9.672321	9.945733	9.726588	10.273412	57
4	9.672558	9.945666	9.726892	10.273107	56
5	9.672795	9.945598	9.727197	10.272803	55
6	9.673032	9.945531	9.727501	10.272499	54
7	9.673268	9.945463	9.727805	10.272195	53
8	9.673505	9.945396	9.728109	10.271891	52
9	9.573741	9.945328	9.728412	10.271587	51
10	9.673977	9.945261	9.728716	10.271284	50
11	9.674213	9.945193	9.729020	10.270980	49
12	9.674448	9.945125	9.729323	10.270677	48
13	9.674684	9.945058	9.729626	10.270374	47
14	9.674919	9.944990	9.729929	10.270070	46
15	9.675154	9.944922	9.730232	10.269767	45
16	9.675389	9.944854	9.730535	10.269464	44
17	9.675623	9.944786	9.730838	10.269162	43
18	9.675859	9.944718	9.731141	10.268859	42
19	9.676094	9.944650	9.731443	10.268559	41
20	9.676328	9.944582	9.731746	10.268254	40
21	9.676562	9.944514	9.732048	10.267952	39
22	9.676796	9.944446	9.732351	10.267649	38
23	9.677030	9.944377	9.732653	10.267347	37
24	9.677264	9.944309	9.732955	10.267045	36
25	9.677497	9.944241	9.733257	10.266743	35
26	9.677731	9.944172	9.733558	10.266441	34
27	9.677964	9.944104	9.733860	10.266140	33
28	9.678197	9.944016	9.734162	10.265838	32
29	9.678430	9.943967	9.734463	10.265537	31
30	9.678663	9.943898	9.734764	10.265236	30
		Sine		Tangent	M

Degree 61.

Degree 28.

M	Sine		Tangent		
30	9.678663	9.943898	9.734764	10.265236	30
31	9.678895	9.943830	9.735666	10.264934	29
32	9.679128	9.943761	9.735362	10.264633	28
33	9.679360	9.943692	9.735668	10.264332	27
34	9.679592	9.943624	9.735968	10.264031	26
35	9.679824	9.943555	9.736269	10.263731	25
36	9.680056	9.943486	9.736570	10.263430	24
37	9.680288	9.943417	9.736870	10.263130	23
38	9.680519	9.943348	9.737171	10.262829	22
39	9.680750	9.943279	9.737471	10.262529	21
40	9.680982	9.943210	9.737771	10.262229	20
41	9.681213	9.943141	9.738071	10.261929	19
42	9.681443	9.943072	9.738371	10.261629	18
43	9.681674	9.943003	9.738671	10.261329	17
44	9.681904	9.942933	9.738971	10.261029	16
45	9.682135	9.942864	9.739271	10.260729	15
46	9.682365	9.942795	9.739570	10.260430	14
47	9.682595	9.942725	9.739870	10.260130	13
48	9.682825	9.942656	9.740169	10.259831	12
49	9.683055	9.942587	9.740468	10.259532	11
50	9.683284	9.942517	9.740767	10.259233	10
51	9.683514	9.942448	9.741066	10.258934	9
52	9.683743	9.942378	9.741365	10.258635	8
53	9.683972	9.942308	9.741664	10.258336	7
54	9.684201	9.942239	9.741962	10.258038	6
55	9.684430	9.942169	9.742261	10.257739	5
56	9.684658	9.942099	9.742559	10.257441	4
57	9.684887	9.942029	9.742858	10.257142	3
58	9.685115	9.941959	9.743156	10.256844	2
59	9.685343	9.941889	9.743454	10.256546	1
60	9.685571	9.941819	9.743751	10.256248	0
		Sine		Tangent	M

Degree 61.

Degree 29.

M	Sine		Tangent		
0	9.685571	9.941819	9.743752	10.256248	60
1	9.685799	9.941749	9.744050	10.255950	59
2	9.686027	9.941679	9.744348	10.255652	58
3	9.686254	9.941609	9.744645	10.255355	57
4	9.686482	9.941539	9.744943	10.255057	56
5	9.686709	9.941468	9.745240	10.254760	55
6	9.686936	9.941398	9.745538	10.254462	54
7	9.687163	9.941328	9.745835	10.254165	53
8	9.687389	9.941257	9.746132	10.253868	52
9	9.687616	9.941187	9.746429	10.253571	51
10	9.687842	9.941116	9.746726	10.253274	50
11	9.688069	9.941046	9.747023	10.252977	49
12	9.688295	9.940975	9.747319	10.252680	48
13	9.688523	9.940905	9.747616	10.252384	47
14	9.688747	9.940834	9.747912	10.252087	46
15	9.688972	9.940763	9.748209	10.251791	45
16	9.689198	9.940693	9.748505	10.251495	44
17	9.689421	9.940622	9.748801	10.251199	43
18	9.689648	9.940551	9.749097	10.250902	42
19	9.689873	9.940480	9.749393	10.250607	41
20	9.690098	9.940409	9.749689	10.250311	40
21	9.690323	9.940338	9.749985	10.250015	39
22	9.690548	9.940267	9.750281	10.249719	38
23	9.690772	9.940196	9.750576	10.249424	37
24	9.690996	9.940125	9.750872	10.249128	36
25	9.691220	9.940053	9.751167	10.248833	35
26	9.691444	9.939982	9.751462	10.248538	34
27	9.691668	9.939911	9.751757	10.248243	33
28	9.691892	9.939840	9.752052	10.247948	32
29	9.692115	9.939768	9.752347	10.247653	31
30	9.692339	9.939697	9.752642	10.247358	30
		Sine		Tangent	M

Degree 60.

Degree 29.

M	Sine		Tangent		M
30	9.692339	9.939697	9.752642	10.247358	30
31	9.692562	9.939625	9.752937	10.247063	29
32	9.692785	9.939554	9.753231	10.246769	28
33	9.693008	9.939482	9.753526	10.246474	27
34	9.693231	9.939410	9.753820	10.246180	26
35	9.693453	9.939339	9.754115	10.245885	25
36	9.693676	9.939267	9.754409	10.245591	24
37	9.693898	9.939195	9.754703	10.245297	23
38	9.694120	9.939123	9.754997	10.245003	22
39	9.694342	9.939051	9.755291	10.244709	21
40	9.694564	9.938980	9.755584	10.244415	20
41	9.694786	9.938908	9.755878	10.244122	19
42	9.695007	9.938835	9.756172	10.243828	18
43	9.695229	9.938763	9.756465	10.243535	17
44	9.695450	9.938691	9.756759	10.243241	16
45	9.695671	9.938619	9.757052	10.242948	15
46	9.695892	9.938547	9.757345	10.242655	14
47	9.696113	9.938475	9.757638	10.242362	13
48	9.696334	9.938402	9.757931	10.242069	12
49	9.696554	9.938330	9.758224	10.241776	11
50	9.696774	9.938257	9.758517	10.241483	10
51	9.696995	9.938185	9.758810	10.241190	9
52	9.697215	9.938112	9.759102	10.240898	8
53	9.697435	9.938040	9.759395	10.240605	7
54	9.697654	9.937967	9.759687	10.240313	6
55	9.697874	9.937895	9.759979	10.240021	5
56	9.698093	9.937822	9.760271	10.239728	4
57	9.698313	9.937749	9.760564	10.239436	3
58	9.698532	9.937676	9.760856	10.239144	2
59	9.698751	9.937603	9.761147	10.238852	1
60	9.698970	9.937531	9.761439	10.238561	0
		Sine		Tangent	M

Degree 60.

Degree 30.

M	Sine		Tangent		
0	9.698970	9.937531	9.761439	10.238561	60
1	9.699189	9.937458	9.761731	10.238269	59
2	9.699407	9.937385	9.762023	10.237977	58
3	9.699626	9.937312	9.762314	10.237686	57
4	9.699844	9.937238	9.762606	10.237394	56
5	9.700062	9.937165	9.762897	10.237103	55
6	9.700280	9.937092	9.763188	10.236812	54
7	9.700498	9.937019	9.763479	10.236521	53
8	9.700716	9.936945	9.763770	10.236230	52
9	9.700933	9.936872	9.164061	10.235939	51
10	9.701151	9.936799	9.764352	10.235648	50
11	9.701568	9.936725	9.764643	10.235357	49
12	9.701585	9.936652	9.764933	10.235067	48
13	9.701802	9.936578	9.765224	10.234776	47
14	9.702019	9.936505	9.765514	10.234486	46
15	9.702236	9.936431	9.765805	10.234195	45
16	9.702452	9.936357	9.766095	10.233905	44
17	9.702669	9.936284	9.766385	10.233615	43
18	9.702885	9.936210	9.766675	10.233325	42
19	9.703101	9.936136	9.766965	10.233035	41
20	9.703317	9.936062	9.767255	10.232745	40
21	9.703533	9.935988	9.767545	10.232455	39
22	9.703748	9.935914	9.767834	10.232166	38
23	9.703964	9.935840	9.768124	10.231876	37
24	9.704179	9.935766	9.768413	10.231587	36
25	9.704395	9.935692	9.768703	10.231297	35
26	9.704610	9.935618	9.768992	10.231008	34
27	9.704820	9.935543	9.769281	10.230719	33
28	9.705040	9.935469	9.769570	10.230430	32
29	9.705254	9.935395	9.769859	10.230141	31
30	9.705469	9.935320	9.770148	10.229852	30
		Sine		Tangent	M

Degree 59.

Degree 30.

M	Sine		Tangent		
30	9.705469	9.935320	9.770148	10.229852	30
31	9.705683	9.935246	9.770437	10.229563	29
32	9.705897	9.935171	9.770726	10.229274	28
33	9.706112	9.935097	9.771015	10.228985	27
34	9.706327	9.935022	9.771303	10.228696	26
35	9.706539	9.934948	9.771592	10.228408	25
36	9.706753	9.934873	9.771880	10.228120	24
37	9.706967	9.934798	9.772168	10.227832	23
38	9.707180	9.934723	9.772456	10.227543	22
39	9.707393	9.934649	9.772745	10.227255	21
40	9.707606	9.934574	9.773033	10.226967	20
41	9.707819	9.934499	9.773321	10.226679	19
42	9.708032	9.934424	9.773608	10.226391	18
43	9.708245	9.934349	9.773896	10.226104	17
44	9.708457	9.934274	9.774184	10.225816	16
45	9.708670	9.934199	9.774471	10.225529	15
46	9.708882	9.934123	9.774759	10.225241	14
47	9.709094	9.934048	9.775046	10.224954	13
48	9.709306	9.933973	9.775333	10.224666	12
49	9.709518	9.933897	9.775621	10.224379	11
50	9.709730	9.933822	9.775908	10.224092	10
51	9.709941	9.933747	9.776195	10.223805	9
52	9.710153	9.933671	9.776482	10.223518	8
53	9.710364	9.933596	9.776768	10.223232	7
54	9.710575	9.933520	9.777055	10.222945	6
55	9.710786	9.933444	9.777342	10.222658	5
56	9.710997	9.933369	9.777628	10.222372	4
57	9.711208	9.933293	9.777915	10.222085	3
58	9.711418	9.933217	9.778201	10.221799	2
59	9.711629	9.933141	9.778487	10.221513	1
60	9.711839	9.933066	9.778774	10.221226	0
		Sine		Tangent	M

Degree 59.

Degree 31.

M	Sine		Tangent		
0	9.711839	9.932066	9.778774	10.221226	60
1	9.712049	9.932990	9.779060	10.220940	59
2	9.712259	9.932914	9.779346	10.220654	58
3	9.712469	9.932838	9.779632	10.220368	57
4	9.712679	9.932761	9.779918	10.220082	56
5	9.712889	9.932685	9.780203	10.219796	55
6	9.713098	9.932609	9.780489	10.219511	54
7	9.713308	9.932533	9.780775	10.219225	53
8	9.713517	9.932457	9.781060	10.218940	52
9	9.713726	9.932380	9.781346	10.218654	51
10	9.713935	9.932304	9.781631	10.218369	50
11	9.714144	9.932227	9.781916	10.218084	49
12	9.714352	9.932151	9.782202	10.217799	48
13	9.714561	9.932074	9.782480	10.217514	47
14	9.714769	9.931998	9.782771	10.217229	46
15	9.714977	9.931921	9.783056	10.216944	45
16	9.715186	9.931845	9.783341	10.216659	44
17	9.715394	9.931768	9.783626	10.216374	43
18	9.715601	9.931691	9.783910	10.216090	42
19	9.715809	9.931614	9.784195	10.215805	41
20	9.716017	9.931537	9.784479	10.215520	40
21	9.716224	9.931460	9.784764	10.215236	39
22	9.716431	9.931383	9.785048	10.214952	38
23	9.716639	9.931306	9.785332	10.214668	37
24	9.716846	9.931229	9.785616	10.214384	36
25	9.717053	9.931152	9.785900	10.214099	35
26	9.717259	9.931075	9.786184	10.213816	34
27	9.717466	9.930998	9.786468	10.213532	33
28	9.717672	9.930920	9.786752	10.213248	32
29	9.717879	9.930843	9.787036	10.212964	31
30	9.718085	9.930766	9.787319	10.212681	30
		Sine		Tangent	M

Degree 58.

Degree 31.

M	Sine		Tangent		
30	9.718085	9.930766	9.787319	10.212681	30
31	9.718291	9.930688	9.787603	10.212397	29
32	9.718497	9.930611	9.787886	10.212114	28
33	9.718703	9.930533	9.788170	10.211830	27
34	9.718909	9.930456	9.788453	10.211547	26
35	9.719114	9.930378	9.788736	10.211264	25
36	9.719320	9.930300	9.789019	10.210981	24
37	9.719525	9.930223	9.789302	10.210698	23
38	9.719730	9.930145	9.789585	10.210415	22
39	9.719935	9.930067	9.789868	10.210132	21
40	9.720140	9.929989	9.790151	10.209849	20
41	9.720345	9.929911	9.790433	10.209566	19
42	9.720549	9.929833	9.750716	10.209284	18
43	9.720754	9.929755	9.790999	10.209001	17
44	9.720958	9.929677	9.790281	10.208719	16
45	9.721162	9.929599	9.791563	10.208436	15
46	9.721366	9.929521	9.791846	10.208154	14
47	9.721570	9.919442	9.792128	10.207872	13
48	9.721774	9.929364	9.792410	10.207590	12
49	9.721978	9.929286	9.792692	10.207308	11
50	9.722181	9.929207	9.792974	10.207024	10
51	9.722385	9.929129	9.793256	10.206744	9
52	9.722588	9.929050	9.793538	10.206462	8
53	9.722791	9.928972	9.793819	10.206180	7
54	9.722994	9.928893	9.794101	10.205899	6
55	9.723197	9.928814	9.794383	10.205617	5
56	9.723400	9.928736	9.794664	10.205336	4
57	9.723603	9.928657	9.794945	10.205054	3
58	9.723805	9.928578	9.795227	10.204773	2
59	9.724007	9.928499	9.795508	10.204492	1
60	9.724210	9.928420	9.795789	10.204211	0
		Sine		Tangent	M

Degree 58.

Degree 32.

M	Sine		Tangent		
0	9.724210	9.928420	9.795789	10.204211	60
1	9.724412	9.928341	9.796070	10.203930	59
2	9.724614	9.928262	9.796351	10.203649	58
3	9.724816	9.928183	9.796632	10.203368	57
4	9.725017	9.928104	9.796913	10.203087	56
5	9.725219	9.928025	9.797194	10.202806	55
6	9.725420	9.927946	9.797474	10.202522	54
7	9.725622	9.927867	9.797755	10.202245	53
8	9.725823	9.927787	9.798036	10.201964	52
9	9.726024	9.927708	9.798316	10.201684	51
10	9.726225	9.927628	9.798596	10.201404	50
11	9.726426	9.927549	9.798877	10.201123	49
12	9.726626	9.927469	9.799157	10.200843	48
13	9.726827	9.927390	9.799437	10.200563	47
14	9.727027	9.927310	9.799717	10.200283	46
15	9.727228	9.927231	9.799997	10.200003	45
16	9.727428	9.927151	9.800277	10.199723	44
17	9.727628	9.927071	9.800557	10.199443	43
18	9.727828	9.926991	9.800836	10.199163	42
19	9.728027	9.926911	9.801116	10.198884	41
20	9.728227	9.926831	9.801396	10.198604	40
21	9.728427	9.926751	9.801675	10.198325	39
22	9.728626	9.926641	9.801955	10.198045	38
23	9.728825	9.926591	9.802234	10.197766	37
24	9.729024	9.926511	9.802513	10.197487	36
25	9.729223	9.926431	9.802792	10.197207	35
26	9.729422	9.926351	9.803072	10.196928	34
27	9.729621	9.926270	9.803351	10.196649	33
28	9.729820	9.926190	9.803630	10.196370	32
29	9.730018	9.926110	9.803908	10.196091	31
30	9.730216	9.926029	9.804187	10.195813	30
		Sine		Tangent	M

Degree 57.

Degree 32.

M	Sine		Tangent		
30	9.730216	9.926029	9.804187	10.195813	30
31	9.730415	9.925949	9.804466	10.195534	29
32	9.730613	9.925868	9.804745	10.195255	28
33	9.730811	9.925787	9.805023	10.194977	27
34	9.731009	9.925707	9.805302	10.194698	26
35	9.731206	9.925626	9.805580	10.194420	25
36	9.731404	9.925545	9.805859	10.194141	24
37	9.731601	9.925464	9.806137	10.193863	23
38	9.731799	9.925384	9.806415	10.193585	22
39	9.731996	9.925303	9.806693	10.193309	21
40	9.732193	9.925222	9.806971	10.193028	20
41	9.732390	9.925141	9.807249	10.192751	19
42	9.732587	9.925060	9.807527	10.192433	18
43	9.732784	9.924978	9.807805	10.192195	17
44	9.732980	9.924897	9.808083	10.191917	16
45	9.733177	9.924816	9.808361	10.191639	15
46	9.733373	9.924735	9.808638	10.191362	14
47	9.733569	9.924653	9.808916	10.191084	13
48	9.733765	9.924572	9.809193	10.190807	12
49	9.733961	9.924491	9.809471	10.190529	11
50	9.734157	9.924409	9.809748	10.190252	10
51	9.734353	9.924328	9.810025	10.189975	9
52	9.734548	9.924246	9.810302	10.189697	8
53	9.734744	9.924164	9.810580	10.189420	7
54	9.734939	9.924083	9.810857	10.189143	6
55	9.735134	9.924001	9.811134	10.188866	5
56	9.735330	9.923919	9.811410	10.188589	4
57	9.735525	9.923837	9.811687	10.188313	3
58	9.735719	9.923755	9.811964	10.188036	2
59	9.735914	9.923673	9.812241	10.187759	1
60	9.736109	9.923591	9.812517	10.187483	0
		Sine		Tangent	M

Degree 57.

Degree 33.

M	Sine		Tangent		
0	9.736109	9.923591	9.812517	10.187483	60
1	9.736309	9.923509	9.812794	10.187206	59
2	9.736497	9.923427	9.813070	10.186930	58
3	9.736692	9.923345	9.813347	10.186653	57
4	9.736886	9.923263	9.813623	10.186377	56
5	9.737080	9.923180	9.813899	10.186101	55
6	9.737274	9.923098	9.814175	10.185824	54
7	9.737467	9.923016	9.814452	10.185548	53
8	9.737661	9.922933	9.814728	10.185272	52
9	9.737854	9.922851	9.815004	10.184996	51
10	9.738048	9.922768	9.815279	10.184720	50
11	9.738241	9.922686	9.815555	10.184445	49
12	9.738434	9.922603	9.815831	10.184169	48
13	9.738627	9.922520	9.816107	10.183893	47
14	9.738820	9.922438	9.816382	10.183617	46
15	9.739013	9.922355	9.816658	10.183342	45
16	9.739205	9.922272	9.816933	10.183066	44
17	9.739398	9.922189	9.817209	10.182791	43
18	9.739590	9.922106	9.817484	10.182516	42
19	9.739783	9.922023	9.817759	10.182240	41
20	9.739975	9.921940	9.818035	10.181965	40
21	9.740167	9.921857	9.818310	10.181690	39
22	9.740359	9.921774	9.818585	10.181415	38
23	9.740550	9.921691	9.818860	10.181140	37
24	9.740742	9.921607	9.819135	10.180865	36
25	9.740934	9.921524	9.819410	10.180590	35
26	9.741125	9.921441	9.819684	10.180315	34
27	9.741316	9.921357	9.819959	10.180041	33
28	9.741507	9.921274	9.820234	10.179766	32
29	9.741698	9.921190	9.820508	10.179492	31
30	9.741889	9.921107	9.820783	10.179217	30
		Sine		Tangent	M

Degree 56.

Degree 33.

M	Sine		Tangent		
30	9.741889	9.921107	9.820783	10.179217	30
31	9.742080	9.921023	9.821057	10.178943	29
32	9.742271	9.920939	9.821332	10.178668	28
33	9.742461	9.920855	9.821606	10.178394	27
34	9.742652	9.920772	9.821880	10.178120	26
35	9.742842	9.920688	9.822154	10.177846	25
36	9.743032	9.920604	9.822429	10.177571	24
37	9.743223	9.920520	9.822703	10.177297	23
38	9.743412	9.920436	9.822977	10.177023	22
39	9.743602	9.920352	9.823250	10.176739	21
40	9.743792	9.920268	9.823524	10.176476	20
41	9.743982	9.920184	9.823798	10.176202	19
42	9.744171	9.920099	9.824072	10.175928	18
43	9.744361	9.920015	9.824345	10.175655	17
44	9.744550	9.919931	9.824619	10.175381	16
45	9.744739	9.919846	9.824892	10.175108	15
46	9.744928	9.919762	9.825166	10.174834	14
47	9.745117	9.919677	9.825439	10.174560	13
48	9.745306	9.919593	9.825713	10.174287	12
49	9.745494	9.919508	9.825986	10.174014	11
50	9.745683	9.919424	9.826259	10.173741	10
51	9.745871	9.919339	9.826532	10.173468	9
52	9.746059	9.919254	9.826805	10.173195	8
53	9.746248	9.919169	9.827078	10.172922	7
54	9.746436	9.919084	9.827351	10.152649	6
55	9.746624	9.918999	9.827624	10.172376	5
56	9.746811	9.918915	9.827897	10.172103	4
57	9.746999	9.918830	9.828170	10.171830	3
58	9.747187	9.918744	9.828442	19.171558	2
59	9.747374	9.918659	9.828715	10.171285	1
60	9.747562	9.918574	9.828987	10.171012	0
		Sine		Tangent	M

Degree 56.

Degree 34.

M	Sine		Tangent		
0	9.747562	9.918574	9.828987	10.171012	60
1	9.747749	9.918489	9.829260	10.170740	59
2	9.747936	9.918404	9.829532	10.170468	58
3	9.748123	9.918318	9.829805	10.170195	57
4	9.748310	9.918233	9.830077	10.169923	56
5	9.748497	9.918147	9.830349	10.169651	55
6	9.748683	9.918062	9.830621	10.169379	54
7	9.748870	9.917975	9.830893	10.169156	53
8	9.749056	9.917891	9.831165	10.168834	52
9	9.749242	9.917805	9.831437	10.168563	51
10	9.749429	9.917719	9.831709	10.168291	50
11	9.749615	9.917634	9.831981	10.168019	49
12	9.749801	9.917548	9.832253	10.167747	48
13	9.749986	9.917462	9.832525	10.167475	47
14	9.750172	9.917376	9.832796	10.167204	46
15	9.750358	9.917290	9.833068	10.166932	45
16	9.750543	9.917204	9.833339	10.166660	44
17	9.750729	9.917118	9.833611	10.166389	43
18	9.750914	9.917032	9.833882	10.166118	42
19	9.751099	9.916945	9.834154	10.165846	41
20	9.751284	9.916819	9.834425	10.165575	40
21	9.751469	9.916773	9.834696	10.165304	39
22	9.751654	9.916686	9.834967	10.165033	38
23	9.751838	9.916600	9.835238	10.164762	37
24	9.752023	9.916514	9.835509	10.164491	36
25	9.752207	9.916427	9.835780	10.164220	35
26	9.752392	9.916340	9.836051	10.163949	34
27	9.752576	9.916254	9.836322	10.163678	33
28	9.752760	9.916167	9.836593	10.163407	32
29	9.752944	9.916080	9.836864	10.163136	31
30	9.753128	9.915994	9.837134	10.162866	30
		Sine		Tangent	M

Degree 55.

Degree 34.

M	Sine		Tangent		
30	9.753128	9.915994	9.837134	10.162866	30
31	9.753312	9.915507	9.837405	10.162595	29
32	9.753495	9.915820	9.837675	10.162325	28
33	9.753679	9.915733	9.837946	10.162054	27
34	9.753862	9.915646	9.838216	10.161784	26
35	9.754046	9.915559	9.838487	10.161513	25
36	9.754229	9.915472	9.838757	10.161243	24
37	9.754412	9.915385	9.839027	10.160973	23
38	9.754595	9.915297	9.839297	10.160702	22
39	9.754778	9.915210	9.839568	10.160432	21
40	9.754960	9.915123	9.839838	10.160162	20
41	9.755143	9.915035	9.840108	10.159892	19
42	9.755325	9.914948	9.840378	10.159622	18
43	9.755508	9.914860	9.840647	10.159352	17
44	9.755690	9.914773	9.840917	10.159083	16
45	9.755872	9.914685	9.841187	10.158813	15
46	9.756054	9.914597	9.841457	10.158543	14
47	9.756236	9.914510	9.841726	10.158273	13
48	9.756418	9.914422	9.841996	10.158004	12
49	9.756600	9.914334	9.842266	10.157734	11
50	9.756781	9.914246	9.842505	10.157465	10
51	9.756963	9.914158	9.842804	10.157195	9
52	9.757144	9.914070	9.843074	10.156926	8
53	9.757316	9.913982	9.843343	10.156657	7
54	9 757507	9.913894	9.843612	10.156387	6
55	9.757688	9.913806	9.843882	10.156118	5
56	9.757869	9.913718	9.844151	10.155849	4
57	9.758049	9.913630	9.844420	10.155580	3
58	9.758230	9.913541	9.844689	10.155311	2
59	9.758411	9.913453	9.844958	10.155041	1
60	9.758591	9.913361	9.845227	10.154773	0
		Sine		Tangent	M

Degree 55.

Degree 35.

M	Sine		Tangent		
0	9.758591	9.913364	9.845227	10.154774	60
1	9.758772	9.913276	9.845496	10.154504	59
2	9.758952	9.913187	9.845764	10.154235	58
3	9.759132	9.913099	9.846033	10.153967	57
4	9.759312	9.913010	9.846302	10.153698	56
5	9.759492	9.912921	9.846570	10.153429	55
6	9.759672	9.912833	9.846839	10.153161	54
7	9.759851	9.912744	9.847107	10.152892	53
8	9.760031	9.912655	9.847376	10.152624	52
9	9.760210	9.912566	9.847644	10.152356	51
10	9.760390	9.912477	9.847913	10.152087	50
11	9.760569	9.912388	9.848181	10.151819	49
12	9.760748	9.912299	9.848449	10.151551	48
13	9.760927	9.912210	9.848717	10.151283	47
14	9.761106	9.912121	9.848985	10.151015	46
15	9.761285	9.912031	9.849254	10.150746	45
16	9.761464	9.911942	9.849522	10.150478	44
17	9.761642	9.911853	9.849789	10.150214	43
18	9.761821	9.911763	9.850057	10.149943	42
19	9.761999	9.911674	9.850325	10.149675	41
20	9.762177	9.911584	9.850593	10.149407	40
21	9.762356	9.911495	9.850861	10.149139	39
22	9.762534	9.911405	9.851128	10.148872	38
23	9.762712	9.911315	9.851396	10.148604	37
24	9.762889	9.911226	9.851664	10.148336	36
25	9.763067	9.911136	9.851931	10.148069	35
26	9.763245	9.911046	9.852199	10.147801	34
27	9.763422	9.910956	9.852466	10.147534	33
28	9.763599	9.910866	9.852731	10.147267	32
29	9.763777	9.910776	9.853001	10.146999	31
30	9.763954	9.910686	9.853268	10.146732	30
		Sine		Tangent	M

Degree 54.

Degree 35.

M	Sine		Tangent		M
30	9.763954	9.910686	9.853208	10.146732	30
31	9.764131	9.910596	9.853532	10.146465	29
32	9.764308	9.910506	9.853802	10.146198	28
33	9.764485	9.910415	9.854069	10.145930	27
34	9.764662	9.910325	9.854336	10.145664	26
35	9.764838	9.910235	9.854603	10.145397	25
36	9.765015	9.910144	9.854870	10.145130	24
37	9.765191	9.910054	9.855137	10.144863	23
38	9.765367	9.909963	9.855404	10.144596	22
39	9.765544	9.909873	9.855671	10.144329	21
40	9.765720	9.909782	9.855937	10.144063	20
41	9.765896	9.909691	9.856204	10.143796	19
42	9.766071	9.909601	9.856471	10.143529	18
43	9.766247	9.909510	9.856737	10.143263	17
44	9.766423	9.909419	9.857004	10.142996	16
45	9.766598	9.909328	9.857270	10.142730	15
46	9.766774	9.909237	9.857537	10.142463	14
47	9.766949	9.909146	9.857803	10.142197	13
48	9.767124	9.909055	9.858069	10.141931	12
49	9.767299	9.908964	9.858336	10.141664	11
50	9.767474	9.908873	9.858602	10.141398	10
51	9.767649	9.908781	9.858868	10.141132	9
52	9.767824	9.908690	9.859134	10.140866	8
53	9.767997	9.908599	9.859400	10.140600	7
54	9.768173	9.908507	9.859666	10.140334	6
55	9.768348	9.908416	9.859932	10.140068	5
56	9.768522	9.908324	9.860198	10.139802	4
57	9.768696	9.908233	9.860464	10.139536	3
58	9.768871	9.908141	9.860730	10.139270	2
59	9.769045	9.908049	9.860995	10.139005	1
60	9.769219	9.907958	9.861261	10.138739	0
		Sine		Tangent	M

Degree 54.

Degree 36.

M	Sine		Tangent		
0	9.769219	9.907958	9.861261	10.138739	60
1	9.769392	9.907866	9.861527	10.138473	59
2	9.769566	9.907774	9.861792	10.138208	58
3	9.769740	9.907682	9.862058	10.137942	57
4	9.769913	9.907590	9.862323	10.137677	56
5	9.770087	9.907498	9.862589	10.137411	55
6	9.770260	9.907406	9.862854	10.137146	54
7	9.770433	9.907314	9.863119	10.136880	53
8	9.770606	9.907221	9.863385	10.136615	52
9	9.770779	9.907129	9.863650	10.136350	51
10	9.770952	9.907037	9.863915	10.136085	50
11	9.771125	9.906945	9.864180	10.135820	49
12	9.771298	9.906852	9.864445	10.135554	48
13	9.771470	9.906760	9.864710	10.135289	47
14	9.771643	9.906667	9.864975	10.135024	46
15	9.771815	9.906574	9.865240	10.134759	45
16	9.771987	9.906482	9.865505	10.134495	44
17	9.772159	9.906389	9.865770	10.134230	43
18	9.772331	9.906296	9.866035	10.133965	42
19	9.772503	9.906203	9.866300	10.133700	41
20	9.772675	9.906111	9.866564	10.133436	40
21	9.772847	9.906018	9.866829	10.133171	39
22	9.773018	9.905925	9.867094	10.132906	38
23	9.773190	9.905832	9.867358	10.132642	37
24	9.773361	9.905738	9.867623	10.132377	36
25	9.773533	9.905645	9.867887	10.132113	35
26	9.773704	9.905552	9.868152	10.131848	34
27	9.773875	9.905459	9.868416	10.131584	33
28	9.774046	9.905365	9.868680	10.131320	32
29	9.774217	9.905272	9.868945	10.131055	31
30	9.774388	9.905179	9.869209	10.130791	30
		Sine		Tangent	M

Degree 53.

Degree 36.

M	Sine		Tangent		
30	9.774388	9.905179	9.869209	10.130791	30
31	9.774558	9.905085	9.864773	10.130527	29
32	9.774729	9.904992	9.867337	10.130263	28
33	9.774899	9.904898	9.870001	10.129999	27
34	9.775070	9.904804	9.870265	10.129735	26
35	9.775240	9.904711	9.870529	10.129471	25
36	9.775410	9.904617	9.870793	10.129207	24
37	9.775580	9.904523	9.871057	10.128943	23
38	9.775750	9.904429	9.871321	10.128679	22
39	9.775920	9.904335	9.871585	10.128415	21
40	9.776090	9.904241	9.871849	10.128151	20
41	9.776259	9.904147	9.872112	10.127888	19
42	9.776429	9.904053	9.872376	10.127624	18
43	9.776598	9.903959	9.872640	10.127360	17
44	9.776768	9.903864	9.872903	10.127097	16
45	9.776937	9.903770	9.873167	10.126833	15
46	9.777106	9.903676	9.873430	10.126570	14
47	9.777275	9.903581	9.873694	10.126306	13
48	9.777444	9.903486	9.873957	10.126043	12
49	9.777613	9.903392	9.874220	10.125780	11
50	9.777781	9.903298	9.874484	10.125516	10
51	9.777950	9.903203	9.874747	10.125253	9
52	9.778119	9.903108	9.875010	10.124990	8
53	9.778287	9.903013	9.875273	10.124727	7
54	9.778455	9.902919	9.875536	10.124464	6
55	9.778623	9.902824	9.875799	10.124201	5
56	9.778792	9.902729	9.876063	10.123937	4
57	9.778960	9.902634	9.876326	10.123674	3
58	9.779129	9.902539	9.876589	10.123411	2
59	9.779295	9.902444	9.876851	10.123149	1
60	9.779463	9.902349	9.877114	10.122886	0
		Sine		Tangent	M

Degree 53.

Degree 37.

M	Sine		Tangent		
0	9.779463	9.902349	9.877114	10.122885	60
1	9.779631	9.902253	9.877377	10.122623	59
2	9.779798	9.902158	9.877640	10.122360	58
3	9.779965	9.902063	9.877903	10.122097	57
4	9.780133	9.901967	9.878165	10.121834	56
5	9.780300	9.901872	9.878428	10.121572	55
6	9.780467	9.901776	9.878691	10.121309	54
7	9.780634	9.901681	9.878953	10.121047	53
8	9.780801	9.901585	9.879216	10.120784	52
9	9.780968	9.901488	9.879478	10.120522	51
10	9.781134	9.901391	9.879741	10.120259	50
11	9.781301	9.901298	9.880003	10.119997	49
12	9.781467	9.901202	9.880265	10.119734	48
13	9.781634	9.901106	9.880528	10.119472	47
14	9.781800	9.901010	9.880790	10.119210	46
15	9.781966	9.900914	9.881052	10.118948	45
16	9.782132	9.900828	9.881314	10.118686	44
17	9.782298	9.900722	9.881576	10.118424	43
18	9.782464	9.900626	9.881839	10.118161	42
19	9.782690	9.900529	9.882101	10.117899	41
20	9.782796	9.900433	9.882363	10.117637	40
21	9.782961	9.900337	9.882625	10.117375	39
22	9.783127	9.900240	9.882886	10.117114	38
23	9.783292	9.900144	9.883148	10.116852	37
24	9.783457	9.900047	9.883410	10.116590	36
25	9.783623	9.899951	9.883672	10.116328	35
26	9.783788	9.899854	9.883934	10.116066	34
27	9.783953	9.899757	9.884195	10.115805	33
28	9.784118	9.899660	9.884457	10.115543	32
29	9.784282	9.899563	9.884719	10.115281	31
30	9.784447	9.899467	9.884980	10.115020	30
		Sine		Tangent	M

Degree 52.

Degree 37.

M	Sine		Tangent		
30	9.784447	9.899467	9.884980	10.115025	30
31	9.784616	9.899370	9.885242	10.114758	29
32	9.784776	9.899273	9.885503	10.114497	28
33	9.784941	9.899175	9.885765	10.114235	27
34	9.785105	9.899078	9.886026	10.113974	26
35	9.785269	9.898981	9.886288	10.113712	25
36	9.785433	9.898884	9.886549	10.113451	24
37	9.785591	9.898787	9.886810	10.113190	23
38	9.785761	9.898689	9.887072	10.112928	22
39	9.785925	9.898592	9.887333	10.112667	21
40	9.786088	9.898494	9.887594	10.112406	20
41	9.786252	9.898397	9.887855	10.112145	19
42	9.786416	9.898299	9.888116	10.111884	18
43	9.786579	9.898201	9.888377	10.111623	17
44	9.786742	9.898104	9.888638	10.111362	16
45	9.786909	9.898006	9.888899	10.111101	15
46	9.787069	9.897908	9.889160	10.110840	14
47	9.787232	9.897810	9.889421	10.110579	13
48	9.787395	9.897712	9.889682	10.110318	12
49	9.787557	9.897614	9.889943	10.110057	11
50	9.787720	9.897516	9.890204	10.109796	10
51	9.787883	9.897418	9.890465	10.109535	9
52	9.788045	9.897320	9.890725	10.109275	8
53	9.788208	9.897222	9.890986	10.109014	7
54	9.788370	9.897123	9.891248	10.108753	6
55	9.788532	9.897025	9.891507	10.108493	5
56	9.788694	9.896926	9.891768	10.108232	4
57	9.788856	9.896828	9.892028	10.107972	3
58	9.789018	9.896729	9.892289	10.107711	2
59	9.789180	9.896631	9.892549	10.107451	1
60	9.789342	9.896532	9.892810	10.107190	0
		Sine		Tangent	M

Degree 52.

Degree 38.

M	Sine		Tangent		
0	9.789342	9.896532	9.892810	10.107190	60
1	9.789504	9.896433	9.893070	10.106930	59
2	9.789665	9.896335	9.893330	10.106669	58
3	9.789827	9.896236	9.893591	10.106408	57
4	9.789988	9.896137	9.893851	10.106149	56
5	9.790149	9.896038	9.894111	10.105889	55
6	9.790310	9.895939	9.894371	10.105628	54
7	9.790471	9.895840	9.894632	10.105368	53
8	9.790632	9.895741	9.894892	10.105108	52
9	9.790793	9.895641	9.895152	10.104844	51
10	9.790954	9.895542	9.895412	10.104588	50
11	9.791115	9.895443	9.895672	10.104328	49
12	9.791275	9.895343	9.895932	10.104068	48
13	9.791436	9.895244	9.896192	10.103808	47
14	9.791596	9.895144	9.896452	10.103548	46
15	9.791756	9.895045	9.896712	10.103288	45
16	9.791917	9.894945	9.896971	10.103028	44
17	9.792077	9.894846	9.897231	10.102769	43
18	9.792237	9.894746	9.897491	10.102509	42
19	9.792397	9.894646	9.897751	10.102249	41
20	9.792557	9.894546	9.898010	10.101990	40
21	9.792716	9.894446	9.898270	10.101730	39
22	9.792876	9.894346	9.898530	10.101470	38
23	9.793035	9.894246	9.898789	10.101211	37
24	9.793195	9.894146	9.899049	10.100951	36
25	9.793354	9.894046	9.899308	10.100692	35
26	9.793513	9.893946	9.899568	10.100432	34
27	9.793673	9.893845	9.899827	10.100173	33
28	9.793832	9.893745	9.900086	10.099913	32
29	9.793991	9.893645	9.900346	10.099654	31
30	9.794149	9.893544	9.900605	10.099395	30
		Sine		Tangent	M

Degree 51.

Degree 38.

M	Sine		Tangent	Tangent	
30	9.794149	9.893544	9.900605	10.099395	30
31	9.794308	9.893444	9.900864	10.099135	29
32	9.794467	9.893343	9.901124	10.098876	28
33	9.794626	9.893243	9.901383	10.098617	27
34	9.794784	9.893142	9.901643	10.098358	26
35	9.794942	9.893041	9.901901	10.098099	25
36	9.795101	9.892940	9.902160	10.097839	24
37	9.795259	9.892839	9.902419	10.097580	23
38	9.795417	9.892738	9.902678	10.097321	22
39	9.795575	9.892637	9.902937	10.097062	21
40	9.795733	9.892536	9.903196	10.096803	20
41	9.795891	9.892435	9.903455	10.096544	19
42	9.796049	9.892334	9.903714	10.096285	18
43	9.796206	9.892233	9.903973	10.096027	17
44	9.796364	9.892132	9.904232	10.095768	16
45	9.796521	9.892030	9.904491	10.095509	15
46	9.796678	9.891929	9.904750	10.095250	14
47	9.796836	9.891827	9.905008	10.094991	13
48	9.796993	9.891726	9.905267	10.094733	12
49	9.797150	9.891624	9.905526	10.094474	11
50	9.797307	9.891522	9.905784	10.094215	10
51	9.797464	9.891421	9.906043	10.093957	9
52	9.797621	9.891319	9.906302	10.093698	8
53	9.797777	9.891217	9.906560	10.093440	7
54	9.797934	9.891115	9.906819	10.093181	6
55	9.798091	9.891013	9.907077	10.092923	5
56	9.798247	9.890911	9.907336	10.092664	4
57	9.798403	9.890809	9.907594	10.092406	3
58	9.798560	9.890707	9.907852	10.092147	2
59	9.798716	9.890605	9.908111	10.091889	1
60	9.798872	9.890503	9.908369	10.091631	0
M		Sine			M

Degree 51.

Degree 39.

M	Sine		Tangent		M
0	9.798872	9.890503	9.908369	10.091631	60
1	9.799028	9.890400	9.908627	10.091373	59
2	9.799184	9.890298	9.908886	10.091114	58
3	9.799339	9.890195	9.909144	10.090856	57
4	9.799495	9.890093	9.909402	10.090598	56
5	9.799651	9.889990	9.909660	10.090340	55
6	9.799806	9.889888	9.909918	10.090081	54
7	9.799961	9.889785	9.910176	10.089823	53
8	9.800117	9.889682	9.910435	10.089565	52
9	9.800272	9.889579	9.910693	10.089307	51
10	9.800427	9.889476	9.910951	10.089049	50
11	9.800582	9.889374	9.911209	10.088791	49
12	9.800737	9.889271	9.911467	10.088533	48
13	9.800892	9.889167	9.911724	10.088275	47
14	9.801047	9.889064	9.911982	10.088017	46
15	9.801201	9.888961	9.912240	10.087760	45
16	9.801356	9.888858	9.912498	10.087502	44
17	9.801510	9.888755	9.912756	10.087244	43
18	9.801665	9.888651	9.913014	10.086986	42
19	9.801819	9.888548	9.913271	10.086729	41
20	9.801973	9.888444	9.913529	10.086471	40
21	9.802127	9.888341	9.913787	10.086213	39
22	9.802282	9.888237	9.914044	10.085956	38
23	9.802437	9.888133	9.914302	10.085698	37
24	9.802589	9.888030	9.914560	10.085440	36
25	9.802743	9.887926	9.914817	10.085183	35
26	9.802897	9.887822	9.915075	10.084925	34
27	9.803050	9.887718	9.915332	10.084668	33
28	9.803204	9.887614	9.915590	10.084410	32
29	9.803357	9.887510	9.915847	10.084153	31
30	9.803510	9.887406	9.916104	10.083895	30
		Sine		Tangent	M

Degree 50.

Degree 39.

M	Sine		Tangent		
30	9.803510	9.887406	9.916104	10.083895	30
31	9.803664	9.887302	9.916362	10.083638	29
32	9.803817	9.887198	9.916619	10.083381	28
33	9.803970	9.887093	9.916876	10.083123	27
34	9.804123	9.887989	9.917134	10.042866	26
35	9.804276	9.886884	9.917391	10.082609	25
36	9.804428	9.886780	9.917648	10.082352	24
37	9.804581	9.886675	9.917905	10.082094	23
38	9.804734	9.886571	9.918162	10.081837	22
39	9.804886	9.886466	9.918420	10.081580	21
40	9.805038	9.886361	9.918677	10.081323	20
41	9.805191	9.886257	9.918934	10.081066	19
42	9.805343	9.886152	9.919191	10.080809	18
43	9.805495	9.886047	9.919448	10.080552	17
44	9.805647	9.885942	9.919705	10.080295	16
45	9.805799	9.885837	9.919962	10.080038	15
46	9.805951	9.885732	9.920219	10.079781	14
47	9.806103	9.885627	9.920476	10.079524	13
48	9.806254	9.885521	9.920733	10.079267	12
49	9.806406	9.885416	9.920990	10.979010	11
50	9.806557	9.885311	9.921247	10.078753	10
51	9.806709	9.885205	9.921503	10.078496	9
52	9.806860	9.885100	9.921760	10.078240	8
53	9.807011	9.884994	9.922017	10.077983	7
54	9.807162	9.884889	9.922274	10.077726	6
55	9.807314	9.884783	9.922530	10.077469	5
56	9.807464	9.884677	9.922787	10.077213	4
57	9.807615	9.884572	9.923044	19.076956	3
58	9.897766	9.884466	9.923300	10.076699	2
59	9.807917	9.884360	9.923557	10.076443	1
60	9.808067	9.884254	9.923813	10.076186	0
		Sine		Tangent	M

Degree 50.

Degree 40.

M	Sine		Tangent		
0	9.808067	9.8842 54	9.923813	10.076186	60
1	9.808218	9.884148	9.924070	10.075930	59
2	9.808368	9.884042	9.924327	10.075673	58
3	9.808519	9.883936	9.924583	10.075417	57
4	9.808669	9.883829	9.924839	10.075160	56
5	9.808819	9.883723	9.925096	10.074904	55
6	9.808969	9.883617	9.925352	10.074647	54
7	9.809119	9.883510	9.925609	10.074391	53
8	9.809269	9.883404	9.925865	10.074135	52
9	9.809419	9.883297	9.926121	10.073878	51
10	9.809569	9.883191	9.926378	10.073622	50
11	9.809718	9.883084	9.926634	10.073366	49
12	9.809868	9.882977	9.926890	10.073110	48
13	9.810017	9.882871	9.927147	10.072853	47
14	9.810166	9.882764	9.927403	10.072597	46
15	9.810316	9.882657	9.927659	10.072341	45
16	9.810465	9.882550	9.927915	10.072085	44
17	9.810614	9.882443	9.928171	10.071829	43
18	9.810763	9.882336	9.928427	10.071573	42
19	9.810912	9.882228	9.928683	10.071317	41
20	9.811061	9.882121	9.928940	10.071060	40
21	9.811210	9.882014	9.929196	10.070804	39
22	9.811358	9.881907	9.929452	10.070548	38
23	9.811506	9.881799	0.929708	10.070292	37
24	9.811655	9.881692	9.929964	10.070036	36
25	9.811804	9.881584	9.930219	10.069781	35
26	9.811952	9.881477	9.930475	10.069525	34
27	9.812100	9.881369	9.930731	10.069269	33
28	9.812248	9.881261	9.930987	10.069013	32
29	9.812396	9.881153	9.931243	10.068757	31
30	9.812544	9.881045	9.931499	10.068501	30
		Sine		Tangent	M

Degree 55.

Degree 40.

M	Sine		Tangent		
30	9.812544	9.881045	9.931499	10.068501	30
31	9.812692	9.880937	9.931755	10.068245	29
32	9.812840	9.880829	9.932010	10.067989	28
33	9.812988	9.880721	9.932266	10.067734	27
34	9.813135	9.880613	9.932522	10.067478	26
35	9.813283	9.880505	9.932778	10.067222	25
36	9.813430	9.880397	9.933033	10.066567	24
37	9.813578	9.880289	9.933289	10.066711	23
38	9.813725	9.880180	9.933545	10.066455	22
39	9.813872	9.880072	9.933800	10.066200	21
40	9.814019	9.879963	9.934056	10.065944	20
41	9.814166	9.879855	9.934311	10.065688	19
42	9.814313	9.879746	9.934567	10.065433	18
43	9.814460	9.879637	9.934822	10.065177	17
44	9.814607	9.879529	9.935078	10.064922	16
45	9.814753	9.879420	9.935333	10.064666	15
46	9.814900	9.879311	9.935589	10.064411	14
47	9.815046	9.879202	9.935844	10.064156	13
48	9.815193	9.879093	9.936100	10.063900	12
49	9.815329	9.878984	9.936355	10.063645	11
50	9.815485	9.878875	9.936610	10.063389	10
51	9.815631	9.878766	9.936866	10.063134	9
52	9.815777	9.878656	9.937121	10.062879	8
53	9.815923	9.878547	9.937376	10.062623	7
54	9.816069	9.878438	9.937632	10.062368	6
55	9.816215	9.878328	9.937887	10.062112	5
56	9.816361	9.878219	9.938142	10.061858	4
57	9.816506	9.878109	9.938397	10.061602	3
58	9.816652	9.877999	9.938653	10.061347	2
59	9.816797	9.877890	9.938903	10.061092	1
60	9.816943	9.877780	9.939163	10.060837	0
		Sine		Tangent	M

Degree 49.

Degree 41.

M	Sine		Tangent		M
0	9.816943	9.877780	9.939163	10.060837	60
1	9.817088	9.877670	9.939418	10.060582	59
2	9.817233	9.877560	9.939673	10.060327	58
3	9.817378	9.877450	9.939928	10.060072	57
4	9.817522	9.877340	9.940182	10.059816	56
5	9.817668	9.877230	9.940438	10.059562	55
6	9.817813	9.877120	9.940693	10.059307	54
7	9.817958	9.877009	9.940948	10.059052	53
8	9.818103	9.876899	9.941203	10.058797	52
9	9.818247	9.876789	9.941458	10.058542	51
10	9.818392	9.876678	9.941713	10.058287	50
11	9.818536	9.876568	9.941968	10.058032	49
12	9.818681	9.876457	9.942223	10.057777	48
13	9.818825	9.876347	9.942478	10.057522	47
14	9.818969	9.876236	9.942733	10.057267	46
15	9.819113	9.876125	9.942988	10.057012	45
16	9.819257	9.876014	9.943243	10.056757	44
17	9.819401	9.875904	9.943498	10.056502	43
18	9.819545	9.875793	9.943752	10.056248	42
19	9.819689	9.875682	9.944007	10.055993	41
20	9.819832	9.875571	9.944262	10.055738	40
21	9.819976	9.875459	9.944517	10.055483	39
22	9.820119	9.875348	9.944771	10.055229	38
23	9.820263	9.875237	9.945026	10.054974	37
24	9.820406	9.875125	9.945281	10.054719	36
25	9.820549	9.875014	9.945535	10.054464	35
26	9.820693	9.874903	9.945790	10.054210	34
27	9.820836	9.874791	9.946045	10.053955	33
28	9.820979	9.874679	9.946299	10.053701	32
29	9.821122	9.874568	9.946554	10.053446	31
30	9.821264	9.874456	9.946808	10.053192	30
		Sine		Tangent	M

Degree 48.

Degree 41.

M	Sine		Tangent		
30	9.821264	9.874456	9.946308	10.053102	30
31	9.821407	9.874344	9.947063	10.052937	29
32	9.821550	9.874232	9.947317	10.052682	28
33	9.821692	9.874120	9.947572	10.052428	27
34	9.821835	9.874008	9.947826	10.052173	26
35	9.821977	9.873896	9.948081	10.051919	25
36	9.822120	9.873784	9.948335	10.051664	24
37	9.822262	9.873672	9.948590	10.051410	23
38	9.822404	9.873560	9.948844	10.051156	22
39	9.822546	9.873447	9.949099	10.050901	21
40	9.822688	9.873335	9.949353	10.050647	20
41	9.822830	9.873223	9.949607	10.050393	19
42	9.822972	9.873110	9.949862	10.050138	18
43	9.823114	9.872998	9.950116	10.049884	17
44	9.823255	9.872885	9.950370	10.049630	16
45	9.823397	9.872772	9.950625	10.049375	15
46	9.823538	9.872659	9.950879	10.049121	14
47	9.823680	9.872546	9.951133	10.048867	13
48	9.823821	9.872434	9.951388	10.048612	12
49	9.823962	9.872321	9.951642	10.048358	11
50	9.824104	9.872208	9.951896	10.048104	10
51	9.824245	9.872094	9.952150	10.047850	9
52	9.824386	9.871981	9.952404	10.047575	8
53	9.824527	9.871868	9.952659	10.047341	7
54	9.824667	9.871755	9.952913	10.047087	6
55	9.824808	9.871641	9.953167	10.046833	5
56	9.824949	9.871528	9.953421	10.046579	4
57	9.825090	9.871414	9.953675	10.046325	3
58	9.825230	9.871301	9.953929	10.046071	2
59	9.825370	9.871187	9.954183	10.045817	1
60	9.825511	9.871073	9.954437	10.045563	0
		Sine		Tangent	M

Degree 48.

Degree 42.

M	Sine		Tangent		
0	9.825511	9.871073	9.954437	10.045562	60
1	9.825651	9.870960	9.954691	10.045308	59
2	9.825791	9.870846	9.954945	10.045054	58
3	9.825931	9.870732	9.955199	10.044800	57
4	9.826071	9.870618	9.955453	10.044546	56
5	9.826211	9.870504	9.955707	10.044292	55
6	9.826351	9.870390	9.955561	10.043038	54
7	9.826491	9.870275	9.956215	10.043784	53
8	9.826631	9.870161	9.956469	10.043531	52
9	9.826770	9.870047	9.956723	10.043276	51
10	9.826910	9.869933	9.956977	10.043023	50
11	9.827049	9.869818	9.957231	10.042769	49
12	9.827189	9.869704	9.957485	10.042515	48
13	9.827328	9.869589	9.957739	10.042261	47
14	9.827467	9.869474	9.957993	10.042007	46
15	9.827606	9.899360	9.958246	10.041753	45
16	9.827745	9.869245	9.958500	10.041500	44
17	9.827884	9.869130	9.958754	10.041246	43
18	9.828023	9.869015	9.959008	10.040992	42
19	9.828162	9.868900	9.959262	10.040738	41
20	9.828301	9.868785	9.959515	10.040485	40
21	9.828439	9.868670	9.959769	10.040231	39
22	9.828578	9.868555	9.960023	10.039977	38
23	9.828716	9.868439	9.960277	10.039723	37
24	9.828855	9.868324	9.960530	10.039469	36
25	9.828993	9.868209	9.960784	10.039216	35
26	9.829131	9.868093	9.961038	10.038962	34
27	9.829269	9.867978	9.961291	10.038608	33
28	9.829406	9.867862	9.961545	10.038451	32
29	9.829545	9.867747	9.961799	10.038201	31
30	9.829683	9.867631	9.962052	10.037947	30
		Sine		Tangent	M

Degree 47.

Degree 42.

M	Sine		Tangent		
30	9.829683	9.867631	9.962052	10.037977	30
31	9.829821	9.867515	9.962306	10.037694	29
32	9.829959	9.867399	9.962560	10.037440	28
33	9.830096	9.867283	9.962813	10.037187	27
34	9.830234	9.867167	9.963067	10.036933	26
35	9.830372	9.867051	9.963320	10.036680	25
36	9.830509	9.866935	9.963574	10.036426	24
37	9.830646	9.866819	9.963827	10.036173	23
38	9.830784	9.866702	9.964081	10.035919	22
39	9.830921	9.866586	9.964335	10.035665	21
40	9.831058	9.866470	9.964588	10.035412	20
41	9.831195	9.866353	9.964842	10.035158	19
42	9.831332	9.866237	9.965095	10.034905	18
43	9.831469	9.866120	9.965348	10.034652	17
44	9.831606	9.866004	9.965602	10.034398	16
45	9.831742	9.865887	9.965855	10.034144	15
46	9.831879	9.865770	9.966109	10.033891	14
47	9.832015	9.865653	9.966362	10.033638	13
48	9.832152	9.865536	9.966616	10.033384	12
49	9.832288	9.865419	9.966869	10.033131	11
50	9.832425	9.865302	9.967122	10.032878	10
51	9.832561	9.865185	9.967376	10.032624	9
52	9.832697	9.865068	9.967629	10.032371	8
53	9.832833	9.864950	9.967883	10.032117	7
54	9.832969	9.864832	9.968136	10.031864	6
55	9.833105	9.864716	9.968389	10.031611	5
56	9.833241	9.864598	9.968643	10.031357	4
57	9.833376	9.864480	9.968896	10.031104	3
58	9.833512	9.864363	9.969149	10.030851	2
59	9.833648	9.864245	9.969403	10.030597	1
60	9.833783	9.864127	9.969656	10.030344	0
		Sine		Tangent	M

Degree 47.

Degree 43.

M	Sine		Tangent		
0	9.833785	9.864127	9.969656	10.030344	60
1	9.833919	9.864010	9.969909	10.030091	59
2	9.834054	9.863892	9.970162	10.029838	58
3	9.834189	9.863774	9.970416	10.029584	57
4	9.834324	9.863656	9.970669	10.029331	56
5	9.834460	9.863537	9.970922	10.029078	55
6	9.834595	9.863419	9.971175	10.028827	54
7	9.834730	9.863301	9.971428	10.028571	53
8	9.834865	9.863183	9.971682	10.028318	52
9	9.834999	9.863064	9.971935	10.028065	51
10	9.835134	9.862946	9.972188	10.027812	50
11	9.835269	9.862827	9.972441	10.027559	49
12	9.835503	9.862709	9.972694	10.027306	48
13	9.835538	9.862590	9.972948	10.027052	47
14	9.835672	9.862471	9.973201	10.026799	46
15	9.835806	9.862353	9.973454	10.026546	45
16	9.835941	9.862234	9.973707	10.026293	44
17	9.836075	9.862115	9.973960	10.026040	43
18	9.836209	9.861996	9.974212	10.025787	42
19	9.836343	9.861877	9.974466	10.025533	41
20	9.836477	9.861757	9.974719	10.025280	40
21	9.836611	9.861638	9.974973	10.025027	39
22	9.836745	9.861519	9.975229	10.024774	38
23	9.836878	9.861399	9.975479	10.024521	37
24	9.837012	9.861280	9.975732	10.024268	36
25	9.837146	9.861161	9.975985	10.024015	35
26	9.837279	9.861041	9.976238	10.023762	34
27	9.837412	9.860921	9.976491	10.023509	33
28	9.837546	9.860802	9.976744	10.023256	32
29	9.837679	9.860682	9.976997	10.023003	31
30	9.837812	9.860562	9.977250	10.022750	30
		Sine		Tangent	M

Degree 46.

Degree 43.

M	Sine		Tangent		M
30	9.837812	9.860562	9.977250	10.022750	30
31	9.837945	9.860442	9.977503	10.022497	29
32	9.838078	9.860322	9.977756	10.022244	28
33	9.838211	9.860202	9.978009	10.021991	27
34	9.838344	9.860082	9.978262	10.021738	26
35	9.838477	9.859962	9.978515	10.021485	25
36	9.838609	9.859842	9.978768	10.021232	24
37	9.838742	9.859721	9.979021	10.020979	23
38	9.838875	9.859601	9.979274	10.020726	22
39	9.839007	9.859480	9.979527	10.020473	21
40	9.839140	9.859360	9.979780	10.020220	20
41	9.839272	9.859239	9.980033	10.019967	19
42	9.839404	9.859118	9.980285	10.019714	18
43	9.839536	9.858998	9.980538	10.019461	17
44	9.839668	9.858877	9.980791	10.019209	16
45	9.839800	9.858756	9.981044	10.018956	15
46	9.839932	9.858639	9.981297	10.018703	14
47	9.840064	9.858514	9.981550	10.018450	13
48	9.840196	9.858393	9.981803	10.018197	12
49	9.840428	9.858272	9.982056	10.017944	11
50	9.840459	9.858150	9.982309	10.017691	10
51	9.840591	9.858029	9.982562	10.017438	9
52	9.840722	9.857908	9.982814	10.017185	8
53	9.840854	9.857786	9.983067	10.016933	7
54	9.840985	9.857665	9.983320	10.016683	6
55	9.841116	9.857543	9.983573	10.016427	5
56	9.841247	9.857421	9.983826	10.016174	4
57	9.841378	9.857300	9.984079	10.015921	3
58	9.841509	9.857178	9.984331	10.015668	2
59	9.841640	9.857056	9.984584	10.015416	1
60	9.841771	9.856934	9.984837	10.015163	0
		Sine		Tangent	M

Degree 46.

Degree 44.

M	Sine		Tangent		
0	9.841771	9.856934	9.984837	10.015162	60
1	9.841902	9.856812	9.985090	10.014910	59
2	9.842033	9.856690	9.985343	10.014657	58
3	9.842163	9.856568	9.985596	10.014404	57
4	9.842294	9.856445	9.985848	10.014151	56
5	9.842424	9.856323	9.986101	10.013899	55
6	9.842555	9.856201	9.986354	10.013646	54
7	9.842685	9.856078	9.986607	10.013393	53
8	9.842815	9.855956	9.986859	10.013140	52
9	9.842945	9.855833	9.987112	10.012888	51
10	9.843076	9.855710	9.987365	10.012635	50
11	9.843206	9.855588	9.987618	10.012382	49
12	9.843336	9.855465	9.987871	10.012129	48
13	9.843465	9.855342	9.988123	10.011877	47
14	9.843595	9.855219	9.988376	10.011624	46
15	9.843725	9.855096	9.988629	10.011371	45
16	9.843855	9.854973	9.988882	10.011118	44
17	9.843984	9.854850	9.989134	10.010866	43
18	9.844114	9.854727	9.989387	10.010613	42
19	9.844243	9.854603	9.989640	10.010360	41
20	9.844372	9.854480	9.989893	10.010107	40
21	9.844502	9.854356	9.990145	10.009855	39
22	9.844631	9.854233	9.990398	10.009602	38
23	9.844760	9.854109	9.990651	10.009349	37
24	9.844889	9.853986	9.990903	10.009096	36
25	9.745018	9.853862	9.991156	10.008844	35
26	9.845147	9.853738	9.991409	10.008591	34
27	9.845276	9.853614	9.991669	10.008338	33
28	9.845404	9.853490	9.991914	10.008086	32
29	9.845533	9.853366	9.992167	10.007833	31
30	9.845662	9.853242	9.992420	10.007580	30
		Sine		Tangent	M

Degree 45.

Degree 44.

M	Sine		Tangent		
30	9.845662	9.853242	9.992420	10.007580	30
31	9.845790	9.853118	9.992672	10.007328	29
32	9.845919	9.852994	9.992925	10.007075	28
33	9.846047	9.852869	9.993178	10.006822	27
34	9.846175	9.852745	9.993430	10.006569	26
35	9.846304	9.852620	9.993683	10.006317	25
36	9.846432	9.852496	9.993936	10.006064	24
37	9.846560	9.852371	9.994189	10.005811	23
38	9.846688	9.852246	9.994441	10.005559	22
39	9.846816	9.852122	9.994694	10.005306	21
40	9.846944	9.851997	9.994947	10.005053	20
41	9.847071	9.851872	9.995199	10.004801	19
42	9.847199	9.851747	9.995452	10.004548	18
43	9.847327	9.851622	9.995701	10.004295	17
44	9.847454	9.851497	9.995957	10.004043	16
45	9.847582	9.851372	9.996210	10.003790	15
46	9.847709	9.851246	9.996463	10.003537	14
47	9.847836	9.851121	9.996715	10.003285	13
48	9.847964	9.850996	9.996968	10.003032	12
49	9.848091	9.850870	9.997220	10.002779	11
50	9.848218	9.850745	9.997473	10.002527	10
51	9.848345	9.850619	9.997726	10.002274	9
52	9.848472	9.850493	9.997979	10.002021	8
53	9.848599	9.850367	9.998231	10.001769	7
54	9.848726	9.850242	9.998484	10.001516	6
55	9.848852	9.850116	9.998737	10.001263	5
56	9.848979	9.849990	9.998989	10.001011	4
57	9.849106	9.849864	9.999242	10.000758	3
58	9.849232	9.849737	9.999495	10.000505	2
59	9.849359	9.849611	9.999747	10.000253	1
60	9.849485	9.849485	10.000000	10.000000	0
		Sine		Tangent	M

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